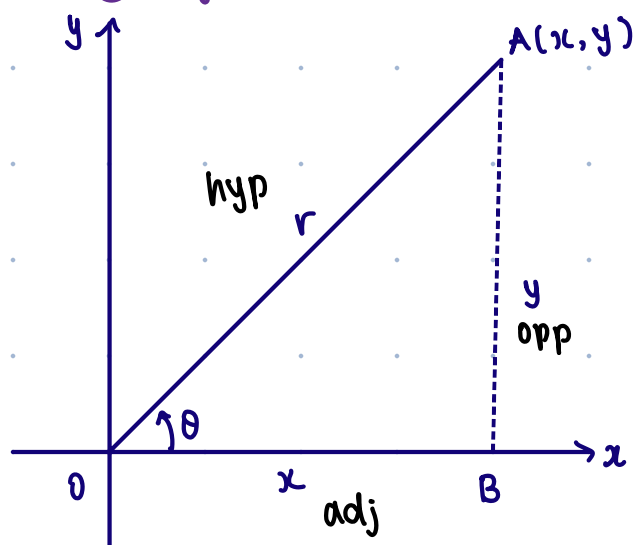


Chapter 9 : Trigonometric Functions & Graphs .

① Trigonometric Ratios of Angle θ



θ : tita

$$\star \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cotan \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

BASIC TRIGO RATIOS

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{r}{y}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}$$

$$\cotan \theta = \frac{1}{\tan \theta} = \frac{x}{y}$$

corresponding Reciprocals.

Recap :

TOA CAH SOH

$$\tan \theta = \frac{o}{a}$$

$$\cos \theta = \frac{a}{h}$$

$$\sin \theta = \frac{o}{h}$$

② Trigonometric Ratios for Special angles .

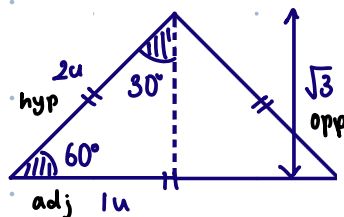
	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

↳ Special angles :

$$180^\circ = \pi \text{ rad} \quad 360^\circ = 2\pi \text{ rad}$$

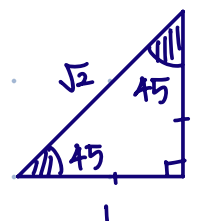
$$90^\circ = \frac{\pi}{2} \text{ rad} \quad 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$45^\circ = \frac{\pi}{4} \text{ rad} \quad 80^\circ = \frac{\pi}{6} \text{ rad}$$



$$h^2 = 2^2 - 1^2$$

$$= \sqrt{3}$$



③ Useful trigonometric Relationships .

#1 Negative angles . #2 Complementary angles . #3 Supplementary angles .

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sin(90^\circ - \theta) = \cos \theta$$

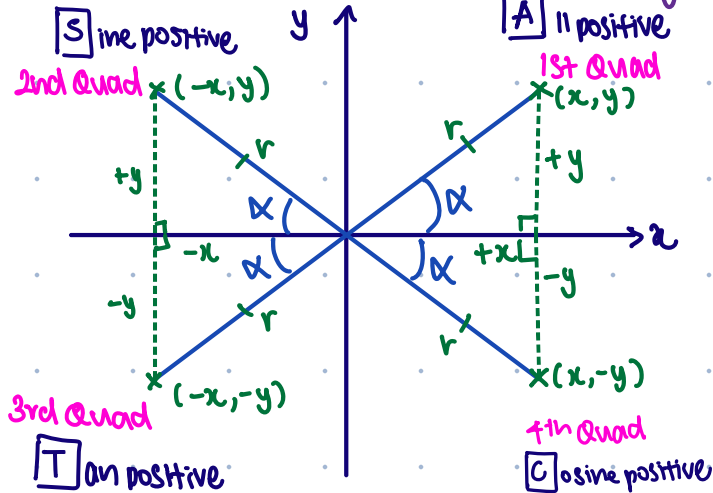
$$\tan(90^\circ - \theta) = \frac{1}{\tan \theta}$$

$$\cos(180^\circ - \theta) = -\cos \theta$$

$$\sin(180^\circ - \theta) = \sin \theta$$

α = reference angle, alpha.

④ Trigonometric Functions of General Angles (ASTC).



1st Quad: All positive

$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \tan \theta = \frac{y}{x}$$

2nd Quad: Sine positive

$$\sin \theta = \frac{y}{r}, \cos \theta = -\frac{x}{r}, \tan \theta = -\frac{y}{x}$$

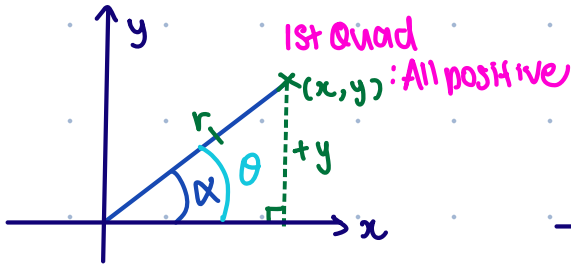
3rd Quad: Tangent positive

$$\sin \theta = -\frac{y}{r}, \cos \theta = -\frac{x}{r}, \tan \theta = \frac{y}{x}$$

