Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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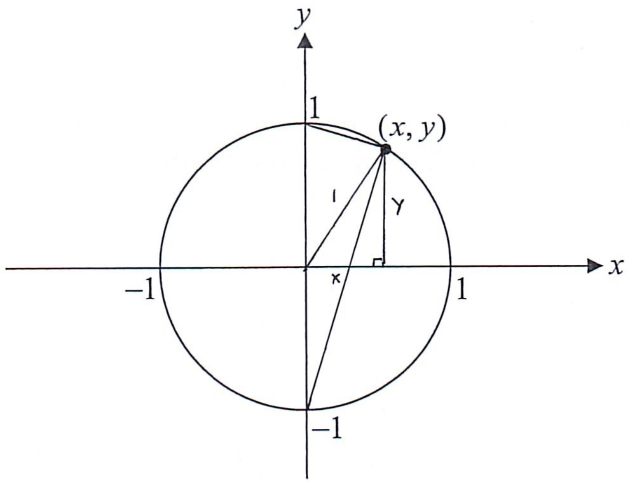
MA303.1.4 – Circles

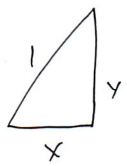
At the end of this unit, you should be able to

- **understand the centre and radius needed for the equation of a circle**
- **write equation of circles and solve problems involving circles**

Consider the unit circle (i.e. a circle of radius 1 unit) with centre at the origin.



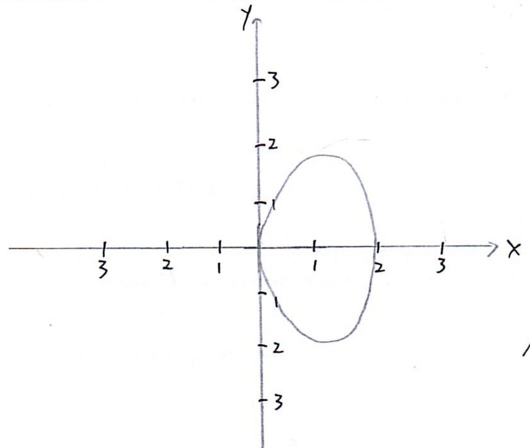
E1. Let the coordinates of a point on the circle be (x, y). Show, using a right-angled triangle, that $x^2 + y^2 = 1$.



$\therefore x^2+y^2=1^2$ (shown)

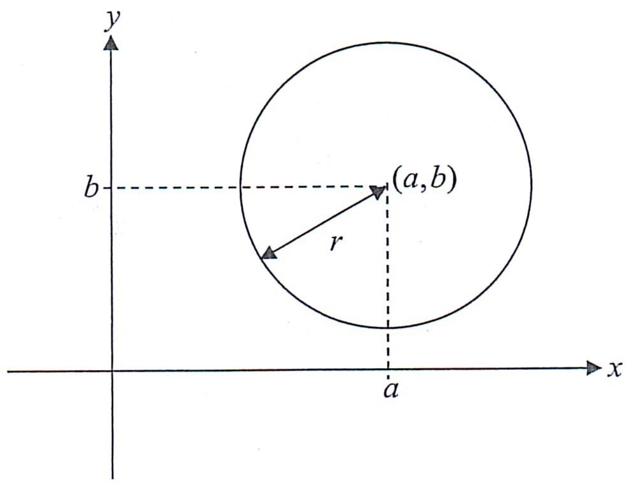
E2. Plot the graph of $(x-1)^2 + y^2 = 1$. What is the radius of the circle plotted? What are the coordinates of the centre of the circle?

x	0	1	2
y	0	1	0



Center of circle: (1, 0)

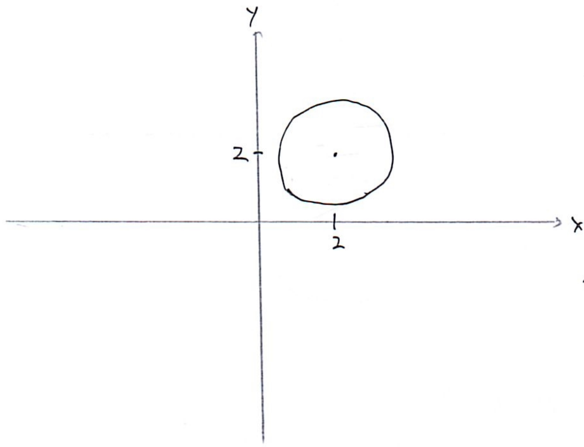
The graph of $(x-a)^2 + (y-b)^2 = r^2$ is that of a circle with radius r units and coordinates of the centre of the circle (a,b).



