ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS 9758/01

Paper 1
QUESTION PAPER

26 August 2025
3 hr

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 **(a)** Without using a calculator, solve the inequality $\frac{x}{x+2} \geq \frac{2}{2-x}$. [4]

(b) Hence, solve the inequality $\frac{|x|}{|x|+2} \geq \frac{2}{2-|x|}$. [2]

2 The graph of $y = \frac{ax^2+bx+c}{x+d}$ has a vertical asymptote with equation $x=-1$ and an oblique asymptote with equation $y=2x+2$. It is given that $c \neq 2$.

(a) Write down the value of d and show that $a=2$ and $b=4$. [3]

(b) Hence find the range of values of c if the graph has no stationary points. [2]

3 It is given that $y = x^{xy}$, where $x>0$, $y>0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2(1+\ln x)}{1-\ln y}$. [3]

(b) Hence find the coordinates of the point on the curve $y = x^{xy}$ whose tangent is parallel to the y-axis. [2]

4 The region R is bounded by the curve with equation $y = x\ln x$, the lines $x=\mathrm{e}$, $x=\mathrm{e}^2$ and the x-axis. Find the exact volume of the solid formed when R is rotated about the x-axis by 2π radians. Give your answer in the form $\frac{\pi\mathrm{e}^3}{27}\left(a\mathrm{e}^3+b\right)$, where a and b are integers to be found. [6]

5 Relative to the origin O, the points A and B have non-zero and non-parallel position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The plane p has equation $\mathbf{r}\cdot\mathbf{n}=0$.

(a) Given that $\mathbf{a}\cdot\mathbf{n}=\mathbf{b}\cdot\mathbf{n}\neq 0$, show that $\overrightarrow{AB}$ is perpendicular to $\mathbf{n}$. Hence, describe the geometrical relationship between $\overrightarrow{AB}$ and the plane p. [2]

(b) Write down the equation of a line parallel to $\overrightarrow{AB}$ that is contained in the plane p. [1]

(c) The point F is the foot of perpendicular from a point C with position vector $\mathbf{c}$ to the plane p. Find the position vector of point F, giving your answer in terms of $\mathbf{c}$ and $\mathbf{n}$. [3]

6

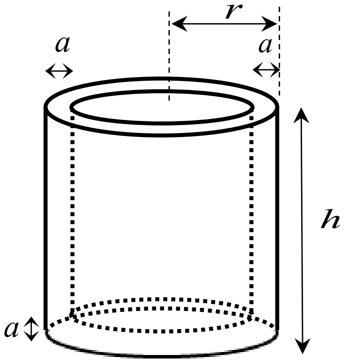


Figure 1

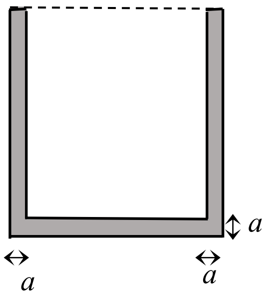


Figure 2

Figure 1 shows an open cylindrical tank. Figure 2 shows the cross-sectional view of the tank. The external radius of the tank is r cm, the external height is h cm and the tank is made with a material of thickness a cm throughout. The internal volume of the tank is fixed at 1000π cm^3.

(a) Show that the volume of the material needed to make the cylindrical tank is given by

$V = k\pi\left[\dfrac{r^2}{(r-a)^2} - 1\right] + \pi r^2 a$, where k is a constant to be determined. [3]

(b) Find, in terms of a, the value of r that minimises V. (You need not show that your answer gives a minimum.) [3]

7 A freshly brewed cup of tea, initially at a temperature of 85°C, is left out to cool in a room with a room temperature of 25°C. After 20 minutes, the temperature of the tea is 55°C.

It is known that the rate of change of the temperature of the tea is proportional to the difference between the temperature of the tea and the room temperature.

