Revision Timed Practice for DRV, Binomial, Geometric, Poisson

1 Christopher plays a computer game with Yi Han, where they each shoot at a target. For each shot, the probability that Christopher and Yi Han hitting the target is 0.25 and 0.35 respectively. This is independent of all other shots for both players and they each take turns to shoot with Christopher starting first. Find the

(i) most likely number of shots, out of 20 shots attempted, that hit the target for Christopher, [2]

(ii) probability that, out of 20 shots attempted, Yi Han hits the target at least five times given that he hits the target only at most ten times, [3]

(iii) probability that Christopher and Yi Han take the same number of shots to hit the target and [3]

(iv) probability that Christopher needed more than twice the number of shots than Yi Han to hit the target. [3]

To make things more exciting, Christopher and Yi Han decides to place a wager on the game. If Christopher needed more than twice the number of shots than Yi Han to hit the target, Christopher pays Yi Han \$10. If Christopher and Yi Han used the same number of shots Christopher receives \$5 from Yi Han. If neither of the two cases, Yi Han win \$2 from Christopher. Find the expected winning of Christopher after 10 games. [2]

Solution:

(i) Let X be the random variable denoting the number of shots hitting the target out of 20 for Christopher.

$X \sim \mathrm{B}(20, 0.25)$ **B1: Define r.v and state distribution with parameters**

X	Y1
0	.00317
1	.02114
2	.06695
3	.1339
4	.18969
5	.20233
6	.16861

Most likely number of shots is 5. **B1: Show table + final answer**

(ii) Let Y be the r.v. denoting the number of shots hitting the target out of 20 for Yi Han.

$Y \sim \mathrm{B}(20, 0.35)$ **B1: Define r.v and state distribution with parameters**

$$\mathrm{P}(Y \geq 5 \mid Y \leq 10) = \frac{\mathrm{P}(5 \leq Y \leq 10)}{\mathrm{P}(Y \leq 10)} = \frac{\mathrm{P}(Y \leq 10) - \mathrm{P}(Y \leq 4)}{\mathrm{P}(Y \leq 10)} = 0.875 \text{ (3 sf)}$$

M1: Conditional Probability
A1: Final Answer

(iii) Let X_1 and Y_1 be the number of shots up to and including the first shot that hits target needed by Christopher and Yi Han respectively.

P(same number of shots each to hit target)

$= \mathrm{P}(X_1 = Y_1 = 1) + \mathrm{P}(X_1 = Y_1 = 2) + \mathrm{P}(X_1 = Y_1 = 3) + \ldots$

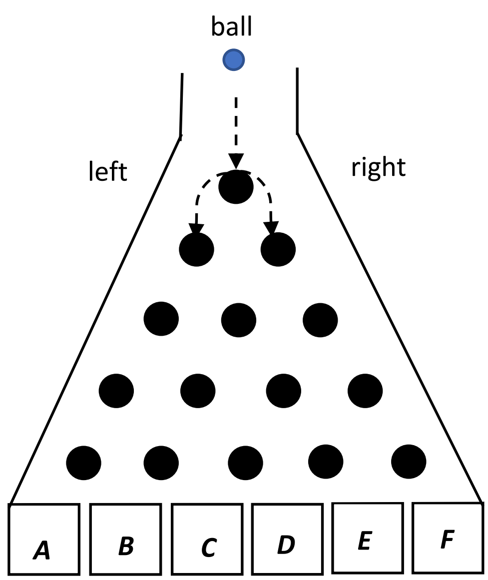
$= (0.25\times0.35) + (0.75 \times 0.65)(0.25 \times 0.35) + (0.75 \times 0.65)^2 (0.25 \times 0.35) + \ldots$

M1: Correct sequences of probabilities with values substituted in

$= \dfrac{0.0875}{1-0.4875}$ **M1: Apply Sum to infinity for GP formula**

$= \dfrac{7}{41}$ or 0.171 **A1: Final Answer**

	(iv) Probability required $= \mathrm{P}(X_1 > 2Y_1)$ $= \mathrm{P}(Y_1 = 1, X_1 > 2) + \mathrm{P}(Y_1 = 2, X_1 > 4) + \mathrm{P}(Y_1 = 3, X_1 > 6) + \ldots$ M1 $= \sum_{r=1}^{\infty} \mathrm{P}(Y_1 = r)\mathrm{P}(X_1 > 2r)$ $= \sum_{r=1}^{\infty} (0.65^{r-1} 0.35) 0.75^{2r}$ $= \frac{0.35}{0.65} \sum_{r=1}^{\infty} \left((0.65)(0.75^2)\right)^r$ M1 $= \frac{0.196875}{1 - 0.365625}$ $= 0.31034 \text{ or } \frac{9}{29}$ A1 Expected winning of Christopher $= 10\left[5\left(\frac{7}{41}\right) - 10\left(\frac{9}{29}\right) - 2\left(1 - \frac{7}{41} - \frac{9}{29}\right)\right] = -\$32.88$ **M1 A1**

