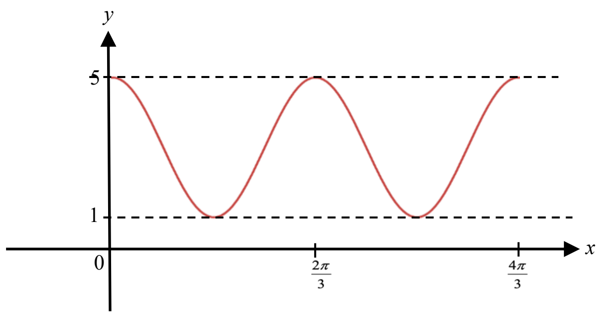
2025 Add Math Prelims Marking Scheme

Qn	Solution	Remarks
1a	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2e^x+1\right)^{-1}$ $=-\frac{2e^x}{\left(2e^x+1\right)^2}$	B1 chain Rule B1 diff^n of Expo
1b	$\int -\frac{2e^x}{\left(2e^x+1\right)^2}\mathrm{d}x=\frac{1}{2e^x+1}\ (+c)$ $\int \frac{e^x}{\left(2e^x+1\right)^2}\mathrm{d}x=\frac{-1}{4e^x+2}+c$	M1 integrate their (a) answer to get $\frac{1}{2e^x+1}$ A1 o.e
		4
2	angle TPR = angle TRP ---(1) ($TP=TR, \Delta$TPR is an isosceles) angle TPQ = angle TSP ---(2) (alternate segment thm.) angle QPR = angle TPR – angle TPQ angle TRP = angle TSP + angle RPS (exterior angle of Δ) angle RPS = angle TRP – angle TSP = angle TPR – angle TPQ = angle QPR $\therefore$ line PR bisects the angle SPQ. (proven)	B1 (isosceles) B1 (tangent chord or alternate segment theorem) B1 (exterior angle of Δ) B1 correct answer
		4
3a	$V=\frac{1}{12}\pi h^3$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{4}\pi h^2$ $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $-10\pi=\frac{1}{4}\pi h^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{40}{h^2}$ cm/s	B1 M1 with correct subst^n A1

3b	$A=\frac{1}{4}\pi h^2$ $\frac{dA}{dh}=\frac{1}{2}\pi h$ $\frac{dA}{dt}=\frac{dA}{dh}\times\frac{dh}{dt}$ $\frac{dA}{dt}=\frac{1}{2}\pi(10)\times-\frac{40}{10^2}$ $\frac{dA}{dt}=-2\pi\text{ cm}^2/\text{s}$ Rate of decreasing = 2π cm^2/s	M1 with correct subst^n A1 $\frac{dA}{dt}$ must be correct
		5
4a	$-2x^2+20x+12$ $=-2\left(x^2-10x\right)+12$ $=-2\left[x^2-10x+\left(\frac{10}{2}\right)^2-\left(\frac{10}{2}\right)^2\right]+12$ $=-2\left[\left(x-5\right)^2-25\right]+12$ $=-2\left(x-5\right)^2+62$	B2 –1 for each error
b	Height = $-2\left(x-5\right)^2+62$ $\left(x-5\right)^2\geq 0$ $-2\left(x-5\right)^2\leq 0$ $-2\left(x-5\right)^2+62\leq 62$ Height $\leq$ 62m Since height is $\leq$ 62m, the maximum height reach by the ball is 62 metres.	B1 Use of $\geq$ B1 Conclusion
c	Height = $-2\left(x-5\right)^2+62$ When x = 10, y = 12 >2 The pellet will not hit the object.	B1
		5
5	Let h be the height of prism $V=\frac{1}{2}\left(1+\sqrt{5}\right)^2\left(\sin 60°\right)h$ $2\sqrt{3}+2\sqrt{15}=\frac{1}{2}\left(1+\sqrt{5}\right)^2\left(\sin 60°\right)h$ $2\sqrt{3}+2\sqrt{15}=\frac{1}{2}\left(6+2\sqrt{5}\right)\frac{\sqrt{3}}{2}h$	B1 Vol of prism o.e $\boxed{\sin 60° \text{ or } \sin\frac{\pi}{3}}$ B1 correct sq of sides

