ST ANDREW'S JUNIOR COLLEGE

General Certificate of Education Advanced Level

Higher 2

MATHEMATICS 9758/01

Paper 1 **1 September 2025 (Monday)**

Preliminary Examination **3 hours**

Additional Materials: **Printed Answer Booklet**

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions. Total marks : **100**

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

1 A function f is defined by $\mathrm{f}(x)=ax^3+bx^2+cx+d$ where a, b, c and d are constants. The graph of $y=\mathrm{f}(x)$ has a turning point at $x=-1$ and passes through the points $(2,10)$ and $(-2,-2)$. Given also that $\int_{-2}^{2}\mathrm{f}(x)\,\mathrm{d}x=16$, find the values of a, b, c and d. [4]

2 A closed cylinder is designed to have a fixed external surface area of p cm^2 such that its volume is maximum. Find the radius of the cylinder in terms of p. [5]

3 The lines l_1 and l_2 have equations

$$l_1:\ \mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix},$$

$$l_2:\mathbf{r}=\begin{pmatrix}0\\5\\-8\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\3\end{pmatrix},$$

where λ and μ are parameters.

(i) Show that l_1 and l_2 are skew lines. [2]

The point P has coordinates $(1,5,0)$ and the plane π_1 has equation $y-z-2=0$.

(ii) Find the coordinates of P', the reflection of P in π_1. [4]

(iii) Find the cartesian equation of the plane π_2 containing P and l_1. [3]

4 **(a)** The function f is defined by

$$\mathrm{f}(x)=\begin{cases} a^2-x^2, & 0<x\le a, \\ a(x-a), & a<x\le 2a, \end{cases}$$

and $\mathrm{f}(x)=\mathrm{f}(x+2a)$ for all real values of x, where a is a positive constant.

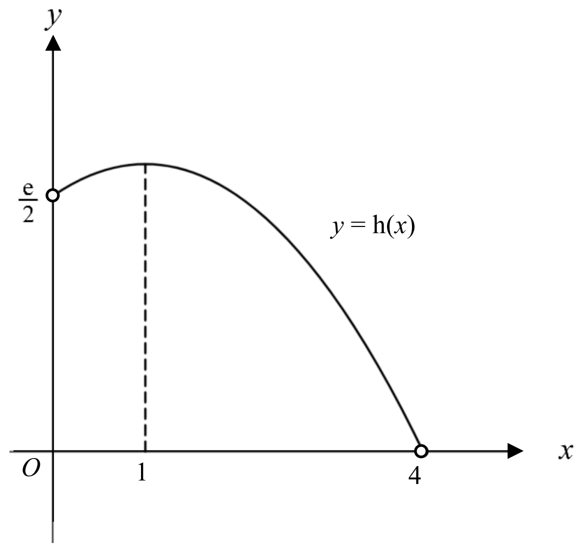
(i) Sketch the graph of $y=\mathrm{f}(x)$ for $-2a\le x\le 3a$. [3]

(ii) Find $\mathrm{f}\left(\dfrac{101a}{2}\right)$ in terms of a. [2]

(b) The function g is defined by

$$\mathrm{g}: x \mapsto \ln x, \text{where } x\in\mathbb{R},\ 0<x\le \mathrm{e}.$$

The diagram below shows the graph of the quadratic function h with domain defined where $0<x<4$. The graph has a maximum point at $x = 1$ and has end-points $\left(0, \dfrac{\mathrm{e}}{2}\right)$ and (4,0).



(i) Determine whether hg exist. Justify your answer. [2]

(ii) State the other value of x for which $\mathrm{h}(x)=\dfrac{\mathrm{e}}{2}$. Given that h^{-1} exists, deduce the restricted domain of h such that domain of h^{-1} is $\left(0, \dfrac{\mathrm{e}}{2}\right]$. [2]

(iii) Using the restricted domain of h found in **(ii)**, find the range of gh. [2]

5. An arithmetic progression has first term a and common difference d, where a and d are non-zero. The 3^{rd}, 9^{th} and 11^{th} terms of the arithmetic progression are three distinct consecutive terms of a geometric progression.

(a) Find d in terms of a. [3]

(b) Let the sum of the first n terms of the arithmetic progression be S_n where $S_1 = 143$.

