Name: _____ ( ) Class: _____

華僑中學
HWA CHONG INSTITUTION
Paper D

INTEGRATED MATHEMATICS

Level : Secondary Three (IP)

Duration : 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions on foolscap paper except

Question 15 on graph paper provided and

Question 16 in the space provided on question paper.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted**.

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 9 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Without the use of calculator, evaluate $\frac{1}{1+\sqrt{3}}+\frac{1}{1-\sqrt{3}}$. [2]

(b) Simplify $\dfrac{\dfrac{2}{x^2+3x+9}}{x-3}-\dfrac{x-3}{x^3-27}$. [3]

(c) A quadratic equation has roots $20\pm\sqrt{17}$. Find the quadratic equation in the form $ax^2+bx+c=0$. [3]

2 **(a)** Solve the simultaneous equations $2(x-y)=x+y-1=2x^2-11y^2$. [5]

(b) Find the range of values of *m* for which the straight line $y=x-m$ meets the curve $y=mx^2+7x$. [3]

3 The function g(x) is defined by $\mathrm{g}: x\mapsto x^2+2x-8$ for $x\geq a$.

(i) State the smallest value of *a* for g^{-1} to exist. [2]

(ii) Sketch the graph of $y=\mathrm{g}^{-1}(x)$ with the value of *a* in (i). [2]

(iii) By completing square, find g^{-1} in similar form, stating its domain. [2]

4 The polynomial $\mathrm{p}(x)=x^4+ax^3+bx^2-2x-4$, where a and b are constants, has factors $x-1$ and $x+2$.

(i) Show that $a = 3$ and $b = 2$. [3]

(ii) Find the third real factor of $\mathrm{p}(x)$. [2]

(iii) Show that this third factor is positive for all real values of *x*. [2]

(iv) Find the range of values of *x* for which $\mathrm{p}(x)$ is positive. [2]

5 **(a)** Solve $\log_5(x-1)+\log_5(x-2)=2\log_5\sqrt{6}$. [4]

(b) Given that $\log_a x+\log_a 3x=\log_a 6+2$, express *x* in terms of *a*. [3]

6 **(a)** Solve $|2-0.5x|=3+x$. [4]

(b) By means of suitable substitution or otherwise, solve the equation $8^x-3\left(2^{2x}\right)=2^x-3$. [5]

7 **(i)** Sketch the graph $y = |1 - 2\cos 2x|$ for $0^{o} \leq x \leq 180^{o}$. [3]

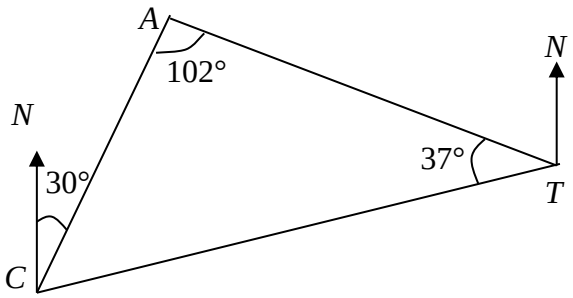
(ii) On your answer to **(i)**, add the graph of $y = 0.5$. [1]

(iii) Hence, state the number of solutions for the equation $2|1 - 2\cos 2x| = 1$. [1]

8 **(a)** Prove the identity $\sec^2 x + \operatorname{cosec}^2 x \equiv \sec^2 x \operatorname{cosec}^2 x$. [3]

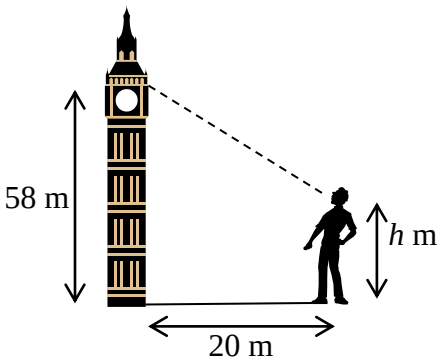
(b) Solve the equation $3\sin x \tan x + 8 = 0$, for values of x between $0°$ and $360°$. [4]

9 The diagram, not drawn to scale, shows a school's clock tower (T), canteen (C) and the administration block (A). Angle $TAC = 102°$, angle $CTA = 37°$ and the bearing of A from C is $030°$.