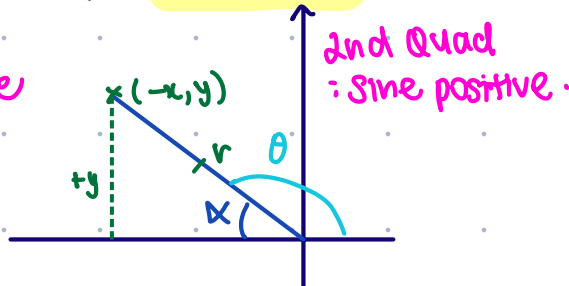
4th Quad: Cosine positive

$$\sin \theta = -\frac{y}{r}, \cos \theta = \frac{x}{r}, \tan \theta = -\frac{y}{x}$$

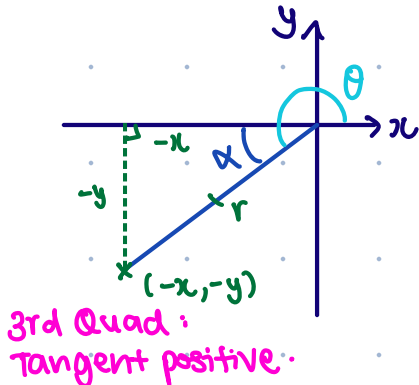
↳ Using the reference angle (α) to find the θ .



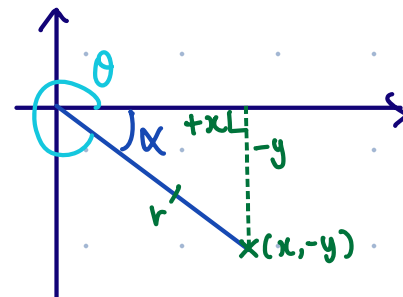
If θ is in 1st Quad, $\theta = \alpha$.



If θ is in 2nd Quad, $\theta = 180^\circ - \alpha$.



If θ is in 3rd Quad, $\theta = 180 + \alpha$.

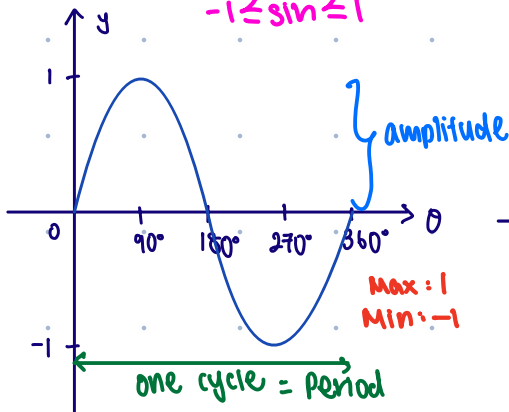


If θ is in 4th Quad, $\theta = 360^\circ - \alpha$.

⑤ Graph of Trigonometric functions.

#1 $y = \sin \theta$

Range: $-1 \leq \sin \leq 1$

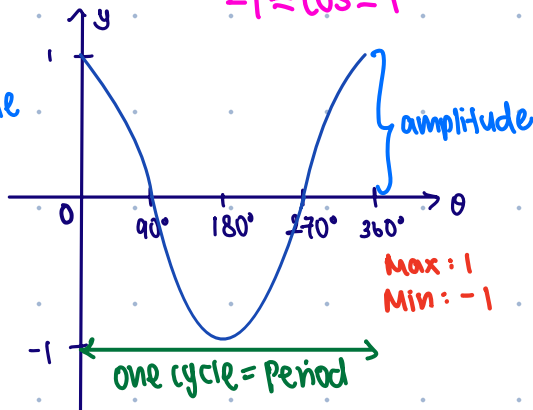


Period = 360° or 2π

Amplitude = 1

#2 $y = \cos \theta$

Range: $-1 \leq \cos \leq 1$

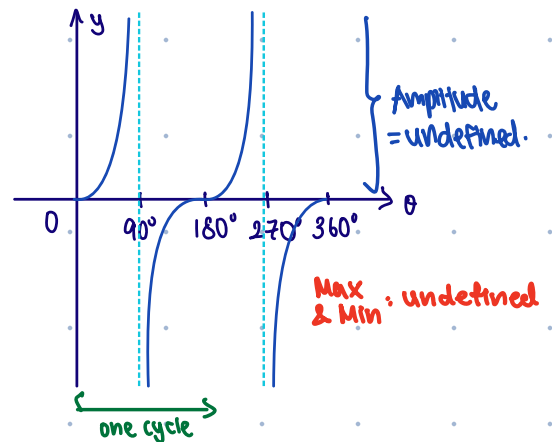


Period = 360° or 2π

Amplitude = 1

#3 $y = \tan \theta$

Range: can be any real number.



Period = 180° or π

Amplitude = undefined

★ Finding Maximum & Minimum values, Amplitude, Vertical shift. ★

#1 Maximum & Minimum values.

- Look at graph & find the highest point (Max). and the lowest point (Min)

#3 Vertical shift (Midline)

$$\text{Vertical shift} = \frac{\text{Max} + \text{Min}}{2}$$

- Represents the vertical distance by which the graph moved up/down from its original position.

#2 Amplitude

$$\text{Amplitude} = \frac{\text{Max} - \text{Min}}{2}$$

- Represents the height of the 'wave'.