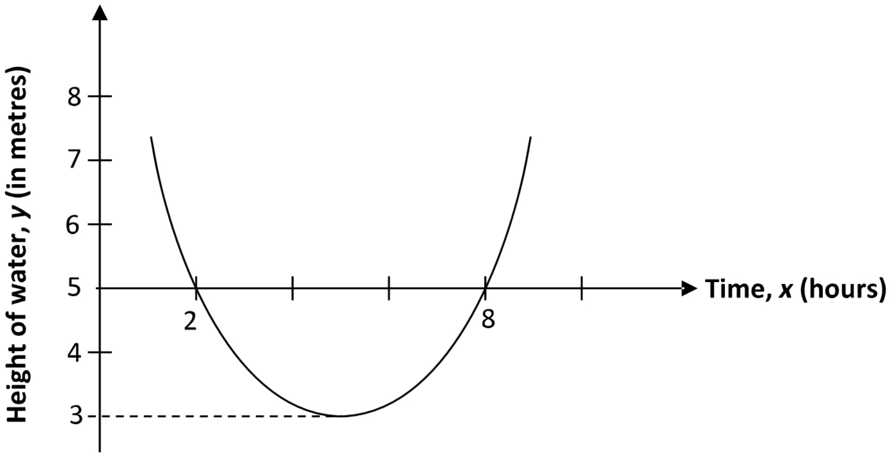
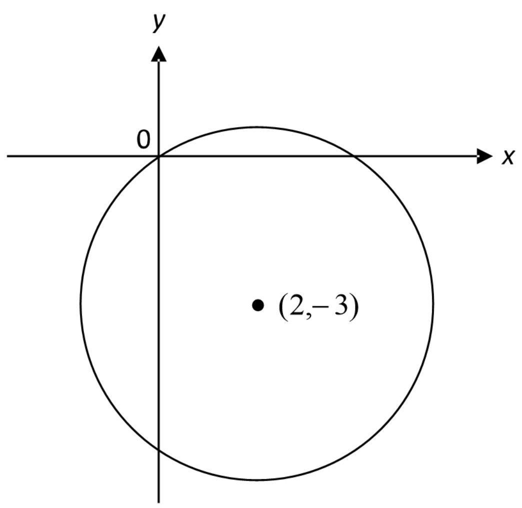
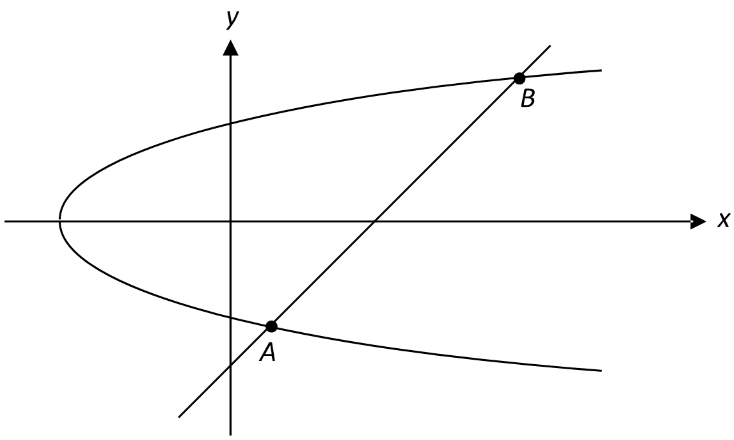
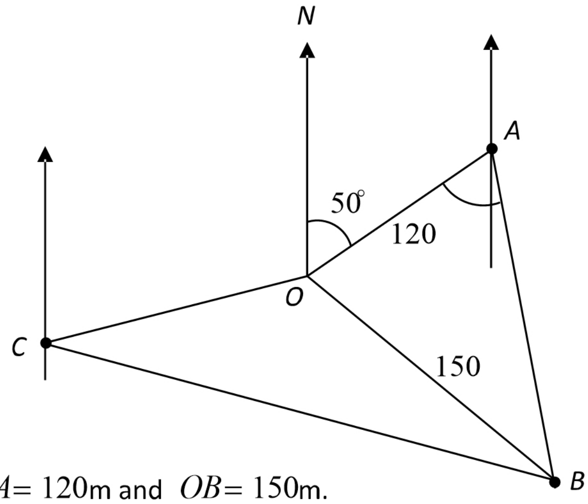
Topic: Coordinate geometry