(i) Find the bearing of T from A, [2]

(ii) Find the distance of AC correct to 1 decimal place, given that $AT = 150$ m. [2]



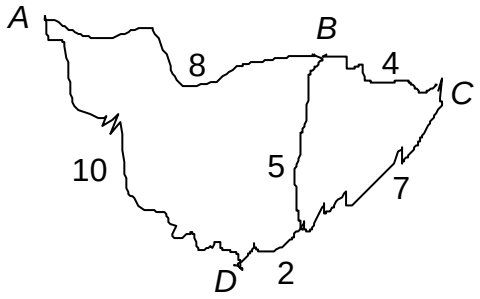
(iii) The diagram above shows a student observing the clock. He is at a distance of 20m from the base of the clock tower and his eye level is h m above the ground. Given that the height of the clock is 58 m and the angle of elevation is $70.5°$, calculate the value of h. [2]

10 Given that $\sin x = \frac{1}{p}$ and that $180° \le x \le 270°$, find the value of:

(i) $\cos(-x)$, [2]

(ii) $\sec(90^o - x)$. [2]

11 The map shows the distance in kilometers between four secondary schools, *A*, *B*, *C* and *D* in Singapore.



(i) Copy and complete the following distance matrix **P** which shows the shortest distance in kilometers between any two schools.

$$\mathbf{P} = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 0 & 8 & & 10 \\ & 0 & & \\ & & 0 & \\ 10 & & & 0 \end{pmatrix} \end{array}$$ [1]

(ii) **Write down** a suitable column matrix which when multiply to **P** will give the total distance of each school to the other schools. [1]

(iii) A combined student councilor activity is to be held in one of the schools so that the sum of its distances to the other schools is a minimum. In which school should the activity be held? Justify your choice using matrix operation. [2]

(iv) Suppose the number of student councilors involved in the combined activity from School *A*, *B*, *C* and *D* are 80, 140, 120 and 90 respectively. Using one matrix operation, calculate which school should the combined activity be held so that councilor movement is kept to a minimum. [2]

12 An experimental function is given by $y = -0.43x^2 + 1.712x + 5.183$.

The list below shows a few methods that can be used to obtain the roots to the function.

- graphing
- factoring
- using the general formula
- checking the discriminant.

Without solving, briefly explain how you would use one of the methods above

(i) to find the root/s to the function, [2]

(ii) to do a quick check whether the function has any *x*-intercept/s. [2]

13 **<u>Answer to this question with accurate drawing will not be accepted.</u>**

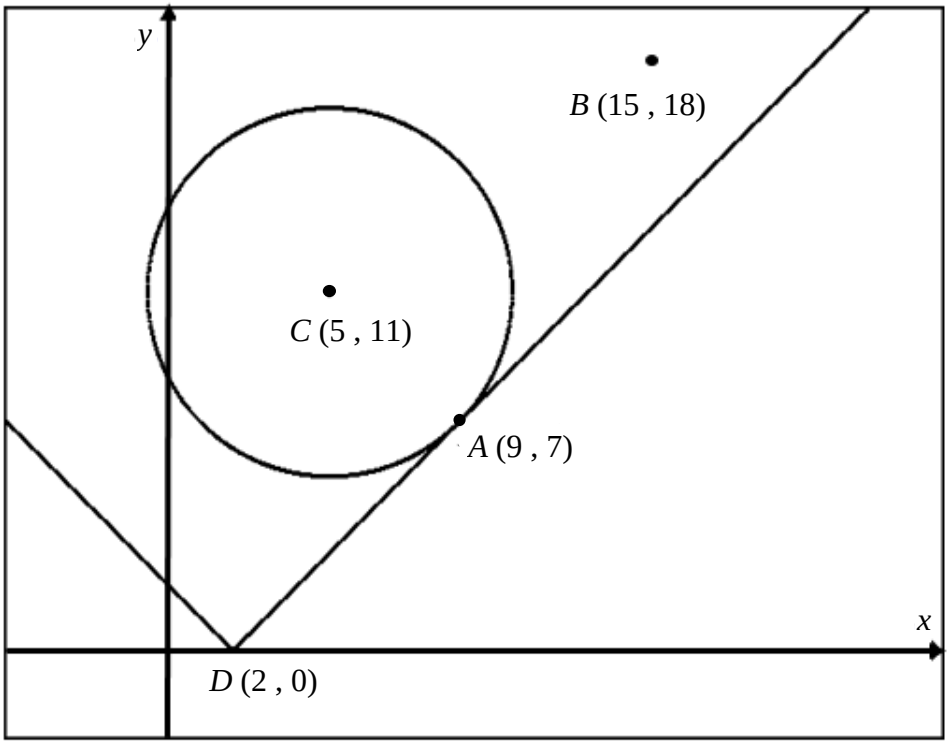
The diagram below shows a circle with centre *C* (5 , 11) and a modulus function. *A* is a point on both the circle and the modulus function. *D* is the point where the modulus function touches the *x*-axis and the coordinates of *B* is (15, 18).

