Year 6 Mathematics: analysis and approaches
Higher level
Paper 1
Preliminary Examinations

Monday 29 August 2022

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name and class on each answer sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper
- The maximum mark for this examination paper is [**110 marks**].

Section A		Section B	
Question	**Marks**	**Question**	**Marks**
1		10	
2		11	
3		12	
4			
5			
6			
7			
8			
9			

Name: ____________________

Class: ____________________

Index: ____________________

/ **110**

16 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x$ and $y = \frac{1}{2}$ when $x = 0$, find y in terms of x.

2. [Maximum mark: 5]

Audrey rolls a **biased** 6-sided dice with faces labelled as 1, 2, 3, 4, 5 and 6. The probabilities of obtaining odd numbers are all the same, and the probabilities of obtaining even numbers are also all the same. The probability of obtaining an even number is twice the probability of obtaining an odd number.

(a) Find the probability of obtaining a 1. [2]

Audrey rolls the die three times.

(b) Find the probability of Audrey obtaining two 1s and one 2. [3]

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3. [Maximum mark 5]

The function f is defined by $f(x)=\dfrac{ax-3}{3x+2}$, where $x\in\mathbb{R}$, $x\neq -\dfrac{2}{3}$ and a is a constant

(a) Write down the equation of the vertical asymptote of the graph of f. [1]

(b) Given that the horizontal asymptote of the graph of f is $y=2$, state the value of a. [1]

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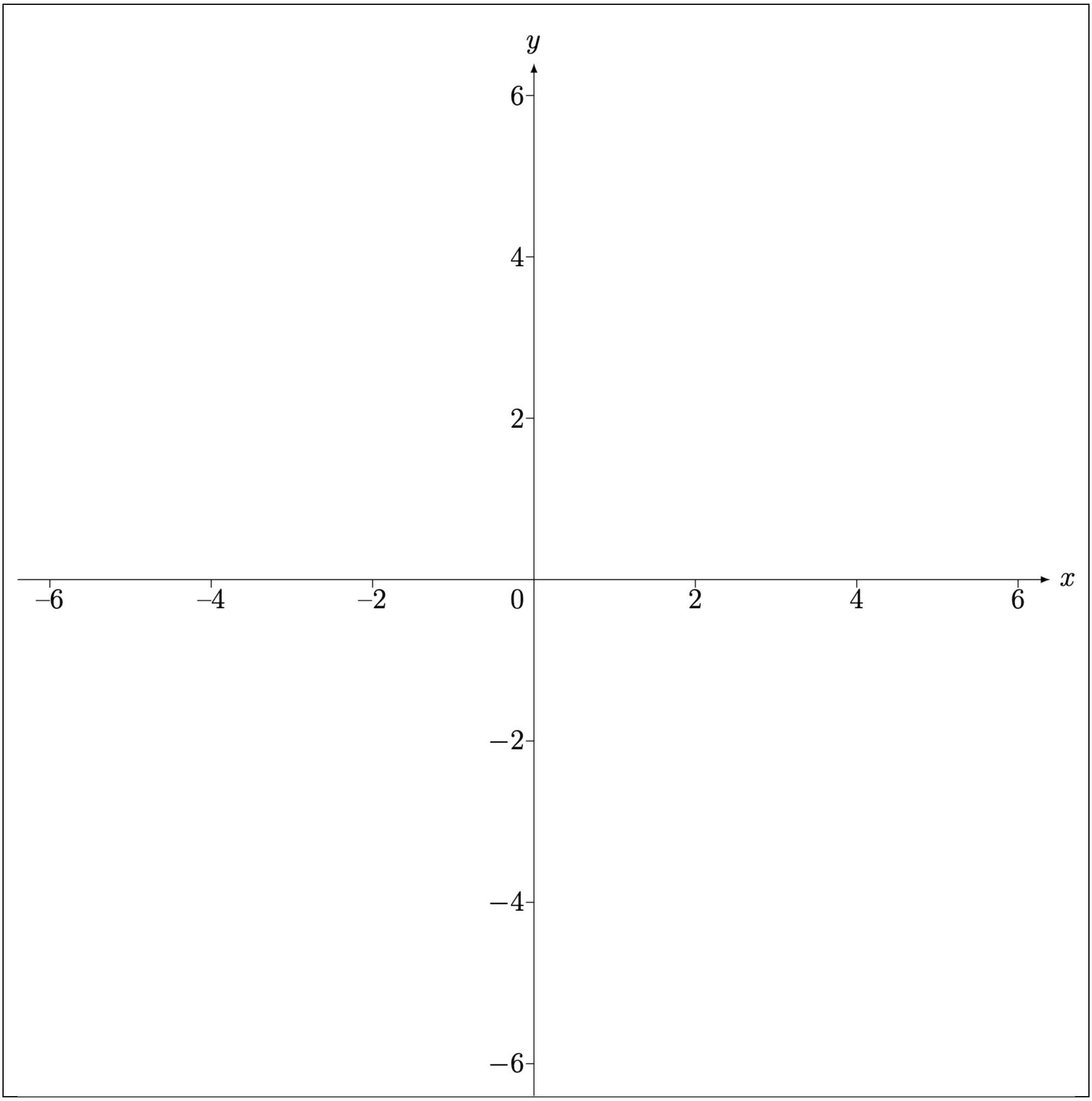
(Question 3 continued)

