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**CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2022
PRELIMINARY EXAMINATION**

**ADDITIONAL MATHEMATICS
Paper 2**

**4049/02
30 August 2022**

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours 15 mins

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										

Table of Penalties		Qn. No.	Parent's/Guardian's Signature	90
Presentation	-1			
Significant Figures/ Units	-1			

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

- 1 Show that $x = -1$ is a solution of the equation $4x^3 - 7x - 3 = 0$ and hence solve the equation completely. [5]

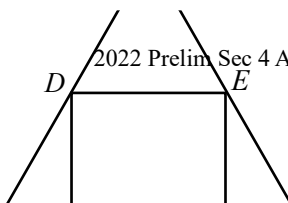
[Turn over

- 2 Show that the equation $e^{x+1} - 6e^{-x} = 2e - 3$ has only one solution and find its exact value.

[5]

3(a) Given that $y = \frac{2x}{\sqrt{3x+1}}$, show that $\frac{dy}{dx} = \frac{3x+2}{\sqrt{(3x+1)^3}}$. [3]

3(b) Hence find the value of $\int_1^5 \frac{6x+3}{\sqrt{(3x+1)^3}} dx$. [5]





In the diagram, $DFGE$ is a rectangle and triangle ABC is an isosceles with $AB = AC = 5$ m. ADB and AEC are straight lines. DE is parallel to BC and $AB = 4AD$. Angle $ABC = \theta$ where $0^\circ < \theta < 90^\circ$.

- (a) Show that the perimeter, P metres, of rectangle $DFGE$ is given by $P = \frac{5}{2}(2 \cos \theta + 3 \sin \theta)$.

[3]

- (b) Express P in the form $\frac{5}{2}R \cos(\theta - \alpha)$, where $R > 0$ and write down the largest possible value of P . [3]

- (c) Find the value of θ for which $P = 6$. [3]

5(a) A student is asked to solve the quadratic inequality $(2x-1)(x-3) < 0$.

The first step of her solution is: $2x - 1 < 0$ and $x - 3 < 0$.

State whether this step is correct and support your answer with a clear explanation. [2]

(b) The equation of a curve is given by $y = 2x^2 - 6x + 7$.

(i) Find the set of values of x for which the curve lies below the line $y = x + 4$. [3]

- (ii) The line $y = 2x + k$ is tangent to the curve at the point Q , where k is a constant.
Find the value of the constant k . [3]

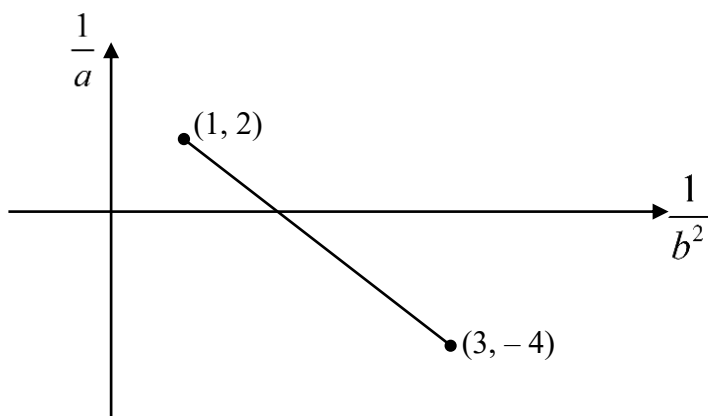
6(a) Write down and simplify the second and fourth term of the expansion $\left(x^3 - \frac{2}{x^2}\right)^n$. [4]

(b) Given that coefficient of the fourth term to the coefficient of the second term is 20 : 1, find the value of n . [3]

- (c) Using the value of n found in part (b) and without expanding $\left(x^3 - \frac{2}{x^2}\right)^n$, explain why there is no constant term in the expansion. [3]

[Turn over]

- 7(a) The diagram shows part of a straight line obtained by plotting $\frac{1}{a}$ against $\frac{1}{b^2}$.
Obtain an equation relating a and b , expressing a in terms of b . [4]



(b) It is given that variable x and y are related by an equation in the form $y = m^{x+c}$, where $m > 1$ and m and c are constants.

(i) Explain why plotting the graph of y against x would not result in a straight line graph. [1]

(ii) Express $y = m^{x+c}$ in a form suitable for drawing a straight line graph and state clearly how the values of m and c can be obtained. [4]

[Turn over

- 8** A particle moves in a straight line such that its velocity, v m/s is given by $v = t^2 - 10t + 9$, where t is the time in seconds after passing a fixed point O .