Question 1 [2008 EOY, Q9]	marks
The graph below shows part of a quadratic curve which represents the water level, *y* metres, of a river over a period of time in hours during the operation of a water dam nearby. At 2 pm and 8 pm, the water level was at the recommended level of 5 metres. Let *x* represents the time in hours since 12pm.  (i) At what time did the water level reaches its minimum?	[1]
(ii) Find the equation of the curve, expressing it in the form $y = a(x - h)^2 + k$ where *a*, *h* and *k* are real numbers.	[4]

(iii) Hence, or otherwise, determine the height of the water level at 12 pm.	[2]
Question 2 [2008 EOY, Q10]	marks
Find the range of values of *k* if the line $2y = x + k$ intersects the curve $y = 2x + \frac{6}{x}$ more than once.	[5]
Question 3 [2008 EOY, Q12]	marks
The diagram below shows a circle with radius *r* units and its centre is $(2, -3)$.  (i) Given that the circle passes through the origin, find the exact value of *r*.	[2]

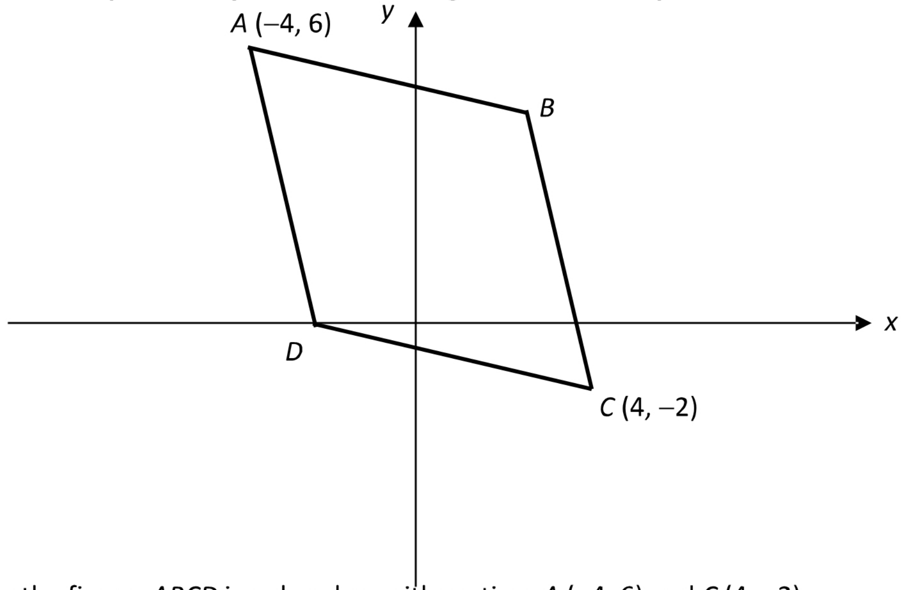
(ii) The circle is reflected about the *y*-axis and then moved 5 units downwards. Write down the coordinates of the new centre of the circle.	[1]
(iii) Find the equation of the new circle in the form $x^2+y^2+2gx+2fy+c=0$ where *g*, *f* and *c* are real constants.	[3]
Question 4 [2008 EOY, Q15]	marks
The figure below shows the graphs of $2y^2=x+7$ and $y=x-3$.  The two curves meet at *A* and *B*. (i) Show that the coordinates of *A* are $(1,-2)$ and find the coordinates of *B*.	[3]

(ii) Find the length of *AB*.	[2]
(iii) Find the equation of the perpendicular bisector of *AB*.	[3]