(a) Given that θ is the temperature of the tea t minutes after the tea is left out to cool, show that $\theta = 25 + 60\mathrm{e}^{-kt}$, where k is a constant to be determined. [5]

(b) Sketch the graph of θ against t. [2]

8 The curve with equation $y=\mathrm{f}(x)$ for $x \le b$, where $\mathrm{f}(x)$ is a **quadratic** function, intersects the x-axis at the points $(a,0)$ and $(b,0)$, and the y-axis at the point $(0,c)$. The graph of $y=\mathrm{f}(x)$ is shown in Figure 1. The scales on the x- and y-axes are the same.

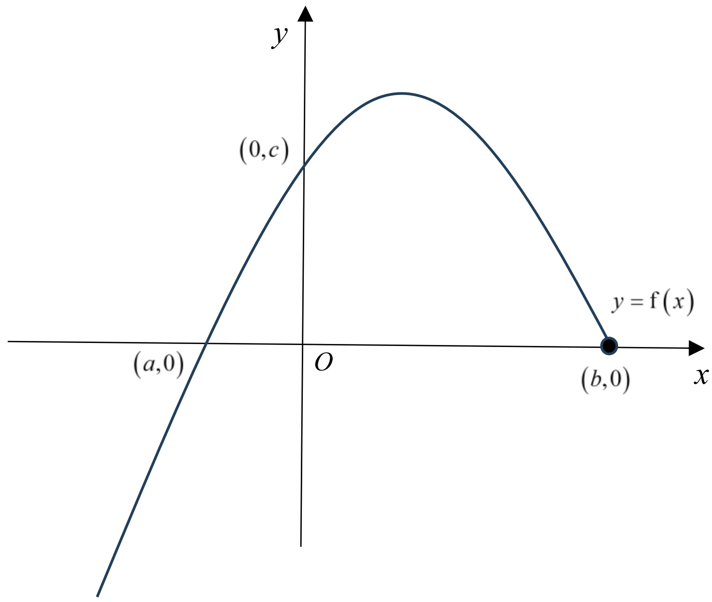


Figure 1

(a) **(i)** Sketch the graph of $y=\mathrm{f}(|x|)$, labelling the points where the graph intersects or touches the axes. [2]

(ii) Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$, labelling the points where the graph intersects or touches the axes, as well as the equations of any asymptotes. [2]

(iii) Describe fully a sequence of transformations which transforms the graph of $y=\mathrm{f}(2x+1)$ onto the graph of $y=\mathrm{f}(x)$. [2]

(b) The function $y=\mathrm{g}(x)$ is such that $\mathrm{g}(x)=\mathrm{f}(x)$ for $x \le k$, and $y=\mathrm{g}^{-1}(x)$ exists.

(i) Find the largest possible value of k in terms of a and b. [1]

(ii) Using the value of k found in **(b)(i)**, sketch the graph of $y=\mathrm{g}^{-1}(x)$ on Figure 1 in the Answer Booklet, labelling the intersections with the axes. Write down the range of g^{-1} in terms of a and b. [2]

(iii) Explain why the solution to $\mathrm{g}^{-1}(x)=x$ satisfies the equation $\mathrm{g}^{-1}(x)=\mathrm{g}(x)$.

[1]

9 The planes π_1 and π_2 have equations $3x + c(y+z) - 2 = 0$ and $\mathbf{r} = (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + s(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - \mathbf{k})$ respectively, where c is a constant, and s and t are parameters. The point $A(1,3,-2)$ lies in both planes.

(a) Show that $c = -1$. [1]

(b) Show that the vector equation of the line of intersection of π_1 and π_2, line l, is given by $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \alpha(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$, where α is a parameter. [3]

(c) Find the position vectors of the points on the line l which are a distance of $3\sqrt{2}$ from the point $B(2,-3,7)$. [4]

(d) Find the equation of the plane π_3 which is parallel to π_2 and contains the point B. Hence show that the distance between the planes π_2 and π_3 is $\dfrac{20}{3\sqrt{3}}$. [3]

10 **Do not use a calculator in answering this question.**

(a) One of the roots of the equation $\omega^4 - 2\omega^3 + 10\omega^2 + p\omega + q = 0$, where p and q are real, is $2 + 3\mathrm{i}$. Find the values of p and q and the other roots of the equation. [6]

(b) The complex numbers u and v are such that $u = -\sqrt{2} + \mathrm{i}\sqrt{2}$, and $|v| = 3$ and $\arg v = \theta$, where $0 < \theta < \dfrac{\pi}{4}$. The points A, B and C represents u, v and $u + v$ respectively on an Argand diagram.