(c) Sketch the graph of f in the axes below, indicating clearly all asymptotes and axial intercepts.. [3]



4. [Maximum mark: 7]

The function f is defined by $f(x) = 2x - \frac{4}{x}$, where $x \in \mathbb{R},\ x \neq 0$.

(a) Show that $f'(2) = 3$. [2]

The values of the function g and its derivative for $x = 2$ and $x = 3$ are shown in the following table.

x	$g(x)$	$g'(x)$
2	1	4
3	6	6

Let $h(x) = g\left(f(x)\right)$ for $x \in \mathbb{R}$.

(b) Find $h(2)$. [2]

(c) Hence, find the equation of the tangent to the graph of h at $x = 2$. [3]

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(Question 4 continued)

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5. [Maximum mark: 5]

The function $f(x) = px^3 - 13x^2 - 8px - 15$ has factors $(x+1)$, $(px+a)$ and $(x+b)$, where a, b and p are constants.

Find the value of abp.

6. [Maximum mark: 7]

Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are given by $\boldsymbol{u} = \boldsymbol{i} - 7\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{v} = a\boldsymbol{i} + b\boldsymbol{j} + 5\boldsymbol{k}$, where a and b are constants. Find a and b given that the magnitude of $\boldsymbol{v}$ is $\sqrt{27}$ and that $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular.

7. [Maximum mark: 6]

Consider the equation $mx^2 + 4x + 4m = 0$, where m is real, and the equation has roots k and $2k$. Find the values of m.

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8 [Maximum mark: 6]

The function is defined as $f(x)=\dfrac{1}{x}-\dfrac{1}{e^x-1}$

(a) Express $f(x)$ as a single fraction. [1]

(b) Hence, or otherwise, find the value of the $\lim\limits_{x\to 0} f(x)$ [5]

9. [Maximum mark: 8]

Find the general solution of the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x}+5y=\frac{\ln x}{x},\ x>0$

giving your answer in the form $y=f(x)$.

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Section B

Answer **all** questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 16]

Consider the function $f(x) = \ln(ax)$ where $x, a \in \mathbb{R}$ and $x > 0$, $a > 0$.

(a) (i) Given that $f^{-1}(1) = \frac{\mathrm{e}}{2}$, show that $a = 2$.

(ii) Find an expression for $f^{-1}(x)$. [5]

(b) Consider the first four consecutive terms of an arithmetic sequence $\ln(54x),\ \ln(px),\ \ln(qx),\ \ln(2x)$, where $x > 0$, p and q are positive integers.

(i) Show that common difference, $d = \ln\left(\frac{1}{3}\right)$.

(ii) Find the value of p and of q.