Question 5 [2008 EOY, Q17]	marks
The diagram below shows the position of three locations *A*, *B* and *C* with respect to a reference point *O*. The bearings of *A*, *B* and *C* from *O* are 050°, 135° and 240° respectively.  It is also given that $OA = 120$m and $OB = 150$m. (i) Find the bearing of *B* from *A*.	[3]
(ii) If the bearing of *B* from *C* is 100°, calculate the length of *OC*.	[3]

(iii) A mast stands vertically at *B* such that the angle of elevation of the top of the mast from *O* is 62°. Find the height of the mast.	[2]
Question 6 [2009 EOY, Q1]	marks
Find the coordinates of the points at which the straight line $y = -x - 5$ intersects the curve $x^2 + (y+2)^2 = 5$.	[4]
Question 7 [2009 EOY, Q9]	marks
The equation of a circle *A* is $x^2 + y^2 + 6x - 4y + 4 = 0$. Find the coordinates of the centre of circle *A* and state its radius.	[3]

Question 8 [2009 EOY, Q11]	marks
Solutions to this question by accurate drawing will not be accepted.	



In the figure, *ABCD* is a rhombus with vertices *A* (–4, 6) and *C* (4, –2). [3]

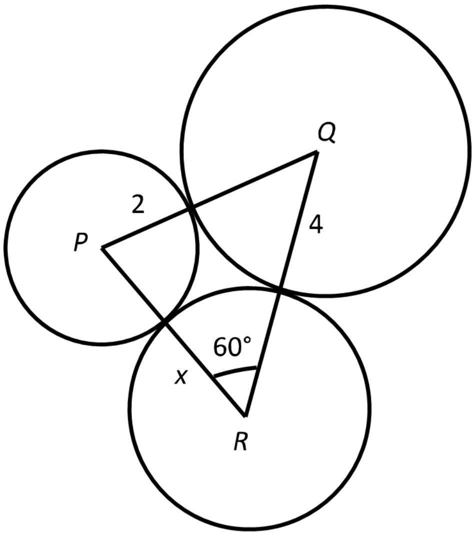
Given that the vertex *B* lies on the line $3x+2y=14$, calculate the

(i) equations of *AC* and *BD*,

.

(ii) coordinates of *B* and *D*,	[3]
(iii) equation of *AB*,	[2]

(iv) area of the rhombus *ABCD* and hence find the perpendicular distance from *B* to the line *CD*.	[4]

Question 9 [2009 EOY, Q14]	marks
The diagram below shows three circular fields with centres *P*, *Q* and *R* tangential to one another. The radii of the three fields are 2 m, 4 m and *x* m respectively and $\angle PRQ = 60^\circ$.  (i) Form an equation in *x* and show that it reduces to $x^2 + 6x - 24 = 0$.	[3]

(ii) Solve the equation, giving your answer in surd form and find the exact area of triangle *PQR*.	[4]
(ii) A pole of y m stands at P. A boy walks from R to Q. Given that the maximum angle of elevation of the top of the pole during the walk is 60°, find y.	[3]

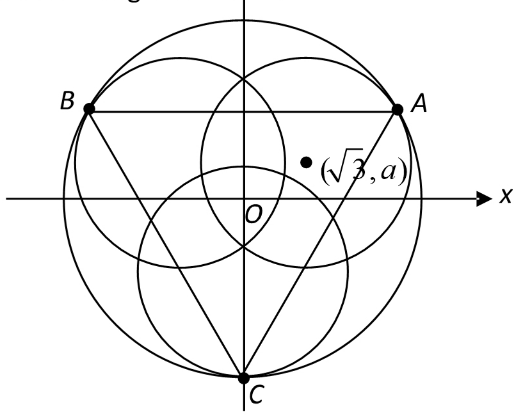
Question 10 [2010 EOY, Q4]	marks
Find the range of values of *k* such that the curve $y+2x^2=k(x-1)$ meets the line $2y=6kx+2k^2-3$ at two distinct points.	[5]