(i) Find the equation of the modulus function. [2]

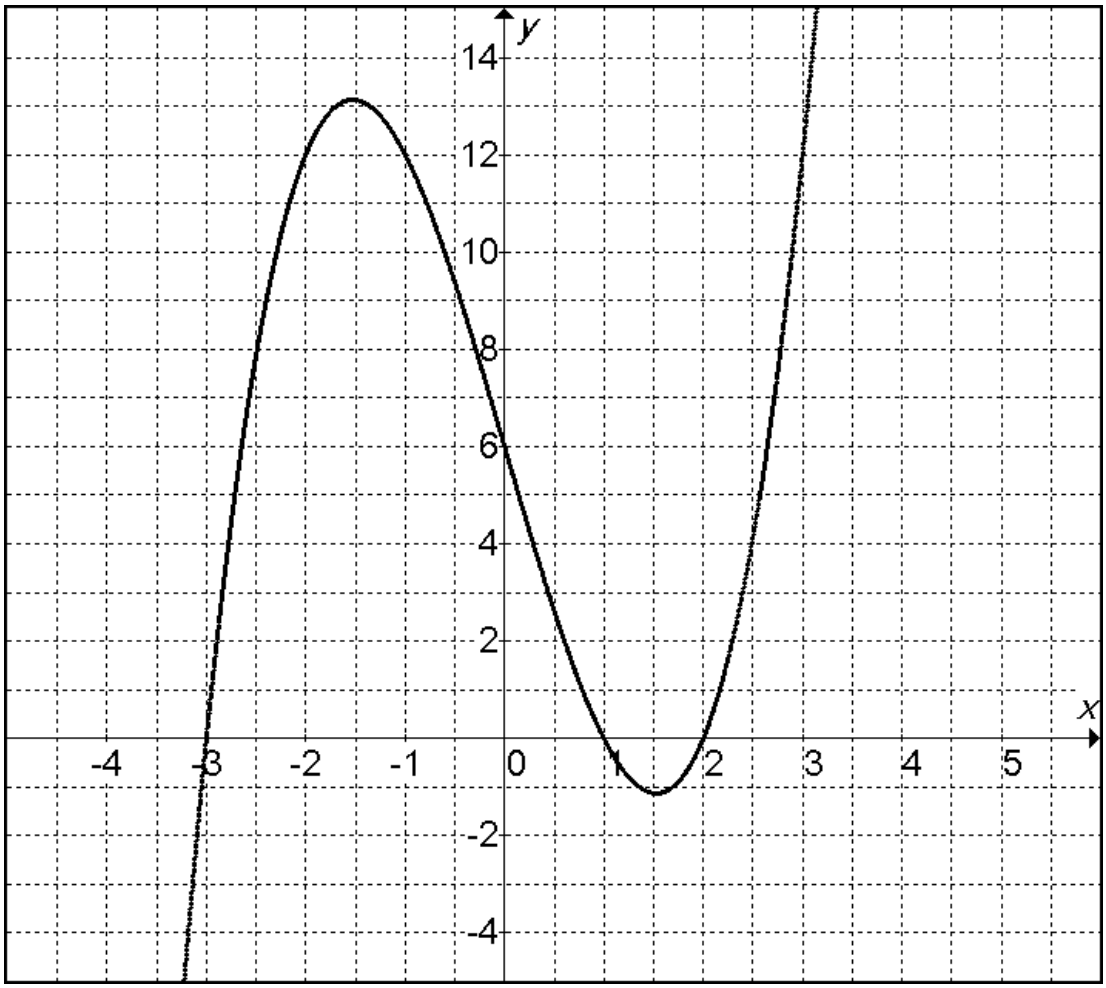
(ii) Find the equation of the circle. [2]

(iii) Find the area of the triangle *CAB*. [2]

(iv) Hence, calculate $\angle CAB$. [3]



