NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **MARKING SCHEME**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8			
2		9		Units	
3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5				Total:	
6					
7					90
Parent's Name & Signature: Date:					

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Show that $\dfrac{4}{\cot\theta - \tan\theta} = 2\tan 2\theta$. [4]

(a)	$\text{LHS} = \dfrac{4}{\cot\theta - \tan\theta}$ $= \dfrac{4}{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}}$ $= \dfrac{4}{\dfrac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}}$ $= \dfrac{4\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta} \div \dfrac{\cos^2\theta}{\cos^2\theta}$ $= \dfrac{4\tan\theta}{1-\tan^2\theta}$ $= 2\tan 2\theta$	M1 for using changing tan and cot M1 M1 A1

(b) Hence, solve the equation $\dfrac{4}{\cot 2x - \tan 2x} = 5$ for $0 < x < \pi$. [3]

(b)	$2\tan 4\theta = 5$ $\tan 4\theta = \dfrac{5}{2}$ basic angle $= 1.1903$ $4\theta = 1.1903,\ \pi + 1.1903,\ 2\pi + 1.1903,\ 3\pi + 1.1903$ $\theta = 0.298,\ 1.08,\ 1.87\ ,\ 2.65$	M1 M1 A1

2 The expression $4x^3 + ax + b$, where a and b are constants, has a factor of $x - 1$, and a remainder of -9 when divided by $x + 2$.

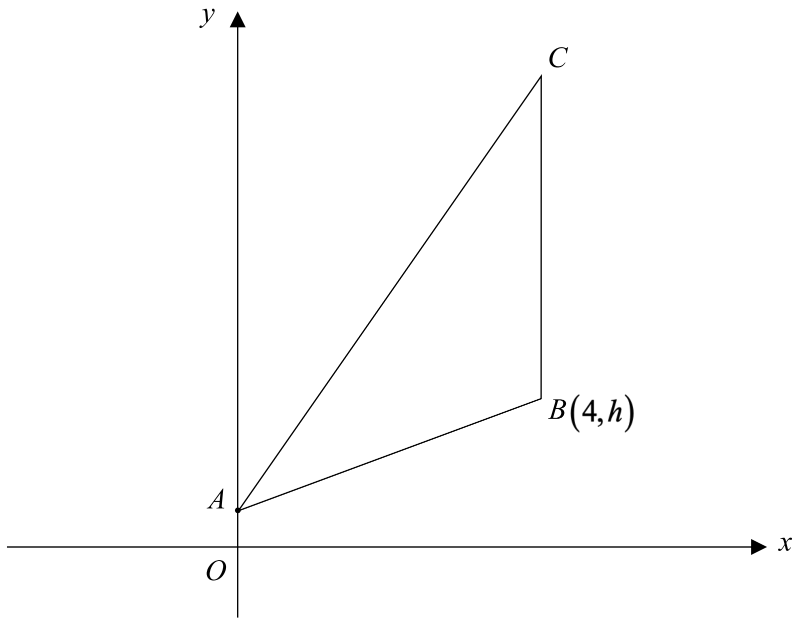
(a) Find the value of a and of b. [4]

Let $\mathrm{f}(x) = 4x^3 + ax + b$	
$\mathrm{f}(1) = 0$	
$4(1)^3 + a(1) + b = 0$	M1 for forming first equation
$4 + a + b = 0$	
$a + b = -4 - - - - - (1)$	
$\mathrm{f}(-2) = -9$	
$4(-2)^3 + a(-2) + b = -9$	M1 for forming second equation
$-32 - 2a + b = -9$	
$-2a + b = 23 - - - - - (2)$	
$(1) - (2)$:	
$3a = -27$	M1 for solving
$a = -9$	
Sub $a = -9$ into (1)	
$-9 + b = -4$	
$b = 5$	A1 for both answers
$\therefore a = -9$ and $b = 5$	

(b) Hence solve the equation $4x^3+ax+b=0$ expressing the non-integer roots in the form of $\dfrac{c+\sqrt{d}}{2}$, where c and d are constants. [4]