Question 11 [2010 EOY, Q5]	marks
A quadratic curve is such that it is symmetrical about the line $x=2k$, where $k>1$. Given that it cuts the *x*-axis when $x=2$, and the maximum value of the curve is $k-1$,	
(i) explain why the other *x*-intercept is $4k-2$, and	[1]
(ii) find, in terms of *x*, *y* and *k*, the equation of the curve	[3]

(cont'd) Given further that the distance between the two x-intercepts is 6 units, find the coordinates of the y-intercept.	[3]
Question 12 [2010 EOY, Q6]	marks
On separate axes, sketch the graphs of (i) $y = a(x-h)^2 + k$ for $a < 0$, $h < 0$ and $k > 0$	[2]
(ii) $y = k(x-\alpha)(x-\beta)$ for $\alpha\beta < 0$, $\lvert\alpha\rvert = \lvert\beta\rvert$ and $k > 0$	[2]

Question 13 [2010 EOY, Q7]	marks

The diagram below shows three small identical circles inscribed in a big circle centred at the origin. Each small circle has radius *r* units (where $r > 2$) and one of the small circles has its centre at the point $(\sqrt{3}ya)$. The small circles are arranged with a rotational symmetry about the origin of order 3.



(i)

By considering the rotational symmetry of the diagram, explain fully why $a = 1$.

[2]

(ii)

Write down the equation of the bigger circle, in terms of *x*, *y* and *r*. [2]

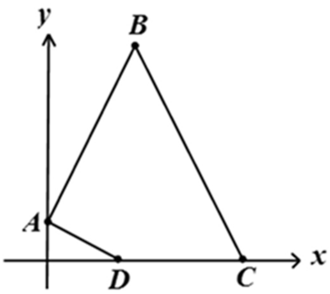
Given further that $r = 3$, (iii) find the coordinates of *A*, the point of contact between the smaller circle and the bigger circle as shown in the diagram, and	[3]
(iv) show that the area of the triangle *ABC* is $\frac{75\sqrt{3}}{4}$ units^2.	[3]

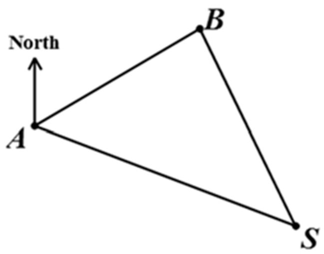
Question 14 [2010 EOY, Q9]	marks
Three points *A*, *B* and *C* on level ground are such that *A* is due north of *B*, $BC = 20$ km, the bearing of *C* from *B* is $042°$ and $AC = 16$ km. (i) Find the possible distances of *A* from *B*.	[5]
(ii) Taking the shorter distance between *A* and *B*, determine the bearing of *A* from *C*.	[2]

Question 15 [2011 EOY, Q6]	marks
The equation of a circle is given by $x^2 + y^2 = 6x - 8y$. (i) Find the centre and the radius of the circle.	[4]
(ii) Hence or otherwise, find the greatest possible value of y.	[2]

Question 16 [2011 EOY, Q7]	marks
The variables x and y are related by the equation $\sqrt{2^y} = ay + \frac{b}{\sqrt{x}}$. A straight line passes through (5, 7) is obtained when $\sqrt{2^y x}$ is plotted against $y\sqrt{x}$. (i) Given that the gradient of this line is 2, calculate the value of a and of b.	[3]
(ii) Given that this line also passes through (k, 13), find the value of k.	[2]

Question 17 [2011 EOY, Q10a]	marks
A quadratic curve has a minimum point at $(1,\ 5)$ and passes through $(0,\ 7)$. If the curve does not meet the line $2(y-mx)=13$, find the range of values of *m*.	[5]