14 State the equation represented by the graph below. [1]

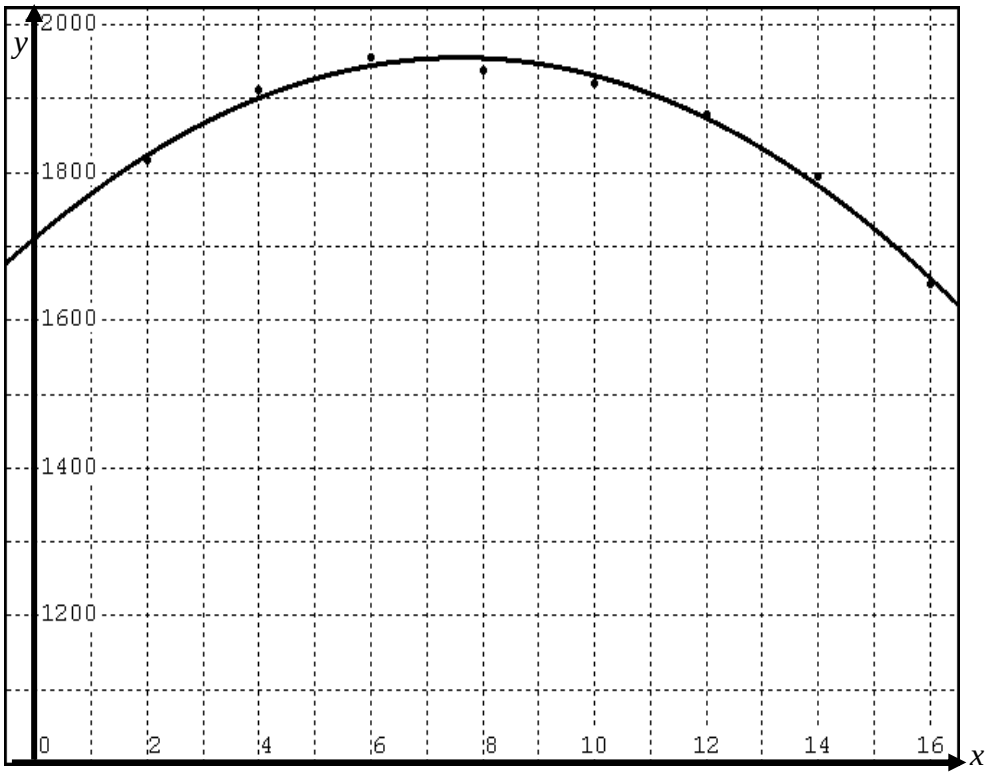


15 Answer the whole of this question on the graph paper provided.

The following table shows the student enrolment in a school over a period of 16 years.

Numbers of years after year 2000, x	Student Enrolment, y
2	1816
4	1911
6	1956
8	1939
10	1921
12	1879
14	1794
16	1650

Below is the graph of student enrolment, y against numbers of years after year 2000, x.



By observing the trend of student enrolment, the school administration decided to use a quadratic function to model the data.

(i) If the modelled quadratic equation is $y = ax^2 + bx + 1712$ where a and b are constants, express the equation in a form that will yield a straight line graph. [1]

(ii) Using a suitable scale, draw the straight line on the graph paper. [3]

(iii) Use your graph to estimate the value of a and of b. [2]

(iv) Use your graph to predict the student enrolment in 2017. [1]

DETACH this page to be submitted together with your solutions.

