

Name: _____ () Class: _____



HIGHER MATHEMATICS

Level : Secondary Three

Duration : 1 hour

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **4 out of 7** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 40.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in a loss of marks.

This question paper consists 4 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer 4 out of 7 questions

- 1** **(a)** Factorise $1 - 2ax - (c - a^2)x^2 + acx^3$ completely. [3]
- (b)** A graph with equation $y = f(x)$ undergoes 3 successive transformations as follows:
- I:** A scaling parallel to x -axis with scale factor $\frac{1}{2}$,
- II:** A translation of 3 units in the direction of the positive x -axis,
- III:** A scaling parallel to y -axis with scale factor 2.
- The resulting equation is $y = 2 \ln(2x - 6)$.
- Determine the equation of the original graph $y = f(x)$, showing your workings clearly. [4]
- (c)** Solve the equation $|3x - 5| = |2x + 4|$. [3]
- 2** **(a)** Find the equation of a parabola which is symmetrical about $x = -2$ and passes through points $(-1, -2)$ and $(2, 43)$. [3]
- (b)** A water tank holds 2 m^3 of water when it is full. A hot water tap supplies water at a rate of $x \text{ m}^3$ per minute and a cold water tap supplies water at a rate of $y \text{ m}^3$ per minute. The water tank can be filled in $3\frac{1}{3}$ minutes using both taps. Filling the tank using only the hot water tap will take 4 minutes longer than using only the cold water tap.
- (i)** Form an equation in x and show that it reduces to $10x^2 - 16x + 3 = 0$. [4]
- (ii)** Solve the equation and explain clearly why one of the answers has to be rejected. [3]
- 3** **(a)** Given that a and b are real numbers such that $a + b - c = 4$ and $ab - 4c = 1$. Find the range of possible values of c . [5]
- (b)** Prove the identity $\frac{\cos A - \cos B}{\sin A + \sin B} = \frac{\sin B - \sin A}{\cos A + \cos B}$. [5]

- 4 (i) In the Cartesian plane, circle C_1 with centre $(3, -1)$ passes through a point $(7, 2)$. Find the equation of the circle. [2]
- (ii) Another circle C_2 has equation $(x-11)^2 + (y-15)^2 = r^2$, where r is positive. Find the range of values of r for which the two circles do not intersect. Leave your answer in the exact form. [3]
- (iii) If $r = 13$, find the coordinates of the points where the two circles intersect. [5]
- 5 (a) In triangle ABC , let $AB = c$, $AC = b$, $BC = a$.
If $\sin \angle A : \sin \angle B : \sin \angle C = 2 : 3 : 4$, find the ratio $a : b : c$. [2]
Hence or otherwise, find $\angle C$ in degrees. [3]
- (b) Given that $\log_{40} x = a$ and $\log_{50} x = b$, find $\log_x 2$ and $\log_x 5$ in terms of a and b . [5]
- 6 A function f is defined as $f(x) = ax + b$, $x \in \mathbb{R}$, where a and b are positive constants. Let $f^n(x)$ denote the composite function where f is iterated n times on x . For example, $f^2(x) = f(f(x))$.
- (i) Find $f^3(x)$. [2]
- (ii) Given that $f^3(x) = 64x + 21$, find the values of a and b . [2]
- (iii) With the values of a and b found in (ii), determine $f^n(x)$, leaving your answer in the form $px + q$, where p and q are in terms of n . [3]
- (iv) Another function g is defined as $g(x) = e^x$, $x < 0$. Find, in terms of a and b , the range of composite function fg . [3]
- 7 (a) When the expression $3x^3 + px^2 + qx + 8$ is divided by $x^2 - 3x + 2$, the remainder is $5x + 6$. Find the value of p and of q . [3]
- (b) Find the range of values of x satisfying $x^2 - 9 \geq (x+3)(x^2 - 3x + 1)$. [5]
Hence, solve the inequality $x^2 - 9 \geq (|x|+3)(x^2 - |3x| + 1)$. [2]

End of Paper