(a) Calculate the values of t when the particle is instantaneously at rest. [2]

(b) Calculate the stationary value of the velocity and explain whether this velocity is a maximum or a minimum. [4]

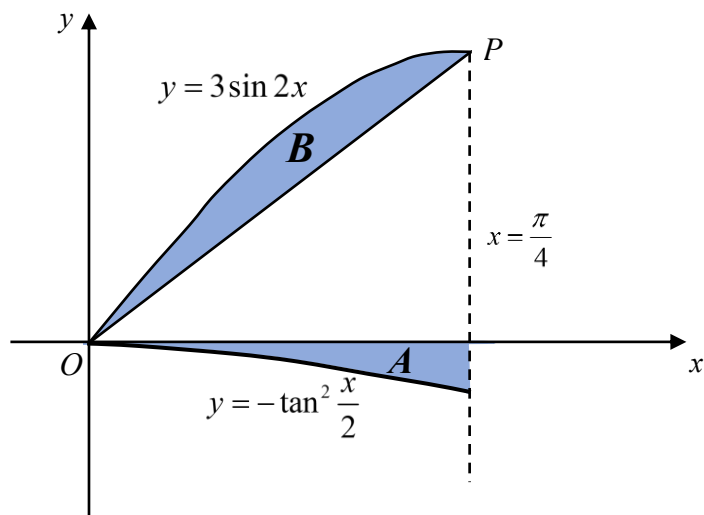
- (c) Calculate the velocity of the particle when it first returns to O . [6]

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- (d) Find the total distance travelled in the first 3 seconds. [2]

- 9 The diagram below, not drawn to scale, shows part of the graphs of $y = 3\sin 2x$ and

$$y = -\tan^2 \frac{x}{2} \text{ for } 0 \leq x \leq \frac{\pi}{4} \text{ and } OP \text{ is a straight line.}$$



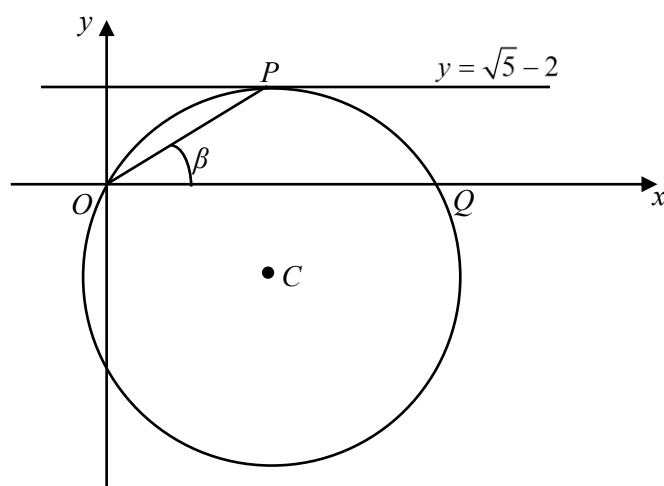
- (a) Find the area of shaded region A , correct to 2 significant figures.

[5]

- (b) Calculate the exact area of the shaded region B .

[7]

10



In the diagram, the line $y = \sqrt{5} - 2$ is tangent to the circle C_1 with centre C at the point P . The circle passes through the origin O and the point Q on the x -axis. The line OP makes an angle β with the x -axis, where $\tan \beta = \sqrt{5} - 2$.

- (a) Find the exact coordinates of points P , Q and C .

[5]

(b)

[2]

- (c) Circle C_2 is the reflection of circle C_1 about the line $y = \sqrt{5} - 2$. Find the equation of circle C_2 . [3]

End of Paper

Answer Key

1 $x = -1, -\frac{1}{2}, \frac{3}{2}$

2 Show $x = \ln 2$ as the only solution

3(a) $\frac{3x+2}{\sqrt{(3x+1)^3}}$ **(b)** $2\frac{5}{6}$

4(b) $P = \frac{5}{2}(\sqrt{13} \cos(\theta - 56.3^\circ))$; Largest $P = \frac{5\sqrt{13}}{2}$ **(c)** $\theta = 8.0^\circ$

5(a) No **(b)(i)** $\frac{1}{2} < x < 3$ **(ii)** $k = -1$

6(a) $T_2 = -2nx^{3n-5}$; $T_4 = -\frac{4n(n-1)(n-2)}{3}x^{3n-15}$ **(b)** $n = 7$

(c) $r = 4.2$ (not a positive integer)

7(a) $a = \frac{b^2}{5b^2 - 3}$ **(b)(ii)** $m = 10^{\text{gradient}}$; $c = \frac{\lg y - \text{intercept}}{\text{gradient}}$

8(a) $t = 1$ or $t = 9$ **(b)** $t = 5$ **(c)** -7.54 m/s **(d)** $17\frac{2}{3} \text{ m}$

9(a) 0.043 units^2 **(b)** $\left(\frac{3}{2} - \frac{3\pi}{8}\right) \text{ sq units}$

10(a) $P = (1, \sqrt{5} - 2)$; $Q = (2, 0)$; $C = (1, -2)$ **(b)** $(x-1)^2 + (y+2)^2 = 5$

(c) $(x-1)^2 + (y-2\sqrt{5}+2)^2 = 5$