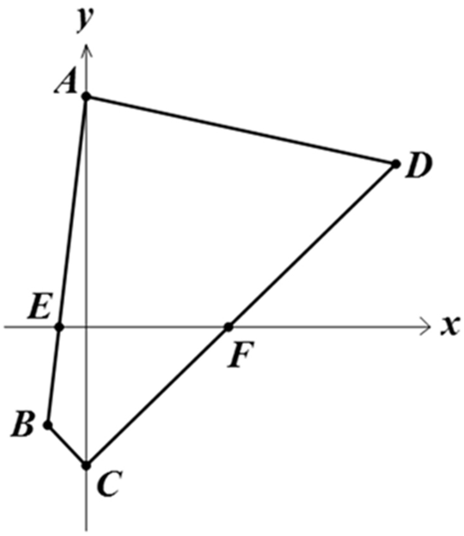
Question 18 [2011 EOY, Q14]	marks
Solutions to this question by accurate drawing will not be accepted. The diagram shows the quadrilateral *ABCD*. The coordinates of *A* and *B* are $(0,1)$ and $(3,7)$ respectively. Points *C* and *D* lie on the *x*-axis and $\angle BAD = 90°$.  (i) Find the equation of the perpendicular bisector of line *AB*.	[3]

(ii) Find the equation of *AD* and hence find the coordinates of *D*.	[2]
(iii) If the area of ΔBCD is 14 square units, find the coordinates of *C*	[1]
(iv) Find all the possible coordinates of *E* for which *A*, *B*, *C* and *E* are vertices of a parallelogram.	[3]
Question 19 [2011 EOY, Q15]	marks
In the diagram *A*, *B* and *S* represent three points in the sea. *B* is 3 km from *A* on a bearing of 060°. The bearings of *S* from *A* and *B* are 110° and 155° respectively.  (i) Calculate the distance *AS*.	[3]

(ii) A ship at *S* sails directly to *A* at a steady speed of 2 kmh^{-1}. After 40 minutes, the ship is at *T,* a point on *AS.* (a) Calculate the distance *BT*.	[3]
(b) Find the bearing of *B* from *T*.	[3]
(iii) A hot air balloon, *H,* is lowering at a point 600 m vertically above ***B***. Calculate the greatest angle of elevation of ***H*** from an observer on the ship as the ship sails from ***S*** to ***A***.	[2]

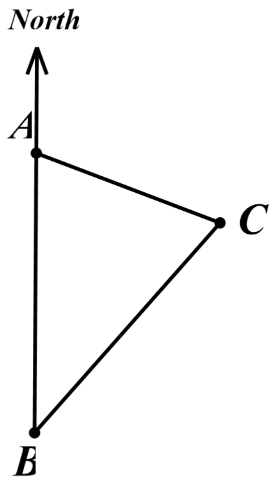
Question 20 [2012 EOY, Q5]	marks
A set of points with coordinates (x, y) lies on the graph whose equation is $\frac{x^2+2x}{4}+\left(\frac{y}{2}-1\right)^2=2$. The points are *d* units from a fixed point *P*. Find the coordinates of *P* and the value of *d*.	[4]

Question 21 [2012 EOY, Q10 c]	marks
(c) Find the coordinates of the points of intersection of the line $10x+5y+4=0$ and the curve $4x^2+y^2=16$.	[5]

Question 22 [2012 EOY Q11]	marks
Solutions to this question by accurate drawing will not be accepted. The diagram shows a quadrilateral *ABCD* with $A(0,6)$, $B(-1,-3)$, $C(0,-4)$, and $D(9,\ 5)$. *AB* and *CD* intersect the *x*-axis at points $E\left(-\frac{2}{3},0\right)$ and $F\ (4,0)$ respectively.  (i) Show that angle *BAD* = 90°.	 [2]

(ii) Find the equation of the perpendicular bisector of *AD*.	[3]
(iii)Given that *ABCD* is a cyclic quadrilateral, find the exact length of the diameter of the circle passing through *A*, *B*, *C* and *D*.	[2]
(iv)A point *P* is such that *BCDP* is a rectangle, find the coordinates of *P*.	[2]
(v)Find the area of the quadrilateral *AEFD*.	[2]