	$h = \dfrac{2\sqrt{3}+2\sqrt{15}}{\frac{1}{2}\left(6+2\sqrt{5}\right)\frac{\sqrt{3}}{2}}$ $= \dfrac{4\sqrt{3}+4\sqrt{15}}{\left(3+\sqrt{5}\right)\sqrt{3}}$ $= \dfrac{4\sqrt{3}+4\sqrt{15}}{3\sqrt{3}+\sqrt{15}} \times \dfrac{3\sqrt{3}-\sqrt{15}}{3\sqrt{3}-\sqrt{15}}$ $= \dfrac{12(3)-4\sqrt{3}\sqrt{15}+12\sqrt{3}\sqrt{15}-60}{27-15}$ $= \dfrac{8\sqrt{45}-24}{12}$ $= -2+2\sqrt{5}\,\text{cm}$	B1 $h = \dfrac{2\sqrt{3}+2\sqrt{15}}{\text{their area}}$ B1 $\times \dfrac{\text{their conjugate}}{\text{their conjugate}}$ M1 expansion of denominator A1
		6
6a	$\left(2-\dfrac{x}{4}\right)^5$ $=(2)^5+\dbinom{5}{1}(2)^4\left(-\dfrac{x}{4}\right)+\dbinom{5}{2}(2)^3\left(-\dfrac{x}{4}\right)^2+\ldots$ $=32-20x+5x^2+\ldots$	B1 B1 B1
b	$\left(4+kx+x^2\right)\left(2-\dfrac{x}{4}\right)^5$ $=\left(4+kx+x^2\right)\left(32-20x+5x^2+\ldots\right)$ Coeff of x $= -80+32k$ Coeff of $x^2 = 52-20k$ $-80+32k-20k+52=-4$ $-28+12k=-4$ $12k=24$ $k=2$	B1 correct coeff x B1 correct coeff x^2 M1 their sum $= -4$ A1
		7
7a	$\sin(A+B)+\sin(A-B)$ $=\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B$ $=2\sin A\cos B$ $k=2$	B1 correct expansion of compound angles B1
b	$P+Q=2A$ $A=\dfrac{P+Q}{2}$ ---(1) $P-Q=2B$ $B=\dfrac{P-Q}{2}$ ---(2) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$	B1 stating of A or B in term of P and Q

	$\sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$	B1 correct subst^{n} of (1) and (2).
c	$\sin 75°+\sin 15°$ $=2\sin\left(\frac{90°}{2}\right)\cos\left(\frac{60°}{2}\right)$ $=2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $=\frac{\sqrt{6}}{2}$	M1 use of (b) or compd $\angle$ B1 Either special angle correct A1
		7
8a	$x-y=0,\ x+3y=20$ $x+3x=20$ $x=5,$ $y=5$ Coord B = (5, 5) $5x-y=4,\ x-y=0,$ $5x-x=4$ $x=1$ $y=1$ Coord A = (1, 1)	M1 Sim Eqn to solve for unknown A1 A1
b	Coord D = (1 – (5 – 2), 1 + 1) = (–2, 2)	M1 correct attempt to find x or y A1
c	Area $=2\times\frac{1}{2}\begin{vmatrix}1 & 5 & 2 & 1\\ 1 & 5 & 6 & 1\end{vmatrix}$ $=5+30+2-5-10-6$ $=16$ units^2	M1 determinant method A1 √
		7
9a	Let h cm be the height of the casing. Base area = x^2 Area of 4 sides = $4xh$ Total material cost = $2x^2 + 4xh$ \$$2x^2$ + 4(\$1)xh = \$600 $h=\frac{600-2x^2}{4x}$ $=\frac{300-x^2}{2x}$	M1 find area of materials/ find cost for rect piece A1 correct equation for h unsimplified/ Correct base area

	Vol of casing, V $= x^2 h$ $= x^2\left(\dfrac{300 - x^2}{2x}\right)$ $= \dfrac{300x - x^3}{2}$ cm^3	B1 AG
b	$\dfrac{dV}{dx} = \dfrac{1}{2}\left(300 - 3x^2\right)$ At stationary value, $\dfrac{dv}{dx} = 0$ $300 - 3x^2 = 0$ $x = 10$ $\dfrac{d^2V}{dx^2} = -3x$ When $x = 10$ $\dfrac{d^2V}{dx^2} < 0$ $\therefore$ Maximum Volume Maximum Volume $= \dfrac{300(10) - (10)^3}{2}$ $= 1000$ cm^3	B1 M1 set to 0 B1 Test and conclusion of nature √ if min A1 correct volume
		7
10a	$a = 2$, $b = 3$, $c = 3$	B1 B1 B1
b		B1 for 2 complete cycles B1 correct maximum and minimum B1 fully correct curve
c	$\cos 3x = d$ $2\cos 3x = 2d$ $2\cos 3x + 3 = 2d + 3$ $y = 2d + 3$ When $y = 1$ $2d + 3 = 1$ $d = -1$ Or min point $\cos 3x = d$ $d = -1$	M1 find eqn of y A1 B1 B1
		8