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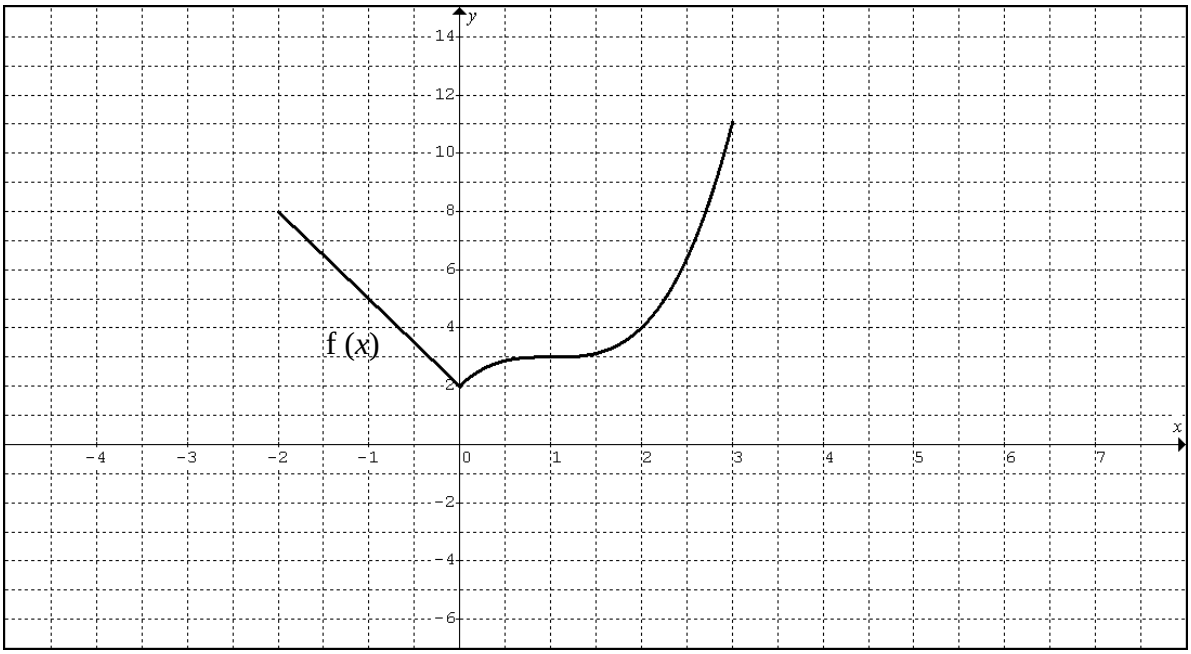
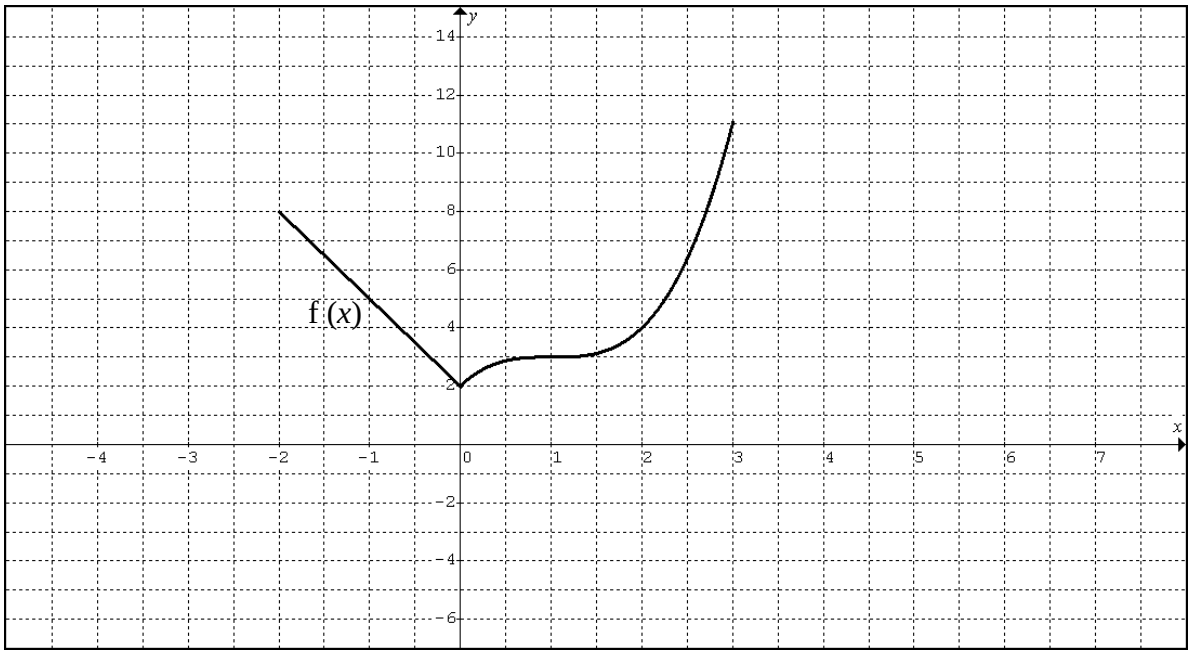
16 The diagram shows the function $y = f(x)$ over the domains $-2 \le x \le 3$. The function is undefined for all other values of x.

(i) In the space provided below, sketch the function given by $f(x - 2) + 3$. [2]



(ii) In the space provided below, sketch the function given by $f(-2x)$. [2]



End of Paper