(i) Find the modulus and argument of u. [2]

(ii) Sketch the points A, B and C on an Argand diagram. [2]

(iii) By finding the angle OAC in terms of θ or otherwise, show that $|u+v|^2 = a + b\cos(\theta + K)$, where a, b and K are constants to be determined. [3]

11 The curve C is defined by the parametric equations $x = at^2$, $y = \frac{a}{t}$, $t \neq 0$, where a is a positive constant.

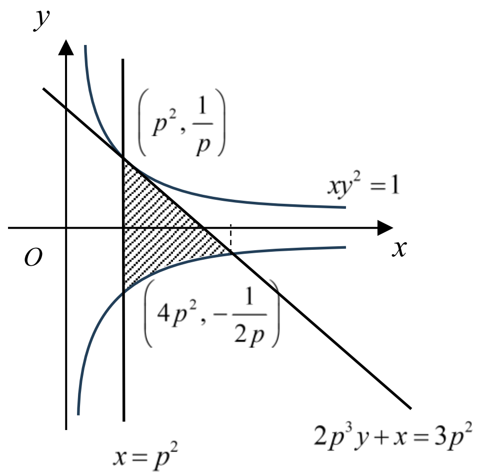
(a) Show that the equation of the tangent to the curve at the point $P\left(ap^2, \frac{a}{p}\right)$ is $2p^3y + x = 3ap^2$. [2]

(b) Find the equations of the tangents to C at the points where $x = a$. Find also the acute angle between these tangents. [3]

(c) **(i)** Show that $(q-p)^2(q+2p) = q^3 - 3p^2q + 2p^3$. [1]

(ii) The tangent to the curve at P cuts the curve again at $Q\left(aq^2, \frac{a}{q}\right)$. Find q in terms of p. [3]

The following diagram shows the graph of C for $a = 1$, and the Cartesian equation of the curve C is $xy^2 = 1$. The tangent to C at the point $\left(p^2, \frac{1}{p}\right)$ cuts the curve again at the point $\left(4p^2, -\frac{1}{2p}\right)$.



The region S is bounded by the tangent to C at the point $\left(p^2, \frac{1}{p}\right)$, the curve C and the line $x = p^2$.

(d) Find the area of S in terms of p. [3]

12 Sarah, a Singaporean Citizen, started a 4-year undergraduate Computing course at the National University of Singapore (NUS) in August 2021. The annual tuition fee, after the MOE Tuition Grant, is \$8,250. Sarah took a Tuition Fee Loan (TFL) covering 90% of her tuition fees. This loan is interest-free during her course of study.

Sarah decided to set aside $\$x$ per month from August 2021. This amount is increased by \$2 each subsequent month. (Sarah set aside $\$x$ in August 2021, $\$(x+2)$ in September 2021, ….) With the amount set aside, she repaid one-fifth of her total TFL at the end of July 2025.

(a) Find the value of x. [3]

Upon Sarah's graduation, the TFL's interest rate becomes effective at a fixed rate of 0.4% per month, calculated on the outstanding balance at the end of the month. The interest is first charged on 31st August 2025.

Sarah plans to make monthly repayments of $\$y$ to the bank on the 1st of every month, starting from 1st September 2025.

(b) Find the amount of outstanding loan after Sarah's first monthly repayment. [1]

(c) Show that the amount of outstanding loan after Sarah's nth monthly repayment is $23760(1.004)^n - 250y(1.004^n - 1)$. [3]

(d) Sarah wants to pay off her remaining loan by the end of 3 years. Find the value of y. [2]

Sarah decided on a different repayment strategy. For the first 24 months, she repays \$500 per month. After these 24 months, she doubles her monthly repayment to \$1000.

(e) Determine the month and year Sarah will clear her loan and calculate the amount of her final repayment. [4]

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