$\mathrm{f}(x)=4x^3-9x+5$ $\mathrm{f}(x)=(x-1)(4x^2+cx-5)$ By comparing coefficients of x, $-9=-5-c$ $c=4$ $\mathrm{f}(x)=(x-1)(4x^2+4x-5)$	M1 for getting quadratic factor
$4x^3-9x+5=0$ $(x-1)(4x^2+4x-5)=0$ $4x^2+4x-5=0$ or $x=1$ $x=\dfrac{-4\pm\sqrt{4^2-4(4)(-5)}}{2(4)}$	M1 for using quadratic formula to solve the quadratic equation
$x=\dfrac{-4\pm\sqrt{96}}{8}$ $x=\dfrac{-4\pm4\sqrt{6}}{8}$ $x=\dfrac{-1\pm\sqrt{6}}{2}$ or $x=1$	A2 for all answers

3 The diagram shows an isosceles triangle ABC, where $AB = BC$ and the line segment BC is parallel to the y-axis. Point A lies on the y-axis and the equation of line AC is $y = 2x + 3$. Point B is $(4, h)$, where h is a positive constant.



(a) Find the coordinates of A, B and C. [4]

(a)	Since A is on the y-axis, $A(0,3)$ Since BC is parallel to the y-axis, and x-coordinate of B is 4, x-coordinate of $C = 4$. $y = 2(4)+3$ $= 11$ $C(4,11)$ $\sqrt{4^2+(h-3)^2} = 11-h$ $16+(h-3)^2 = (11-h)^2$ $16+h^2-6h+9 = 121-22h+h^2$ $16h = 96$ $h = 6$ $B(4,6)$	B1 B1 M1: form equation using length formula A1

Given that the point D is such that $ABCD$ is a kite, and the line segment CD is parallel to the x-axis.

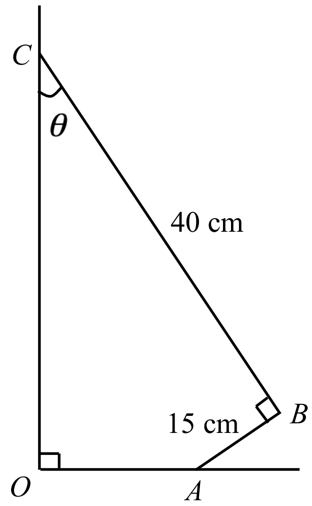
(b) Find the coordinates of D. [4]

(b)	Gradient of $BD = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $6 = -\frac{1}{2}(4) + c$ $c = 8$ Equation of BD: $y = -\frac{1}{2}x + 8$ Since CD is parallel to the x-axis, y-coordinate of $D = 11$. $11 = -\frac{1}{2}x + 8$ $3 = -\frac{1}{2}x$ $x = -6$ $D(-6, 11)$	M1: for finding gradient A1 M1: for subst $y = 11$ A1

(c) Find the area of the kite $ABCD$. [2]

(c)	Area $= \frac{1}{2}\begin{vmatrix} 0 & 4 & 4 & -6 & 0 \\ 3 & 6 & 11 & 11 & 3 \end{vmatrix}$ $= \frac{1}{2}\left[(0 + 44 + 44 - 18) - (12 + 24 - 66 + 0)\right]$ $= 50 \text{units}^2$	M1 in anticlockwise A1

4 In the diagram, it is given that $AB = 15$ cm, $BC = 40$ cm and $\angle OCB = \theta$. The vertical line OC is perpendicular to OA and AB is perpendicular to BC.



(a) Show that $OC = 15\sin\theta + 40\cos\theta$. [2]

(i)	$\sin\theta = \frac{AB}{15}$ $\cos\theta = \frac{BC}{40}$ $OC = AB\sin\theta + BC\cos\theta$ $= 15\sin\theta + 40\cos\theta$	B1 B1

(b) Express OC in the form $R\sin(\theta+\alpha)$, where R is a positive constant and α is an acute angle in degrees. [3]

(ii)	$R = 40^2 + 15^2 = \sqrt{1825}$ $\tan\alpha = \frac{40}{15}$ $\alpha = 69.444...$ $OC = 42.7\sin(\theta + 69.4°)$	M1 M1 A1

(c) Given that OC is at the maximum, determine the shortest distance of B to OC. [3]

(iii)	$\max OC$ = 42.7 cm when $\theta + 69.444...° = 90°$ $\theta = 20.556°$ shortest distance $= 40\sin(20.556°)$ $= 14.0$cm	M1 M1 A1

5 **(i)** Show $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right] = \frac{k}{\sin 6x}$, stating the value of the constant k. [4]