11a	area of the triangle OBX $= \frac{1}{2}(7.2)(9)\sin\theta$ $= 32.4(\sin\theta)\ \text{cm}^2$	B1
b	Area $= 0.5(12)(9)$ $= 54\ \text{cm}^2$ $54 = 32.4\sin\theta + \frac{1}{2}(7.2)(12)\sin(90° - \theta)$ $108 = 64.8\sin\theta + 86.4\cos\theta$ $4\cos\theta + 3\sin\theta = 5$	B1 M1 use of $\frac{1}{2}(7.2)(12)\sin(90° - \theta)$ B1 $\sin(90° - \theta)$ to $\cos\theta$ AG
c	$4\cos\theta + 3\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{4^2 + 3^2}$ $= 5$ $\tan\alpha = \frac{3}{4}$ $\alpha = 38.9°$	M1 $R = \sqrt{a^2 + b^2}$ A1 M1 $\tan\alpha = \frac{b}{a}$ A1
d	$5\cos(\theta - 36.87°) = 5$ $\theta - 36.87° = 0$ $\theta = 36.9°$	B1
		9
12a	When $y = 0$ $\frac{1}{2} = \frac{2}{(x-2)^2}$ $x = 0$ (n.a) or 4 Coord $A = (4, 0)$ When $y = 7.5$ $8 = \frac{2}{(x-2)^2}$ $x = 1.5$ or 2.5 Coord $B = (2.5, 7.5)$ Coord $C = (1.5, 7.5)$	M1 subst^n to find unknowns A1 A1 A1
12b	Area $= 7.5 + 2\int_0^{1.5} 2(x-2)^{-2} - \frac{1}{2}\text{d}y$ $= 7.5 + [-4(x-2)^{-1} - x]_0^{1.5}$ $= 7.5 + \left(-4(1.5-2)^{-1} - 1.5\right) - \left(-4(0-2)^{-1}\right)$ $=12$ units	M1 sum of rectangle and 2 symmetrical parts B1 correct integration to linear x M1 -2 or -4 $(x-2)^{-1}$ M1 Substn of correct limits A1
		9

13a	$$\frac{3x^2+3x+3}{(x+2)(x-1)^2}=\frac{A}{x+2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$ $3x^2+3x+3=A(x-1)^2+B\left(x+2\right)\left(x-1\right)+C\left(x+2\right)$ When $x = 1$, $C = 3$ When $x = -2$, $A = 1$ $3 = A + B$ $B = 2$ $$\frac{3x^2+3x+3}{(x+2)(x-1)^2}=\frac{1}{x+2}+\frac{2}{x-1}+\frac{3}{(x-1)^2}$$	M1 realising the form (C can be linear) M1 remove denominator A1 A1 A1 –1 mark if no final statement
b	$$\int_3^4 \frac{3x^2+3x+3}{\left(x+2\right)\left(x-1\right)^2}\,\mathrm{d}x$$ $$=\int_3^4\left(\frac{1}{x+2}+\frac{2}{x-1}+\frac{3}{(x-1)^2}\right)\mathrm{d}x$$ $$=\int_3^4\left(\frac{1}{x+2}+2\left(\frac{1}{x-1}\right)+3(x-1)^{-2}\right)\mathrm{d}x$$ $$=\left[\ln\left(x+2\right)+2\ln\left(x-1\right)-3(x-1)^{-1}\right]_3^4$$ $$=\left(\ln 6+2\ln 3-1\right)-\left(\ln 5+2\ln 2-1.5\right)$$ $$=1.49$$	M1 integrate to ln A1 2 ln $(x-1)$ B1 $-3(x-1)^{-1}$ M1 substn of limits A1
c	$$\int_{-5}^1 \frac{3x^2+3x+3}{\left(x+2\right)\left(x-1\right)^2}\,\mathrm{d}x$$ When $x = 1$, the <u>denominator of the integral is 0</u> and thus $\frac{3x^2+3x+3}{\left(x+2\right)\left(x-1\right)^2}$ <u>does not exist. As such it is not possible to find the value.</u> or <u>$=\left(\ln 3+2\ln(0-1)\right)-\left(\ln(-3)+2\ln(-6)-1.5\right)$</u> <u>Since ln 0 and ln(–3) and ln(–6) do not exist, it is not possible to find the value.</u>	B1 B1 B1 B1
		12