RAFFLES INSTITUTION
2024 YEAR 5 TIMED PRACTICE (MODIFIED)
Higher 2

MATHEMATICS **9758**

2 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF26)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

RAFFLES INSTITUTION
Mathematics Department

1 A smoothie shop sells three different types of smoothie blends, Berrylicious, Tropicality and Pumptein. The smoothie blends are dispensed from machines and each serving is prepared by blending protein powder, mixed fruits and milk.

Relative to a serving of Berrylicious, a serving of Tropicality uses 50% the amount of protein powder, 30% more the amount of mixed fruits and 75% the amount of milk. Relative to a serving of Berrylicious, a serving of Pumptein uses 50% more the amount of protein powder, 60% the amount of mixed fruits and 25% more the amount of milk.

A serving of Berrylicious, Tropicality and Pumptein weighs 360 grams, 350 grams and 355 grams respectively.

Find the amount of protein powder, mixed fruits and milk required to make a serving of Berrylicious, in grams. [4]

2 A graphic calculator is **not** to be used in answering this question.

Given that $3-4\mathrm{i}$ is a root of the equation

$$5z^3+az^2+bz+25=0,$$

find the values of the real numbers *a* and *b* and the remaining roots of the equation. [4]

3 With reference to the origin *O*, the points *A* and *C* have position vectors $\mathbf{a}$ and $\mathbf{c}$ respectively, where $\mathbf{a}$ and $\mathbf{c}$ are non-zero and non-parallel vectors. The point *E* lies on *AC* such that $AE : AC = \lambda - 2 : \lambda$, where $\lambda > 2$.

(a) Find the area of triangle *OAE* in terms of λ, $\mathbf{a}$ and $\mathbf{c}$. [3]

(b) Given that the area of triangle *OAE* is $\frac{3}{10}|\mathbf{a}\times\mathbf{c}|$, solve for the value of λ. [1]

(c) Given that $\mathbf{a}$ is a unit vector, give the geometrical interpretation of

(i) $|\mathbf{a}.\mathbf{c}|$, [1]

(ii) $|\mathbf{a}\times\mathbf{c}|$. [1]

(d) If $\angle OAC = 90^\circ$, explain why $\mathbf{a}.\mathbf{c}$ is positive. [1]

4 The line l_1 contains the points A and B with coordinates $(-1, 3, 7)$ and $(a, 0, -2)$ respectively, where a is a constant. The line l_2 has equation $-\frac{x+7}{2} = \frac{y-10}{3}$, $z = 10$. It is given that l_1 and l_2 cross at the point C.

(a) Find the value of a and the coordinates of C. [4]

(b) The point P has coordinates $(-2, 9, 8)$. Find the point on l_2 which is closest to P and hence find the exact perpendicular distance between P and l_2. [4]

5 On the same axes, sketch the curves with equations $y = |x^2 - 5x|$ and $y = |16 - 3x|$, stating the exact coordinates of any points of intersection with the axes.
Hence solve exactly the inequality $|x^2 - 5x| \geq |16 - 3x|$. [8]

6 The curve C has equation

$$x^2 - ae^x \tan^{-1} y = 0, \qquad \text{for } x > 0,$$

where a is a positive constant.

(a) Show that $ae^x \frac{dy}{dx} = (2x - x^2)(1 + y^2)$. [2]

It is given that C has one stationary point.

(b) Find the x-coordinate of this stationary point and determine its nature, showing your working clearly. [3]

7 A curve C has parametric equations

$$x = (1+t)^2, \quad y = (1-t)^2.$$

The point A is on C such that the tangent to C at A has gradient 2.

(a) Find the equation of this tangent and hence show algebraically that this tangent never meets C again. [5]

(b) The tangent and the normal to C at A meet the y-axis at points P and Q respectively. Find the area of the triangle APQ. [3]

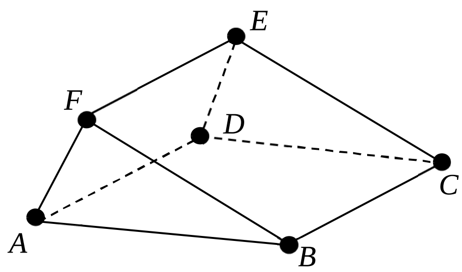
8 The function g is defined by $g: x \mapsto \ln(x^2 - 3) + 1$, for $x \in \mathbb{R}$, $x > \sqrt{3}$.

(a) Explain why g^{-1} exists. [2]

(b) Find $g^{-1}(x)$ and write down the domain and range of g^{-1}. [4]

(c) The function f is defined by $f: x \mapsto a - \left(x - 2\sqrt{3}\right)^2$, for $x \in \mathbb{R}$, $x > 0$ for some real constant a. Show that the composite function fg^{-1} exists. Find $fg^{-1}(x)$ and its range in terms of a. [4]

9 An architect is designing the structure of a slide for an indoor children's playground. As shown in the diagram below, part of the structure has a rectangular base *ABCD* and a rectangular slide surface *BCEF* and two sides *ABF* and *DCE* which are perpendicular to the base *ABCD*.



It is given that the base *ABCD* is part of the plane with equation $-2x - 6y + 7z = 7$, and the line *BF* has equation $\mathbf{r} = \begin{pmatrix} -1 \\ 18 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 8 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$.

(a) Find the position vector of *B*. [3]

(b) Find the angle between the slide surface *BCEF* and its base *ABCD*. [3]

(c) Show that the cartesian equation of the plane containing side *ABF* is $62x + 5y + 22z = 138$. [3]

It is now given that the other side *DCE* is part of the plane with equation $62x + 5y + 22z = 200$.

(d) The playground's theme requires the slide surface width to be between 80 cm and 100 cm. Taking each unit in the architect's design to be a metre, determine if the slide meets the requirement. [3]