(iii) Show that $S_5 = 5\ln(6x)$.

(iv) An infinite geometric sequence has the first term 5 and a common ratio $\frac{1}{3}$.

Given that the sum of the infinite geometric sequence is three times of S_5,

find the value of x [11]

11. [Maximum mark: 17]

(a) (i) For the complex number $z = \cos\theta + i\sin\theta$, show that $\left(z + \frac{1}{z}\right) = 2\cos\theta$

(ii) Hence or otherwise, and considering $\left(z + \frac{1}{z}\right)^5$, show that

$$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$$

(iii) If $\sin\alpha = \frac{1}{3}$, for $0 < \alpha < \frac{\pi}{2}$, find the value of

$$\frac{16\sqrt{2}}{27}\left(\cos 5\alpha + 5\cos 3\alpha + 10\cos\alpha\right).$$

[9]

(b) (i) Obtain the roots to the equation $w^5 = 4\sqrt{2}$. Show the points corresponding to these roots in an Argand diagram.

(ii) For each root w, let $v = w\sqrt{2}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$.

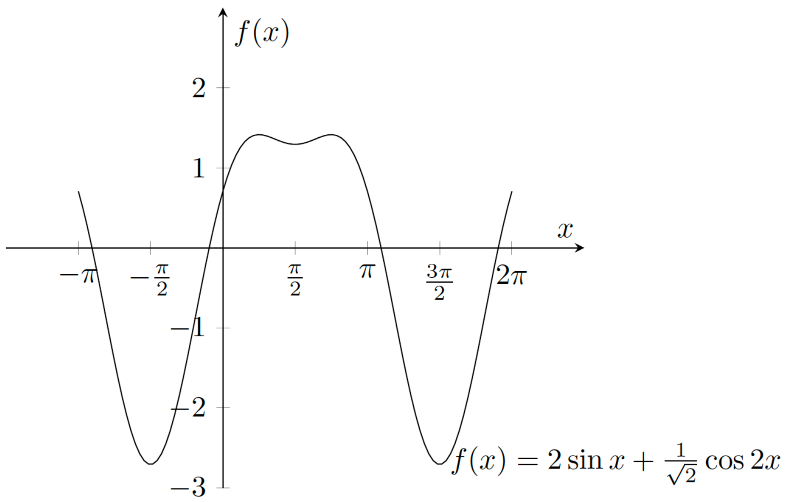
Show the points corresponding to the values of v on your Argand diagram,

(iii) Find, in simplified form, an equation for which the values of v are the roots.

[8]

12. [Maximum mark: 24]

The function $f(x)=2\sin x+\dfrac{1}{\sqrt{2}}\cos 2x$ for $-\pi \le x \le 2\pi$ is shown below,



(a) (i) Show that $f'(x)=2\cos x-\sqrt{2}\sin 2x$.

(ii) Find the values of x for which $f'(x)=0$.

(iii) State the range of the function. [7]

The function $g(x)=2\sin x$, for $-\pi \le x \le 2\pi$.

(b) For $0 \le x \le \pi$ find the values of x when $f(x)=g(x)$. [4]

(c) (i) For the domain $0 \le x \le \dfrac{3\pi}{2}$, sketch on the same axes $f(x)$ and $g(x)$.

(ii) Show that $[g(x)]^2-[f(x)]^2=-2\sqrt{2}\cos 2x\sin x-\dfrac{1}{2}\cos^2 2x$

The enclosed region, R, between $f(x)$, $g(x)$ and $x=a$ and $x=b$, where a and b are the first 2 positive roots, is rotated 2π around the x-axis.

(iii) Show that the equation for the volume can be written as

$$V=\pi\int_a^b (n\cos 4x+p\sin 3x+q\sin x+m)\,\mathrm{d}x$$

where, p, q m and n are constants.

[Hint $2\cos A\sin B=\sin(A+B)-\sin(A-B)$].

(iv) Hence, or otherwise, find the volume of the enclosed region R [13]

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Answers written on this page
will not be marked.