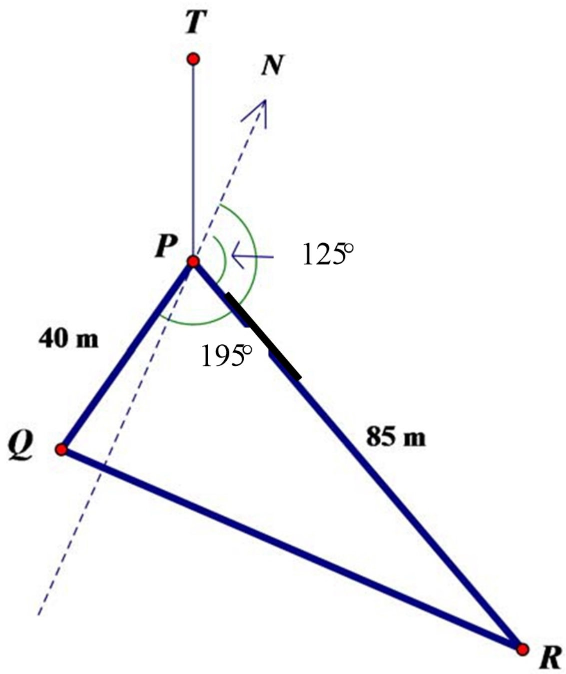
Question 23 [2012 EOY Q13]	marks
ABC represents a horizontal triangular field such that *A* is due north of *B*, *AB* = 240 m, *AC* = 180m and the bearing of *C* from *A* is 110°.	



(i) Find the distance *BC*.	[3]
(ii) Find the bearing of *C* from *B*.	[3]

(iii) A vertical tree, of height 25 m, is planted at *C*. A farmer walks along a footpath from *A* to *B*. Find the maximum angle of elevation of the top of the tree from the farmer during this walk. You may ignore the height of the farmer.	[4]
Question 24 [2013 EOY, Q12]	marks

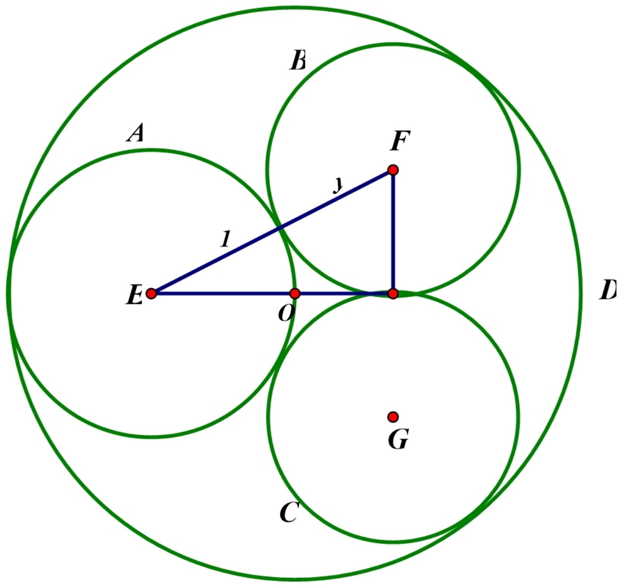
12 In the diagram, *PQR* represents a horizontal triangular field such that *PR* = 85 m and *PQ* = 40 m. The bearing of *R* from *P* and the bearing of *Q* from *P* is $125°$ and $195°$ respectively.