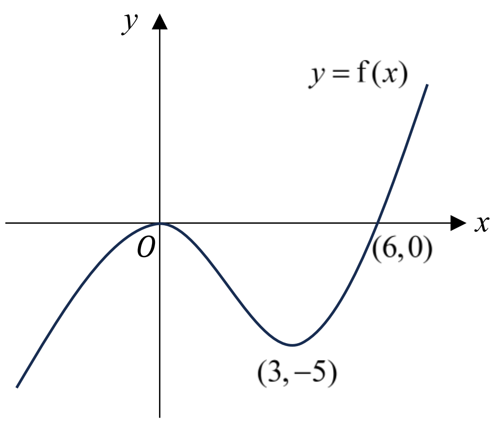
Find the largest possible value of S_n. [3]

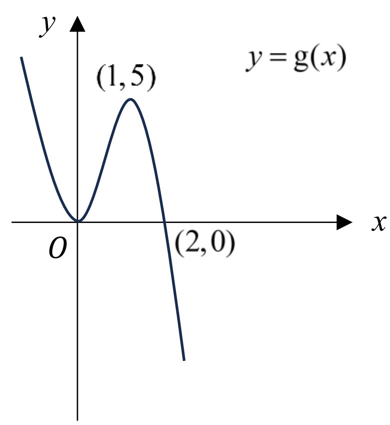
The first term of the geometric progression is b, where b is positive.

(c) Determine whether the geometric series is convergent. [2]

(d) Show that the sum of all terms after the n^{th} term of the geometric progression is at most $\frac{1}{2}b$, where n is a positive integer. [2]

6. The graph of $y = \mathrm{f}(x)$ cuts the x-axis at the point (6,0) and has a maximum point and a minimum point at (0,0) and $(3,-5)$ respectively. The graph of $y = \mathrm{g}(x)$ cuts the x-axis at the point (2,0), and has a minimum point and a maximum point at (0,0) and (1,5) respectively. The diagrams below show the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.





(i) Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{g}(x)$. [2]

It is given that $\mathrm{g}(x)$ can be expressed in the form $a\mathrm{f}(bx + c) + d$ where a, b, c and d are constants.

(ii) State the values of a, b, c and d. [2]

(iii) The derivative of the function $y = \mathrm{f}(x)$ with respect to x is denoted by $\mathrm{f}'(x)$. It is given that $\mathrm{f}'(6) = 4$. Find the exact value of $\mathrm{g}'(2)$. [3]

7. **(a)** **(i)** On a single diagram, sketch the graphs of $y=\dfrac{3x}{x-1}$ and $y=3x$, stating the equation(s) of any asymptote(s) and the coordinates of points of intersection. [4]

(ii) Hence find the area bounded by the curve $y=\dfrac{3x}{x-1}$ and the lines $y=3x$ and $y=8$. [2]

(b) A curve has the equation $y=\sqrt{x^2+1}$ where x is non-negative.

(i) With the aid of a sketch of the above curve, explain why the value of $S=\lim\limits_{n\to\infty}\dfrac{1}{n}\left[1+\sqrt{\dfrac{1}{n^2}+1}+\sqrt{\dfrac{4}{n^2}+1}+\ldots+\sqrt{\dfrac{(n-1)^2}{n^2}+1}\right]$ is $\int_0^1\sqrt{x^2+1}\,\mathrm{d}x$. [4]

(ii) Hence find the value of S. [1]

8 A curve has equation $(x-y)^2=2\mathrm{e}^{xy}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x-y-y\mathrm{e}^{xy}}{x-y+x\mathrm{e}^{xy}}$. [3]

(ii) Given that the curve cuts the positive y-axis at point A, find the equation of the normal to the curve at A. [2]

(iii) The normal to the curve at A meets the curve at another point B. Find the coordinates of B. [3]

9. **(a)** Use the substitution $x=3\sec\theta$ to find $\displaystyle\int\frac{1}{x^2\sqrt{x^2-9}}\,\mathrm{d}x$. [5]

(b) The region R lies in the first quadrant and is bounded by the curve $y^2=\dfrac{1}{x^2\sqrt{x^2-9}}$ and the lines $y=0.1$ and $x=4$. Find the exact volume of the solid generated when R is rotated through 2π radians about the x-axis. [4]

