

2026 4E5N Amath Paper 2 Mark Scheme (4049/02)

Qn	Mark Scheme	
1	$x\sqrt{108} - \sqrt{75} = x\sqrt{5}$ $x\sqrt{108} - x\sqrt{5} = \sqrt{75}$ $x(\sqrt{108} - \sqrt{5}) = \sqrt{75}$ $x = \frac{\sqrt{75}}{\sqrt{108} - \sqrt{5}}$ $x = \frac{5\sqrt{3}}{\sqrt{108} - \sqrt{5}} \times \frac{\sqrt{108} + \sqrt{5}}{\sqrt{108} + \sqrt{5}}$ $x = \frac{5\sqrt{3}(\sqrt{108} + \sqrt{5})}{103}$ $x = \frac{90 + 5\sqrt{15}}{103}$ $p = 90, q = 5$	
2a	$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 17\left(\frac{3}{2}\right)^2 + 41\left(\frac{3}{2}\right) - 30 = 0$ By factor theorem, $(2x - 3)$ is a factor of $f(x)$.	
2b	$2x^3 - 17x^2 + 41x - 30$ $= (2x - 3)(x^2 + kx + 10)$ Comparing coefficients of x^2, $-17 = -3 + 2k$ $2k = -14$ $k = -7$ $f(x) = 2x^3 - 17x^2 + 41x - 30$ $= (2x - 3)(x^2 - 7x + 10)$ $= (2x - 3)(x - 2)(x - 5)$	
2c	$f(x) = (2x - 3)(x - 2)(x - 5) = 0$ $x = \frac{3}{2} \quad \text{or} \quad x = 2 \quad \text{or} \quad x = 5$	

2d	$g(x) = a(x-3)(3x-1) + 2$ Using $a = 1$ (or any other non-zero value of a) $g(x) = 3x^2 - 10x + 5$	
2e	$x^6 + 27y^3 = (x^2 + 3y)(x^4 - 3x^2y + 9y^2)$	
3	$\frac{d^2y}{dx^2} = 6 \sin 2x - 4 \cos x$ $\frac{dy}{dx} = \frac{-6 \cos 2x}{2} - 4 \sin x + C_1$ At $x = 0$, $\frac{dy}{dx} = 2$ $2 = \frac{-6 \cos 0}{2} - 4 \sin 0 + C_1$ $C_1 = 5$ $\frac{dy}{dx} = -3 \cos 2x - 4 \sin x + 5$ $y = -\frac{3}{2} \sin 2x + 4 \cos x + 5x + C_2$ Sub (0,5) $5 = -\frac{3}{2} \sin 2(0) + 4 \cos(0) + 5(0) + C_2$ $C_2 = 1$ Equation: $y = -\frac{3}{2} \sin 2x + 4 \cos x + 5x + 1$	
4ai	$-90^\circ \leq \sin^{-1} x \leq 90^\circ$	
4aia	$-90^\circ < \tan^{-1} x < 90^\circ$	

4b	$\frac{\sin 60^\circ \times \tan 45^\circ}{\cos 30^\circ} + \cos^2 45^\circ$ $= \frac{\frac{\sqrt{3}}{2} \times 1}{\frac{\sqrt{3}}{2}} + \frac{1}{2}$ $= \frac{3}{2}$	
5a	Total cost of reinforced fence $4x \times 3 = 12x$ Total cost of long fence = L $12x + L = 120$ $L = 120 - 12x$ $A = Lx$ $= (120 - 12x)x$ $= 120x - 12x^2$ (shown)	
5b	$\frac{dA}{dx} = 120 - 24x$ For stationary value $\frac{dA}{dx} = 0$ $120 - 24x = 0$ $24x = 120$ $x = 5$ Showing that it is maximum, $\frac{d^2A}{dx^2} = -24 < 0$ Hence, it is a maximum value. Maximum A $A = 120(5) - 12(5)^2$ $= 300 \text{ cm}^2$ Overall units -1	