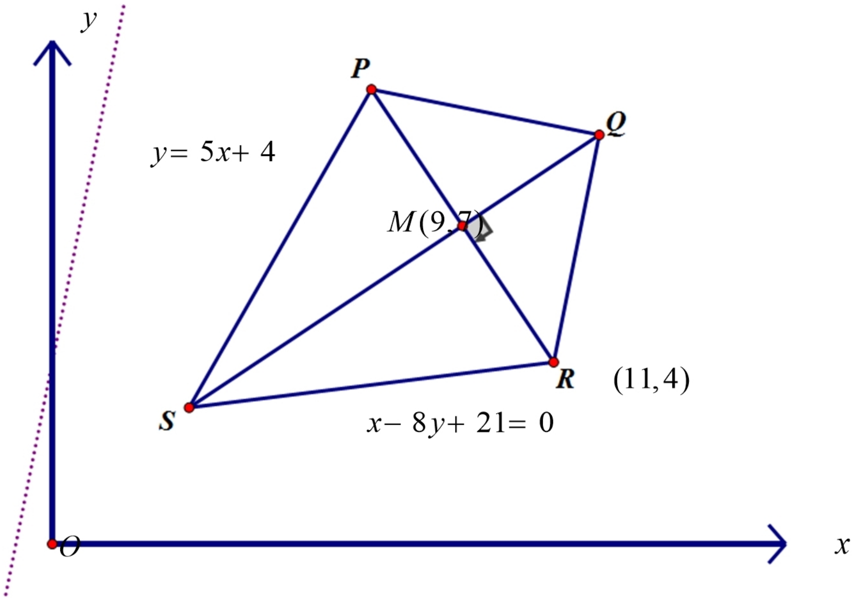


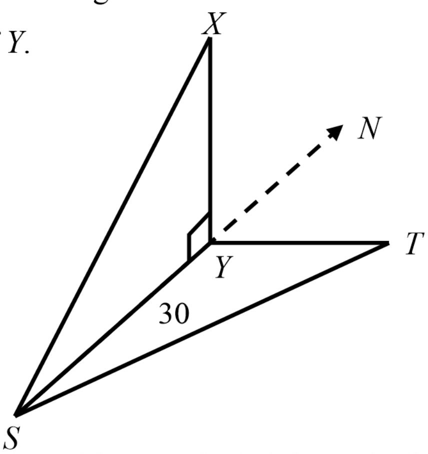
(i) Calculate the distance from *Q* to *R*. [2]

(ii) Calculate the area of the triangular field *PQR*.	[2]
(iii) Calculate the shortest distance from *P* to the road *QR*.	[2]
TP represents a building at the corner *P*. Given that the angle of depression of the top of the building *T* to the corner *R* is $20°$, (iv) calculate the height of the building *TP*.	[2]
A man walks along the road from *Q* to *R*, (v) calculate the greatest angle of elevation of the top of the building.	[2]

Question 25 [2013 EOY, Q13]	marks
(i) The equation of a circle is given by $x^2 + y^2 - 10x + 2y + 1 = 0$. Find the centre and the radius of the circle.	[2]
(ii) The tangent to the circle at a point *A* is parallel to the straight line $x - y + 2 = 0$. Find the equation of the diameter of the circle through *A*.	[2]
(iii) The circle is then reflected on the line $y = x$. State the equation of the new circle formed.	[1]

Question 26 [2013 EOY, Q14]	marks
Circles A, B and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. E, F and G are the centres of circles A, B and C respectively. Circle A has radius 1 and passes through O, the centre of circle D. Both circles B and C have radii y. By considering <u>two right-angled triangles</u>, find the radius y of the circle B. 	[6]

