RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 3: Equations and Inequalities

Section A (Basic Questions)

1 Without using a calculator, solve the following inequalities.

(a) $3x^2+4x+1>0$

(b) $2x^2+2x+1\le 0$

(c) $x^2-9<(x-3)(x^2-4x+9)$

[(a) $x<-1$ or $x>-\frac{1}{3}$ **(b)** No solution **(c)** $2<x<3$ or $x>3$ **]**

2 Without using a calculator, solve the following inequalities.

(a) $\frac{x-5}{1-x}\ge 1$

(b) $\frac{3-x}{x+2}\le\frac{1}{x}$ [4]

[(a) $1<x\le 3$ **(b)** $x<-2$ or $x>0$ **]**

3 Without using a calculator, solve the following inequalities.

(i) $|2x-3|<7$

(ii) $|2+6x|>1$

[(i) $-2<x<5$ **(ii)** $x<-\frac{1}{2}$ or $x>-\frac{1}{6}$ **]**

4 By sketching appropriate graphs on a single diagram, find the exact solution set to the following inequalities.

(i) $|x+3|\ge 3x+1$

(ii) $|11x|>|x-1|$

[(i) $(-\infty,1]$ **(ii)** $\left(-\infty,-\frac{1}{10}\right)\cup\left(\frac{1}{12},\infty\right)$ **]**

5 A cinema sells tickets at three different prices, depending on the age of the customer. The age categories are under 16 years, between 16 and 65 years, and over 65 years. Three groups of people, *A*, *B* and *C*, go to the cinema on the same day. The numbers in each age category for each group, together with the total cost of the tickets for each group, are given in the following table.

Group	Under 16 years	Between 16 and 65 years	Over 65 years	Total cos
A	9	6	4	\$162.03
B	7	5	3	\$128.36
C	10	4	5	\$158.50

Write down and solve the equations to find the cost of a ticket for each of the age categories.

[4]

[x =7.65, y=9.85, z =8.52**]**

Section B (Discussion Questions)

1 Without using a calculator, solve the following inequalities.

(a) $(x+2)^2 \geq 9$

(b) $(2-x)(x^2-x+1) \leq 0$

(c) $2x^2-4x+5>0$

(d) $\dfrac{x^2-x-2}{(2-x)^2} \geq x+1$

(e) $-1 \leq \dfrac{x^2-3x-4}{x^2+2x+3} < 1$

[(a) $x \leq -5$ or $x \geq 1$, **(b)** $x \geq 2$, **(c)** $\mathbb{R}$, **(d)** $x \leq -1$ or $2 < x \leq 3$, **(e)** $-\dfrac{7}{5} < x \leq -\dfrac{1}{2}$ or $x \geq 1$**]**

2 Without using a calculator, solve the inequality $\dfrac{x^2+x+1}{x^2+x-2} < 0$. [4]

[$-2 < x < 1$**]**

3 Without using a calculator, solve the inequality $\dfrac{x^2+2x-9}{x^2-9} \geqslant 1$.

Hence solve the inequalities

(i) $\dfrac{x^2+2|x|-9}{x^2-9} \geqslant 1$,

(ii) $\dfrac{(\ln x)^2+\ln x^2-9}{(\ln x)^2-9} \geqslant 1$.

[$-3 < x \leq 0$ or $x > 3$, **(i)** $x = 0$ or $x < -3$ or $x > 3$, **(ii)** $\mathrm{e}^{-3} < x \leq 1$ or $x > \mathrm{e}^3$**]**

4 **(i)** By using an algebraic method, solve the inequality $\frac{x+3}{x+4} \le \frac{5}{1-2x}$. [4]

(ii) Hence, solve the inequality $\frac{x^2+3}{x^2+4} \le \frac{5}{1-2x^2}$. [4]

[(i) $-4 < x < \frac{1}{2}$, **(ii)** $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ **]**

5 Sketch the graph of $y = 2x^2 - 5x - 3$ for $-2 \le x \le 4$, indicating the axial intercepts. Hence write down the range of values of k for which the equation $|2x^2 - 5x - 3| = k$ has 4 distinct roots.

[$0 < k < \frac{49}{8}$ **]**

6 Find the set of values of x such that $|2-3x| > 2|x+2|$. Solve this using two different methods:

(a) algebraic method,

(b) graphical method.

[$\{x \in \mathbb{R} : x < -0.4 \text{ or } x > 6\}$]

7 The solution set to the inequality $|2x+3| < ax+4$ is $\left(-\frac{9}{5}, b\right)$. Find a and b.

[$a = \frac{17}{9}$, $b = 9$**]**

8 **(i)** On the same axes, sketch the graphs of $y = |x+a|$ and $y = |ax+1|$ where $0 < a < 1$, and hence solve the inequality $|x+a| < |ax+1|$. [4]

(ii) Deduce the solution to the inequality $\left|\frac{1}{x}+a\right| < \left|\frac{a}{x}+1\right|$. [2]

[(i) $-1 < x < 1$, **(ii)** $x < -1$ or $x > 1$**]**

9 **(i)** The first three terms of a sequence are given by $u_1 = 10$, $u_2 = 6$, $u_3 = 5$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [4]

(ii) Find the set of values of n for which u_n is greater than 100. [2]

[$u_n = 1.5n^2 - 8.5n + 17$, $\{n \in \mathbb{Z}^+ : n \ge 11\}$**]**

10 A curve C has equation

$$y = \frac{a}{x^2} + bx + c,$$

where a, b and c are constants. It is given that C passes through the points with coordinates $(1.6, -2.4)$ and $(-0.7, 3.6)$, and that gradient of C is 2 at the point where $x = 1$.

(i) Find the values of a, b and c, giving your answers correct to 3 decimal places. [4]

(ii) Find the x-coordinate of the point where C crosses the x-axis, giving your answer correct to 3 decimal places. [2]

[(i) $a = -3.593, b = -5.187, c = 7.303$, **(ii)** -0.589**]**

11 A movie theatre supports a charity event by providing discounted movie tickets at the following prices based on age group.

Age group	Discounted ticket price
≤ 12 years	\$6
13 to 49 years	\$9
≥ 50 years (senior citizen)	\$4

The event organiser plans to spend \$700 to sponsor 100 participants of which at least 35 of them are senior citizens. Find the possible number of participants for each age group. [4]

[$\begin{cases} x = 5 \\ y = 58 \\ z = 37 \end{cases}$ or $\begin{cases} x = 0 \\ y = 60 \\ z = 40 \end{cases}$ **]**