6	$v = t^2 - 9t + 20$ $s = \frac{t^3}{3} - \frac{9}{2}t^2 + 20t + C$ At $t = 0, s = 0$ $C = 0$ $s = \frac{t^3}{3} - \frac{9}{2}t^2 + 20t$ Finding turning points, $v = 0$ $t^2 - 9t + 20 = 0$ $(t - 4)(t - 5) = 0$ $t = 4 \quad \text{or} \quad t = 5$ $s(0) = 0$ $s(4) = \frac{88}{3}$ $s(5) = \frac{175}{6}$ $s(7) = \frac{203}{6}$ Total distance travelled $= \frac{88}{3} + \frac{1}{6} + \frac{14}{3} = \frac{205}{6} \text{ m or } 34.2 \text{ m}$ <i>Overall units -1</i>	
7a	Let $u = 3^x$ $u^2 - 4u + 3 = 0$ $(u - 1)(u - 3) = 0$ $u = 1 \quad \text{or} \quad u = 3$ $3^x = 1 \Rightarrow x = 0$ $3^x = 3 \Rightarrow x = 1$	

7b	$\frac{y}{x^2} = 2 \Rightarrow y = 2x^2$ $\log_2(y) + \log_2(x+2) - \log_2(x) = 4$ $\log_2\left(\frac{y(x+2)}{x}\right) = 4$ $\frac{y(x+2)}{x} = 16$ $\frac{2x^2(x+2)}{x} = 16$ $2x(x+2) = 16$ $x(x+2) = 8$ $x^2 + 2x - 8 = 0$ $(x+4)(x-2) = 0$ $x = -4 \text{ (rej)} \quad \text{or} \quad x = 2$ $y = 2(2)^2 = 8$	
8a	$\frac{dy}{dx} = \frac{(x-2)^2(2x-4) - (x^2-4x+3)[2(x-2)]}{(x-2)^4}$ $= \frac{2(x-2)[(x-2)^2 - x^2 + 4x - 3]}{(x-2)^4}$ $= \frac{2[x^2 - 4x + 4 - x^2 + 4x - 3]}{(x-2)^3}$ $= \frac{2}{(x-2)^3}$	
8b	For increasing functions, $\frac{dy}{dx} > 0$	

	$\frac{2}{(x-2)^3} > 0$ $x-2 > 0$ $x > 2$	
8c	$\frac{d^2y}{dx^2} = 2[-3(x-2)^{-4}]$ $= \frac{-6}{(x-2)^4}$ Since $(x-2)^4 > 0$, $\frac{-6}{(x-2)^4} < 0$. Hence, the gradient is a decreasing function for $x \neq 2$.	
9a	$\angle XZY = \angle XYM$ (alt. segment theorem) $\angle ZXY = \angle XYM$ (alt. $\angle$ s $ZX \parallel NM$) Hence, $\angle XZY = \angle ZXY$. Triangle XYZ is an isosceles triangle.	
9b	$\angle XYV = \angle WYX$ (common angle) $\angle XWY = \angle XYM$ (alt. segment theorem) $\angle VXY = \angle XYM$ (alt $\angle$ s, $ZX \parallel NM$) $\therefore \angle XWY = \angle VXY$ By AA similarity test, triangle YXW is similar to triangle YVX.	
9c	$WY = \sqrt{3^2 + 4^2} = 5$ $\frac{XY}{WY} = \frac{VX}{WX}$ $\frac{4}{5} = \frac{VX}{3}$ $VX = \frac{12}{5}$ $ZX = 2 \times VX = \frac{24}{5}$ $ZY = XY = 4$ $ZY : ZX$ $= 4 : \frac{24}{5}$ $= 5 : 6$	