(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right] = \frac{1}{\tan 3x}\left(\sec^2 3x\right)(3)$ $= \frac{3\cos 3x}{\sin 3x \cos^2 3x}$ $= \frac{3}{\sin 3x \cos 3x}$ $= \frac{3(2)}{2\sin 3x \cos 3x}$ $= \frac{6}{\sin 6x}$ $k = 6$	M1 – differentiate correctly M1 – simplify trigo functions M1 – correct application of sine double angle formula A1

(ii) Differentiate $\ln\sqrt{\frac{7x}{x^2-1}}$ with respect to x. [3]

(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\sqrt{\frac{7x}{x^2-1}}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\ln\frac{7x}{x^2-1}\right)$ $= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\left[\ln 7x - \ln\left(x^2-1\right)\right]\right)$ $= \frac{1}{2}\left(\frac{7}{7x}\right) - \frac{1}{2}\left(\frac{2x}{x^2-1}\right)$ $= \frac{1}{2}\left(\frac{x^2-1-2x^2}{x\left(x^2-1\right)}\right)$ $= -\frac{x^2+1}{2x\left(x^2-1\right)}$	M1 – use logarithm laws to simplify M1 – differentiate $\ln 7x$ and $\ln\left(x^2-1\right)$ A1

6 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 2e^{x} + e^{-2x}$.

The y-axis is a normal to the curve at the point Q, where $y = 1$.

(a) Show that Q is a stationary point. [2]

(a)	The x-axis is perpendicular to the y-axis. Since the y-axis is a normal, the x-axis is parallel to a tangent at $y = 1$. Since the gradient of the x-axis is 0, $\dfrac{dy}{dx} = 0$ at $y = 1$. Therefore, there is a stationary point.	B1: finding $\dfrac{dy}{dx}$ B1: stating why gradient = 0

(b) Hence, find an expression for $\mathrm{f}(x)$. [6]

(b)	$\mathrm{f}'(x) = \int 2e^{x} + e^{-2x}\,dx$ $= 2e^{x} + \dfrac{e^{-2x}}{-2} + c$ $= 2e^{x} - \dfrac{1}{2e^{2x}} + c$	M1: for correct integration
	At $x = 0$, $\mathrm{f}'(x) = 0$, $2 - \dfrac{1}{2} + c = 0$ $c = -\dfrac{3}{2}$	M1
	$\mathrm{f}'(x) = 2e^{x} - \dfrac{1}{2e^{2x}} - \dfrac{3}{2}$	A1
	$\mathrm{f}(x) = \int 2e^{x} - \dfrac{1}{2e^{2x}} - \dfrac{3}{2}\,dx$ $= 2e^{x} - \dfrac{1}{2}\left(\dfrac{e^{-2x}}{-2}\right) - \dfrac{3}{2}x + c$ $= 2e^{x} + \dfrac{1}{4e^{2x}} - \dfrac{3}{2}x + c$	M1
	At $x = 0$, $y = 1$, $1 = 2 + \dfrac{1}{4} + c$ $c = -\dfrac{5}{4}$	M1
	$\mathrm{f}(x) = 2e^{x} + \dfrac{1}{4e^{2x}} - \dfrac{3}{2}x - \dfrac{5}{4}$	A1

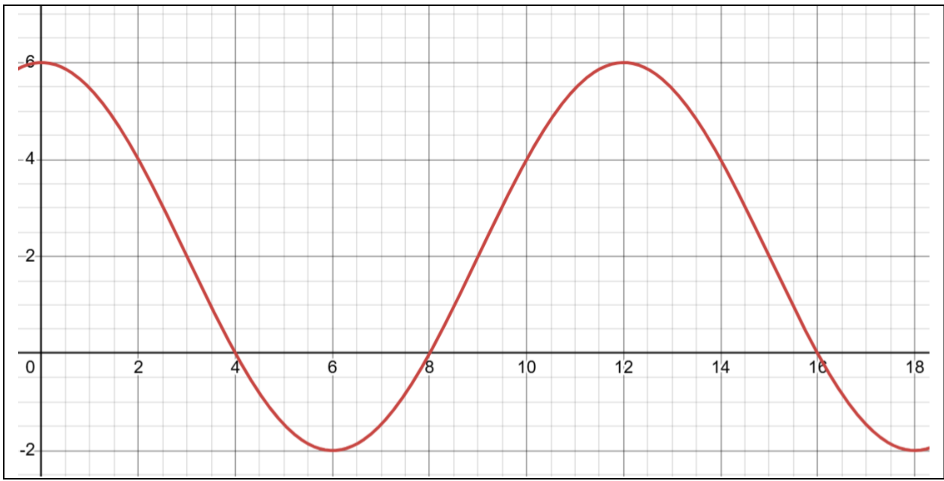
7 A waterwheel rotates at 1 revolution per 12 seconds. Initially, Point A, marked on the highest point of the rim of the wheel is 6 m above the water surface. It reaches 2 m below the water surface 6 seconds later.

