RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial C6A: Differentiation Techniques

Section A (Basic Questions)

1 Differentiate each of the following with respect to x:

(a) $x(x^3+1)^{\frac{1}{3}}$

(b) $\dfrac{3x}{(x+3)(x+1)}$

(c) $\sec^3(2x-3)$

(d) $\sin^{-1}\left(\dfrac{x}{2}\right)$

(e) $\cos^{-1}\sqrt{x^3}$ giving your answer in non-trigonometric form,

(f) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$

(g) $\left(\mathrm{e}^{2x}-\mathrm{e}^{-2x}\right)^2$

(h) $\ln\left((x^3+2)^3(x^2+2)^2\right)$

(i) $\log_2(3x^4-\mathrm{e}^x)$

(j) 7^{x^2}

2 Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y for each of the following.

(a) $x+y+\sin y=2$

(b) $y=\sin\left((x+y)^2\right)$

(c) $\sin x\cos y=\dfrac{1}{2}$

3 Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t for each of the following.

(a) $x=\dfrac{2-3t}{1+t}$, $y=\dfrac{3+2t}{1+t}$

(b) $x=\dfrac{1}{2}(\mathrm{e}^t-\mathrm{e}^{-t})$, $y=\dfrac{1}{2}(\mathrm{e}^t+\mathrm{e}^{-t})$

Section B (Discussion Questions)

1 By considering the derivative as a limit, $\mathrm{f}'(x)=\lim_{\delta x\to 0}\left(\frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}\right)$, show that the derivative of $\frac{1}{x^2}$ is $-\frac{2}{x^3}$. [3]

2 Differentiate each of the following with respect to x:

(a) $x\sqrt{3-4x^2}$

(b) $\frac{x^2-7x+4}{x^3}$

(c) $\frac{x(x-2)}{(x-1)^2}$

(d) $\cos x°$

(e) $\sin^4 x^2$

(f) $\cos^{-1}\sqrt{1-x^2}$

(g) $\sec 3x \sin^{-1} 2x$

(h) $\tan^{-1}\left(\mathrm{e}^{2x}\right)$

(i) $\operatorname{cosec}\left(\ln\left(x^2+3\right)\right)$

(j) $\mathrm{e}^{2x}\tan x$

(k) $\ln\left(9+3\mathrm{e}^{3x}\right)$

(l) $\ln\sqrt{x^2+1}$

(m) $\ln\left(\frac{\mathrm{e}^{\sqrt{x}}}{\cos^3 x}\right)$, where $0<x<\frac{\pi}{2}$

(n) $\ln\sqrt{\frac{x^2+1}{3x^2+a}}$, where a is a positive constant

(o) $x^{\sec 2x}$

(p) $\left(x^2+1\right)^x$

Implicit Differentiation

3 **(a)** Given $x^n y + \ln y^2 = n^3$, where n is a constant, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of n, x and y. [3]

(b) Given that $y^{2y} = \mathrm{e}^{3x+2\mathrm{e}}$, where $y > \mathrm{e}^{-1}$, show that $y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{2}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$. [3]

(c) It is given that $y^2 = \sin x + \cos x$. Show that $y\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + A\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, where A is a real constant to be determined. [2]

(d) Given that $y^{\frac{1}{x}} = x^{\ln x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

4 Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t for each of the following.

(a) $x = a\sec t,\ y = a\tan t$, where a is a non-zero constant

(b) $x = \frac{u}{u-1}, \quad y = \frac{u^2}{u-1}, \quad$ for $u \neq 1$.

5 A curve C has equation $\frac{1 - x^2 y^2}{3y^2 + x^2} = \frac{1}{2}$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-x - 2xy^2}{y(3 + 2x^2)}$. [3]

(b) Find the equation of the tangents to the curve that are parallel to the y-axis. [3]

6 A curve C has parametric equations

$$x = \frac{1}{1-t^2},\ \ y = \frac{t}{1-t^2},\ \ t \neq \pm 1.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [2]

(ii) Show that there are no points on C with tangents inclining at an angle of $\frac{\pi}{4}$ with the positive x-axis. [3]