10a	$h = 15 - 5t^2 + 10t$ $= -5(t^2 - 2t - 3)$ $= -5(t^2 - 2t + 1 - 1 - 3)$ $= -5(t - 1)^2 + 20$	
10b	Maximum height = 20 Time = 1	
10c	$h = -5\left(1 + \frac{k}{5} - 1\right)^2 + 20$ $= -5\left(\frac{k}{5}\right)^2 + 20$ $= -\frac{k^2}{5} + 20$ $-\frac{k^2}{5} + 20 = k + 10$ $k^2 + 5k - 50 = 0$ $(k + 10)(k - 5) = 0$ $k = -10 \text{ (rej) or } k = 5$	
11a	$\sqrt{32x} = -2x + 5$ $32x = 25 - 20x + 4x^2$ $4x^2 - 52x + 25 = 0$ $(2x - 1)(2x - 25) = 0$ $x = \frac{1}{2} \quad \text{or} \quad x = \frac{25}{2} \text{ (rej)}$ $y = -2\left(\frac{1}{2}\right) + 5$ $= 4$ $\therefore A(0.5, 4)$ Sub $x = 1$ into $y = \sqrt{32x}$ $y = \sqrt{32} = 4\sqrt{2}$ $\therefore B(1, 4\sqrt{2})$	

11b	$\int_{0.5}^1 (\sqrt{32x} + 2x - 5) \, dx$ $= \left[\frac{\sqrt{32}x^{\frac{3}{2}}}{\frac{3}{2}} + x^2 - 5x \right]_{0.5}^1$ $= \left[\frac{8\sqrt{2}}{3}x^{\frac{3}{2}} + x^2 - 5x \right]_{0.5}^1$ $= \left(\frac{8\sqrt{2}}{3} + 1 - 5 \right) - \left(\frac{8\sqrt{2}}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} + \frac{1}{4} - 5(0.5) \right)$ $= \frac{8\sqrt{2}}{3} - 4 - \frac{8\sqrt{2}}{3} \left(\frac{1}{2\sqrt{2}} \right) + \frac{9}{4}$ $= \frac{32\sqrt{2}}{12} - \frac{37}{12}$ $= \frac{32\sqrt{2} - 37}{12}$	
11c	Total area $= (4\sqrt{2}) - (0.5 \times 4) - \frac{32\sqrt{2} - 37}{12}$ $= 2.969 \text{ (3 d.p)}$	
12a	$R = \sqrt{6^2 + 8^2} = 10$ $\alpha = \tan^{-1} \left(\frac{8}{6} \right) = 0.92730$ $24 = 15 + 6 \sin \left(\frac{\pi t}{12} \right) + 8 \cos \left(\frac{\pi t}{12} \right)$ $24 = 15 + 10 \sin \left(\frac{\pi t}{12} + 0.92730 \right)$ $\sin \left(\frac{\pi t}{12} + 0.92730 \right) = \frac{9}{10}$ Let reference angle be θ $\theta = \sin^{-1} \left(\frac{9}{10} \right)$ $\theta = 1.1198$	

	$\frac{\pi t}{12} + 0.92730 = 1.1198$ $t = 0.735296 = 44.1 \text{ min}$ It starts to flower at 00 44 (or 0045).	
12b	Maximum temp = 25 degree Celsius. $\sin\left(\frac{\pi t}{12} + 0.92730\right) = 1$ $\frac{\pi t}{12} + 0.92730 = \frac{\pi}{2}$ $t = 2.45797 = 2 \text{ hr } 27 \text{ mins}$ Time = 02 27 or 02 28	
12c	$10 = 15 + 10 \sin\left(\frac{\pi t}{12} + 0.92730\right)$ $\sin\left(\frac{\pi t}{12} + 0.92730\right) = -0.5$ $\alpha = \frac{\pi}{6}$ $\frac{\pi t}{12} + 0.92730$ lies in the third and fourth quadrant. $\frac{\pi t}{12} + 0.92730 = \frac{7\pi}{6}, \frac{11\pi}{6}$ $t = 10.5, 18.5$	