The motion of point A can be modelled using the trigonometric equation $h = b\cos(kt) + 2$, where the height of point A above the water surface, h, is a function of time t, in seconds.

(i) Show that $b = 4$ and $k = \dfrac{\pi}{6}$. [2]

$b = \dfrac{6+2}{2} = 4$	B1
$k = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$	B1

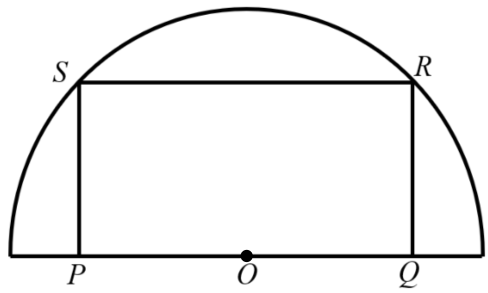
(ii) On the axes below, sketch the graph of h, for $0 \le t \le 18$. [3]

	B1 for shape (regardless no of cycles) B1 for period and correct x values (5 per cycle). Must see all 9 values. B1 for correct max, min and axis of equation Note: zero mark if student give wrong curve totally.

(iii) A similar waterwheel at a different location has a point, B, marked on its rim. The motion of point B is modelled by the equation $h_B = 4\cos(\frac{\pi}{6}t) + 4$. By drawing a suitable line on the same axis above, determine the number of time(s) when point B touches the water surface from $0 \le t \le 18$. [2]

2 times	M1 for drawing line of $y = -2$ A1

8 In the figure, $PQRS$ is a rectangle which fits inside a semicircle of radius 8 cm and centre O.



If $PQ = x$ cm and $QR = y$ cm,

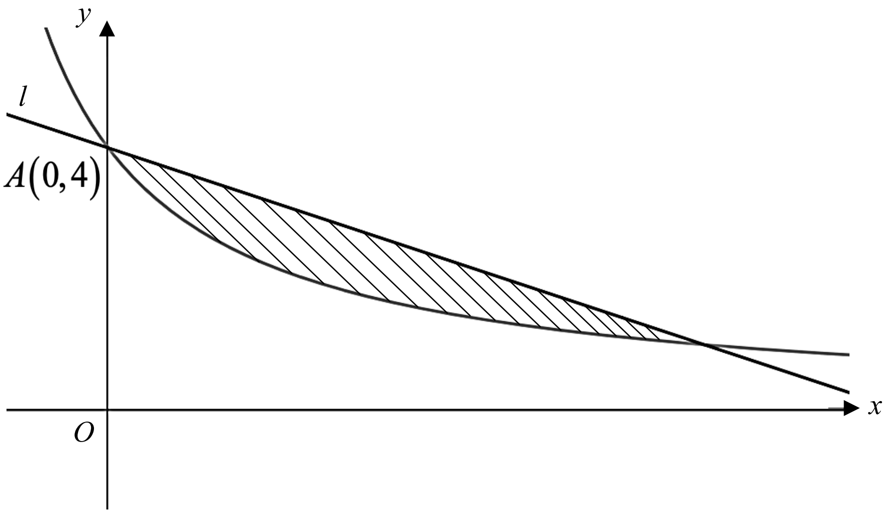
(a) Show that the area of the rectangle, $A\,\text{cm}^2$, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [3]

(a)	$A = xy$ $8^2 = y^2 + \left(\frac{x}{2}\right)^2$ $64 = y^2 + \frac{x^2}{4}$ $y = \sqrt{\frac{256 - x^2}{4}}$ $A = x\left(\sqrt{\frac{256 - x^2}{4}}\right)$ $A = \frac{x}{2}\sqrt{256 - x^2}$	M1 – form Pythagoras Theorem M1 – make y the subject A1