2	In the game of Plinko at a funfair, a ball is dropped down a chute riddled with pegs. When it hits a peg, it has a chance of falling to the left or to the right. It then proceeds down to another row of pegs, hitting one of them only and falling left or right to the next row. The game will stop when the ball landed in one of the slots at the bottom of the chute. The diagram below shows a plinko board with 5 rows and 6 slots; *A*, *B*, *C*, *D*, *E* and *F*.  Given that each peg in the plinko has been "fixed" such that the probability of the ball falling to the left after hitting each peg is 0.7 and the probability of the ball falling to the right after hitting each peg is 0.3. **(i)** Show that the probability of a ball landing in slot *D* is 0.1323. [1]

(ii) Balls are dropped in succession until one ball lands in slot D. The probability that at least n balls are needed is at least 0.5. Find the largest value of n. [3]

Suppose a small gift and a mystery gift will be won if a ball landed in slot E and F respectively. On average, 2.835 small gifts and 0.243 mystery gifts were won per day at a funfair. Let S and M be the number of small gifts and mystery gifts, won per day, respectively.

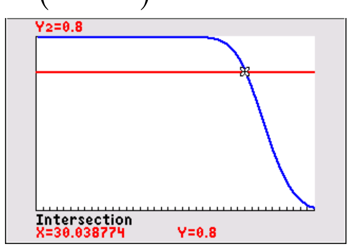
(iii) State in context, two assumptions needed for S and M to be well modelled by Poisson distributions. [2]

Assume now that S and M follow independent Poisson distributions,

(iv) find the largest integer n such that the probability of total number of gifts won in n days is more than 100 is at most 0.2. [3]

(v) find the probability that in a 7-day period, there are at least 15 small gifts won given that the total number gifts won is less than 18. [3]

Solution:

12m	**2020/RI/MYE/9**	
(i)	For the ball to land in slot D it must fall to the left 2 times and fall to the right 3 times in any order. Let X denotes the number of times a ball falls to the left out of 5 hits on the pegs. $X \sim \mathrm{B}(5, 0.7)$ $\mathrm{P}(X=2) = {}^5C_2 \times (0.7)^2 \times (0.3)^3 = 0.1323$	B1: Probability expression to show given result.
(ii)	Let W denotes the number of balls needed to land in slot D for the first time. $W \sim \mathrm{Geo}(0.1323)$ $\mathrm{P}(W \geq n) = 1 - \mathrm{P}(W \leq n-1)$ $= 1 - \left(1 - (0.8677)^{n-1}\right)$ $= 0.8677^{n-1}$ Therefore $0.8677^{n-1} \geq 0.5$ $n \leq 5.884$ Largest n is 5.	M1 M1 A1
(iii)	Assumptions: 1. The small gifts won are independent of one another and the mystery gifts won are independent of one another. 2. The average rate of gifts won is assumed to be constant per day.	B1 B1
(iv)	$S \sim \mathrm{Po}(2.835)$, $M \sim \mathrm{Po}(0.243)$ Let $T(n)$ denotes the total number of gifts won in n days. Then $T(n) \sim \mathrm{Po}(2.835n + 0.243n)$ i.e. $T(n) \sim \mathrm{Po}(3.078n)$ So, $\mathrm{P}(T(n) > 100) \leq 0.2$ $1 - \mathrm{P}(T(n) \leq 100) \leq 0.2$ $\mathrm{P}(T(n) \leq 100) \geq 0.8$  By using GC, $0 \leq n \leq 30.0388$ Thus, the largest value of n is 30.	B1: Distribution M1 A1

(v)	Let $S(7)$ and $M(7)$ be the number of small gifts and mystery gifts won respectively, in a 7-day period. Then, $S(7) \sim \text{Po}(19.845)$, $M(7) \sim \text{Po}(1.701)$ and $T(7) \sim \text{Po}\left(21.546\right)$. Required probability $= \text{P}\left(S_7 \geq 15 \mid T_7 < 18\right)$ $= \dfrac{\text{P}\left(S_7 \geq 15 \cap T_7 \leq 17\right)}{\text{P}\left(T_7 \leq 17\right)}$ $= \dfrac{\text{P}\left(S_7 = 15\right)\text{P}\left(M_7 \leq 2\right) + \text{P}\left(S_7 = 16\right)\text{P}\left(M_7 \leq 1\right) + \text{P}\left(S_7 = 17\right)\text{P}\left(M_7 = 0\right)}{\text{P}\left(T_7 \leq 17\right)}$ $= 0.209595748 + 0.169289797 + 0.0731658431$ $= 0.4520513 = 0.452$ (3.sf)	M1 for 1 correct case. M2 for all correct cases (If all 3 cases are wrong but stated correct distributions and conditional probability result, award 1 mark) A1