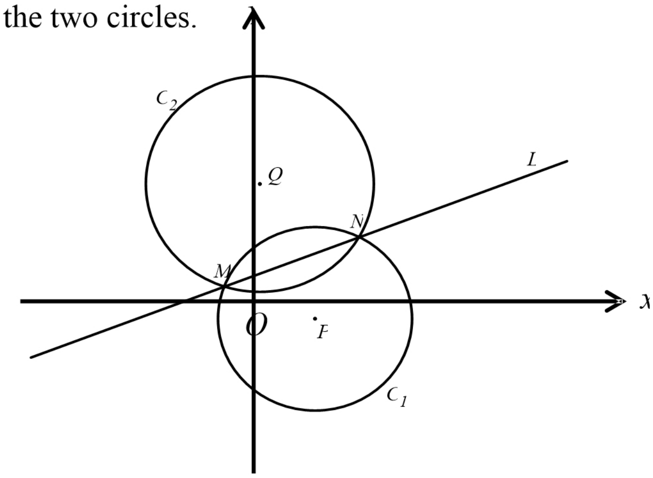
Question 27 [2013 EOY, Q16]	marks
Solutions to this question by accurate drawing will not be accepted The diagram shows a quadrilateral *PQRS* in which *QS* is the perpendicular bisector of *PR*. The mid-point of *PR* is *M*. The coordinates of *R* and *M* are (11, 4) and (9, 7) respectively. *RQ* is parallel to the line $y = 5x + 4$ and the equation of *SR* is $x - 8y + 21 = 0$.  Find (i) the coordinates of *P*,	[2]
(ii) the equations of *QR* and *QS*,	[4]

(iii) the coordinates of Q and of S,	[3]
(iv) the area of $PQRS$.	[2]
Question 28 [2014 EOY, Q14]	marks
1 Three points, S, Y and T are on level ground. Y is the foot of a vertical flagpole XY, and S is 30 m due south of Y.  (i) The angle of elevation of X from S is $25°$. Calculate the height of the flagpole.	[2]

(ii) The bearing of T from S and the bearing of T from Y are $042°$ and $075°$ respectively, calculate the distance ST.	[3]
(iii) A man is standing at the midpoint of ST. Calculate the angle of elevation of X from the man.	[5]

Question 29 [2014 EOY, Q16]	marks

The diagram shows two intersecting circles C_1 and C_2 with centres P and Q respectively. A straight line L passes through M and N, which are the points of intersections of the two circles.



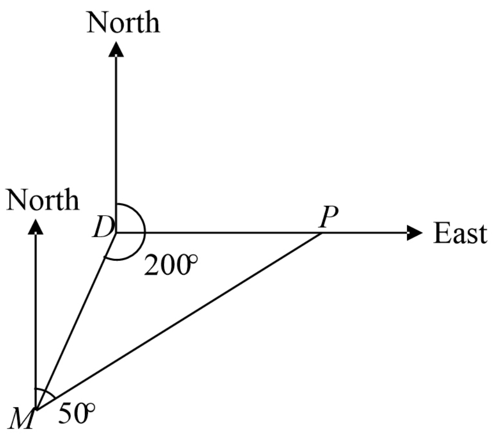
(i)

Given that the equation of the circle C_1 is $x^2 + y^2 - 6x + 4y - 21 = 0$, find the coordinates of P and the radius of C_1. [3]

(ii) It is also given that the equation of the line L is $4y - x = 6$, find the coordinates of M and of N.	[4]
(iii) Find the equation of the perpendicular bisector of MN.	[3]

(iv) Given that the radius of circle C_2 is $\frac{1}{2}\sqrt{221}$ units, find the coordinates of Q.	[4]
(v) Calculate the area of the quadrilateral *PMQN*.	[2]

Question 30 [2015 EOY, Q10]	marks
Two lines l_1 and l_2 are such that they are parallel, and l_2 is the perpendicular bisector of two points $A(-2,4)$ and $B(3,-6)$. (i) Find the equation of l_2.	[3]
(ii) Hence find the equation of l_1 if it passes through the point $C(5,8)$.	[2]
(iii) Find the distance between the two lines.	[4]

Question 31 [2015 EOY, Q11]	marks
A man, at point *M*, is 80 metres at a bearing of $200°$ from an observation deck, *D*, which stands 25 metres tall. He begins to walk to a point *P* which is due East of *D* at a bearing of $050°$.	
	
Ignoring the man's height, calculate	
(i)	
the angle of elevation of the top of the deck from the man at *M*,	[2]

(ii) the distance he walks when the angle of elevation of the top of the deck from his position is the greatest, and	[3]
(iii) the distance DP.	[2]