(b) The maximum area is obtained when the value of x, which can vary, gives a stationary value of A. Find the maximum value of A. [6]

<table>
<tr><td>(b)</td><td>$A=\frac{x}{2}\sqrt{256-x^2}$</td><td></td></tr>
<tr><td></td><td>$\frac{\mathrm{d}A}{\mathrm{d}x}=\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)\left(256-x^2\right)^{-\frac{1}{2}}(-2x)+\left(256-x^2\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)$</td><td>M1 – differentiate correctly</td></tr>
<tr><td></td><td>$=\frac{1}{2}\left(256-x^2\right)^{-\frac{1}{2}}\left[-x^2+256-x^2\right]$</td><td></td></tr>
<tr><td></td><td>$=\frac{-2x^2+256}{2\sqrt{256-x^2}}$</td><td></td></tr>
<tr><td></td><td>$\frac{\mathrm{d}A}{\mathrm{d}x}=0$</td><td>M1 – equate derivative to 0</td></tr>
<tr><td></td><td>$\frac{-2x^2+256}{2\sqrt{256-x^2}}=0$</td><td></td></tr>
<tr><td></td><td>$-2x^2+256=0$</td><td>M1 – find x</td></tr>
<tr><td></td><td>$x^2=128$</td><td></td></tr>
<tr><td></td><td>$x=\sqrt{128}$</td><td></td></tr>
<tr><td></td><td>$A=\frac{\sqrt{128}}{2}\sqrt{256-\left(\sqrt{128}\right)^2}$</td><td>A1</td></tr>
<tr><td></td><td>$A=64$</td><td>M1</td></tr>
<tr><td></td><td>
<table>
<tr><td>x</td><td>$\sqrt{128}^-$</td><td>$\sqrt{128}$</td><td>$\sqrt{128}^+$</td></tr>
<tr><td>Sign of $\frac{\mathrm{d}A}{\mathrm{d}x}$</td><td>-</td><td>0</td><td>+</td></tr>
<tr><td>Sketch of tangent</td><td></td><td></td><td></td></tr>
</table>
</td><td>A1</td></tr>
<tr><td></td><td>By first derivative testing, A is a maximum.</td><td></td></tr>
</table>

9 The diagram shows part of the curve $y = \dfrac{8}{2+3x}$. A line, l, passes through the curve at point $A(0,4)$ and is parallel to the tangent to the curve at $x = -2$.



(a) Show that the equation of line l is $y = -\dfrac{3}{2}x + 4$. [3]

(a)	$\dfrac{dy}{dx} = 8(-1)(2+3x)^{-2}(3)$ $= -\dfrac{24}{(2+3x)^2}$ At x = -2, Gradient $= -\dfrac{24}{(2-6)^2}$ $= -\dfrac{3}{2}$ Equation of l: $y = -\dfrac{3}{2}x + 4$	M1 M1 A1

(b) Find the area of the shaded region. Leave your answer in the form $a + b\ln 2$. [6]

(b)	$\frac{8}{2+3x} = -\frac{3}{2}x + 4$ $\frac{16}{2+3x} = -3x + 8$ $16 = -6x - 9x^2 + 16 + 24x$ $9x^2 - 18x = 0$ $x(x-2) = 0$ $x = 0 \quad or \quad 2$ At $x = 2$, $y = -\frac{3}{2}(2) + 4$ $= 1$ Area $= \frac{1}{2}(4+1)(2) - \int_0^2 \frac{8}{2+3x}\,dx$ $= 5 - \frac{8}{3}\left[\ln(2+3x)\right]_0^2$ $= 5 - \frac{8}{3}[\ln 8 - \ln 2]$ $= 5 - \frac{16}{3}\ln 2$	M1: for equating the equations A1: for coordinates of other intersection point M1: for area of trapezium / integrating line minus integrating curve M1: for integration M1: for definite integrals A1

10 A particle travels in a straight line such that at time t seconds after leaving a point A, which has a displacement of 2 metres from a fixed point O, its velocity, v ms^{-1} is given by

$v = 3t^2 - 21t + 18$.