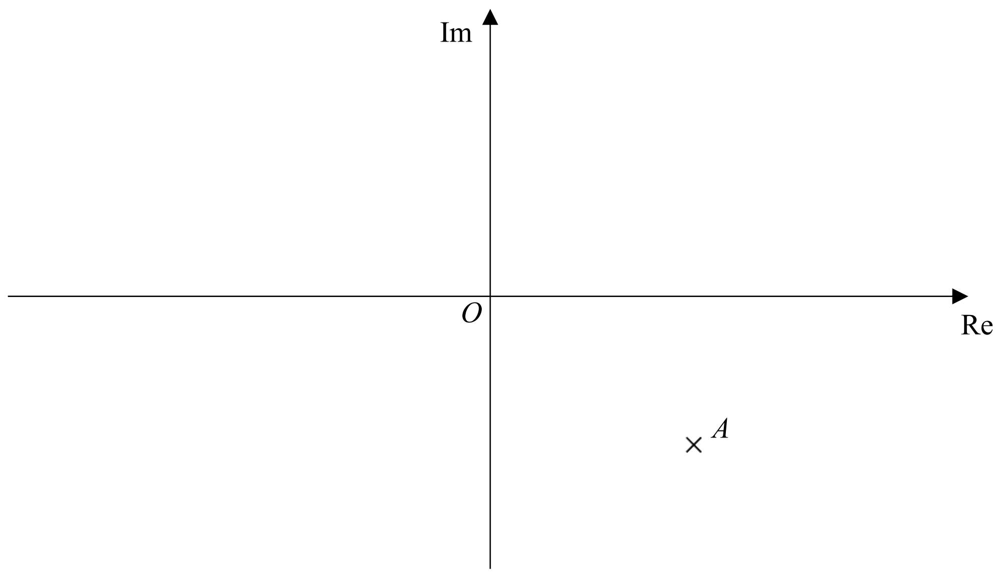
10. **Do not use a calculator in answering this question.**

(a) Find the complex numbers z and w which satisfy the following simultaneous equations.

$$|z| + 5w = 0$$
$$\mathrm{i}z - 4w = -4 + 7\mathrm{i}$$

Give your answers in the form of $a + \mathrm{i}b$, where a and b are real constants. [5]

(b)



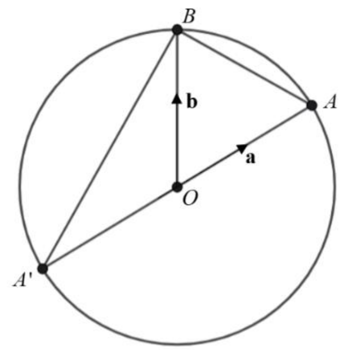
The point A on the Argand diagram represents the complex number u.

(i) On the copy of the Argand diagram in the Printed Answer booklet, plot the point B to represent the complex number $-u$. [1]

The points C and D represent the complex numbers $v - u$ and $(v-u)^*$ respectively, where v is an unknown complex number. It is also given that $\angle CDA = 90^\circ$.

(ii) By using the Argand diagram or otherwise, state the value of $\mathrm{Im}(v)$ and justify your answer. [2]

11. The points A, B and A' lie on a circle with center O such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. It is also given that the line segment AA' is a straight line that passes through O (see diagram).



(i) Using a suitable scalar product, prove that $\measuredangle ABA' = 90^\circ$. [4]

(ii) Express the area of $\Delta ABA'$ in the form $k|\mathbf{a}\times\mathbf{b}|$, where k is a real constant. [2]

12. Welding fumes contain a dangerous amount of particulates. To protect workers in a workshop, an extraction device remove particulates in the air continuously. Let V mg represent the mass of particulates in the air in the workshop at time t min after the extraction device is activated.

Particulates are produced at a constant rate of 0.32 mg/min. The rate at which particulates are removed is proportional to the mass of particulates in the air.
When the mass of particulates in the air in the workshop is 18.75 mg, the mass of particulates in the air increases at a rate of 0.12 mg/min.

(i) Show that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{4}{375}(30 - V)$. [2]

(ii) Solve the differential equation, given that the workshop is initially free of particulates. [5]

(iii) Sketch the graph of V against t. [2]

(iv) It is recommended that the mass of particulates in this workshop should be kept below 32 mg. Comment on whether the extraction device is effective enough to meet the recommendation. [1]

The extraction device becomes less efficient and now removes particulates at **half** its original rate.

(v) Write down a new differential equation to model this. [1]

(vi) The mass of particulates in the air in the workshop is said to reach a steady-state when it remains constant. Find the value of V at steady-state. [1]

End of Paper

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