Find,

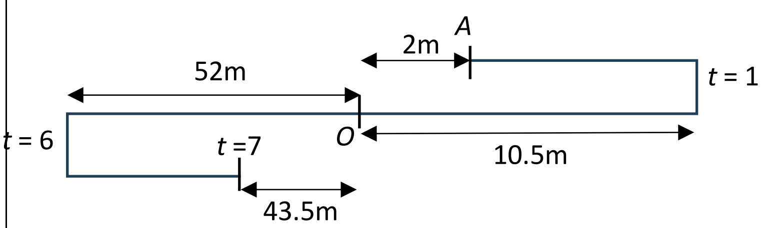
(a) the minimum velocity of the particle, [3]

(a)	$\frac{dv}{dt} = 6t - 21$ $6t - 21 = 0$ $t = \frac{21}{6} s$ $v = 3\left(\frac{21}{6}\right)^2 - 21\left(\frac{21}{6}\right) + 18$ $= -\frac{75}{4} ms^{-1}$ $\frac{d^2v}{dt^2} = 6 > 0$ Therefore, $v = -\frac{75}{4} ms^{-1}$ is the minimum velocity.	M1: for finding t (Award even if have careless mistake) A1 B1: for checking max/min (Award even if value of v is wrong)

(b) the time(s) at which the particle is instantaneously at rest, [1]

(b)	$3t^2 - 21t + 18 = 0$ $t^2 - 7t + 6 = 0$ $(t-6)(t-1) = 0$ $t = 1 \text{ or } 6$	B1

(c) the total distance travelled in the first 7 seconds. [6]

$s=\int 3t^2-21t+18\,dt$ $=t^3-\frac{21}{2}t^2+18t+c$ At $t=0$, $s=2$, therefore, $c=2$ $s=t^3-\frac{21}{2}t^2+18t+2$	B1: for getting s
At $t=1$, $s=1^3-\frac{21}{2}1^2+18(1)+2$ $=10.5m$	M1
At $t=6$, $s=6^3-\frac{21}{2}(6)^2+18(6)+2$ $=-52m$	M1
At $t=7$, $s=7^3-\frac{21}{2}(7)^2+18(7)+2$ $=-43.5m$	M1
 	
Total Distance $=(10.5-2)+10.5+52+(52-43.5)$ $=79.5m$	M1 A1

11 **(a)** Given the function $y = \dfrac{m-2x}{x^2-3}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

$y = \dfrac{m-2x}{x^2-3}$ $\dfrac{dy}{dx} = \dfrac{(x^2-3)(-2)-(m-2x)(2x)}{(x^2-3)^2}$ $= \dfrac{-2x^2+6-2mx+4x^2}{(x^2-3)^2}$ $= \dfrac{2x^2+6-2mx}{(x^2-3)^2}$	M2 – finding numerator and denominator $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ A1

(b) Hence, find the range of values of m such that the function $y = \dfrac{m-2x}{x^2-3}$ is always increasing. [4]

Since $\dfrac{2x^2+6-2mx}{(x^2-3)^2} > 0$ and $(x^2-3)^2 > 0$, $\therefore\ 2x^2-2mx+6>0$ $2x^2-2mx+6 = 2(x^2-mx+3)$ $= 2\left[\left(x-\dfrac{m}{2}\right)^2 - \left(\dfrac{m}{2}\right)^2 + 3\right]$ $= 2\left(x-\dfrac{m}{2}\right)^2 - \dfrac{m^2}{2} + 6$ For $2x^2-2mx+6>0$, $-\dfrac{m^2}{2}+6>0$ $\dfrac{m^2}{2}<6$ $m^2<12$ $m^2-12<0$ $(m+\sqrt{12})(m-\sqrt{12})<0$ $-2\sqrt{3}<m<2\sqrt{3}$	M1 – stating $\dfrac{\mathrm{d}y}{dx}>0$ or wrong expression >0 M1 – completing the square method M1 – stating $-\dfrac{m^2}{2}+6>0$ A1

Alternative Solution Since $\frac{2x^2+6-2mx}{\left(x^2-3\right)^2}>0$ and $\left(x^2-3\right)^2>0$, $\therefore\ 2x^2-2mx+6>0$ $2x^2-2mx+6=0$ discriminant <0 $(-2m)^2-4(2)(6)<0$ $4m^2-48<0$ $m^2-12<0$ $\left(m+\sqrt{12}\right)\left(m-\sqrt{12}\right)<0$ $-2\sqrt{3}<m<2\sqrt{3}$	M1 – stating $\frac{dy}{dx}>0$ or wrong expression > 0 M1 – finding discriminant M1 – stating discriminant < 0 A1

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