E3. Write down the equation of the circle whose radius is 2 units and the centre of the circle is $(1,-3)$.

Equation: $(x-1) + (y+3) = 2^2$

P1. Sketch the graph of $(x-2)^2+(y-2)^2=2$. Write down the coordinates of the points of intersection with the axes, if any.

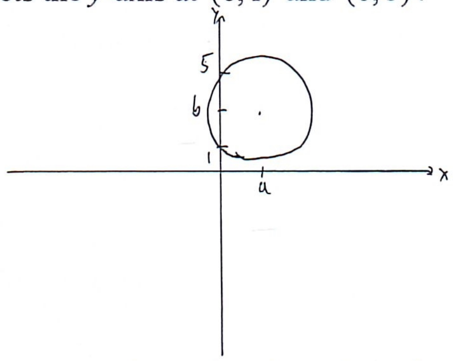


E4. A circle has centre (a,b) such that it intersects the y-axis at $(0,1)$ and $(0,5)$.

(a) Explain why $b=3$.

b = the average of the y-intercepts.

$(5+1) \div 2 = 3$.



(b) Given that the centre lies on the line $2y-x=4$, determine the value of a, and hence find the radius of the circle.

$2y-x=4$

$2y=x+4$

$y=\frac{1}{2}x+2$ —— ①

Sub $(a,3)$ into ①

$3=\frac{1}{2}a+2$

$a=2$

$(0-2)^2+(5-3)^2=r^2$

$r=\sqrt{8}$

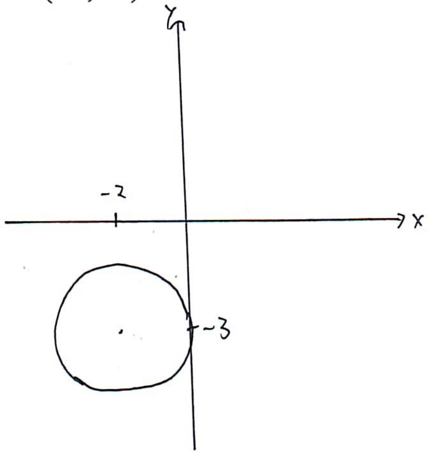
$=2\sqrt{2}$ units

Radius of circle is $2\sqrt{2}$ units.

P2. Find the equation of the circle whose centre is at $(-2,-3)$ and which touches the y-axis.

Radius $= 2$ as it touches the y-axis.

$\therefore$ Equation : $(x+2)^2+(y+3)^2=2^2$



Equations of the form $x^2+y^2+2gx+2fy+c=0$

E5. Expand $(x+2)^2+(y-3)^2=1$ and show that $x^2+y^2+4x-6y+12=0$.

$(x^2+4x+4)+(y^2-6y+9)=1$

$x^2+y^2+4x-6y+12=0$ (shown)

E6. By completing the square, express the following in the form $(x-a)^2+(y-b)^2=r^2$

(a) $x^2+6x+y^2-4y+1=0$

$(x^2+6x+9)-9+(y^2-4y+4)-4+1=0$

$(x+3)^2+(y-2)^2=12$

(b) $2x^2+2y^2-4x+y-3=0$

$2(x^2+y^2)+2(y^2+\frac{y}{2})-3=0$

$2(x-1)^2-2+2(y+\frac{1}{4})^2-\frac{1}{8}-3=0$

$(x-1)^2+(y+\frac{1}{4})^2=\frac{41}{16}$

E7. By comparing

$$(x-a)^2+(y-b)^2=r^2 \text{ and } x^2+y^2+2gx+2fy+c=0$$

(a) Show that $c=a^2+b^2-r^2$.

$x(x-a) - a(x-a) + y(y-b) - b(y-b) = r^2$

$x^2-2ax+a^2+y^2-2by+b^2=r^2$

$x^2+y^2+2(-a)x+2(-b)y+a^2+b^2-r^2=0$

$\therefore x^2+y^2+2gx+2fy \equiv x^2+y^2+2(-a)x+2(-b)y+a^2+b^2-r^2$

$\therefore c=a^2+b^2-r^2$ (shown)

(b) Write down g in terms of a and f in terms of b.

$g=-a$

$f=-b$

(c) Hence, show that $r=\sqrt{f^2+g^2-c}$.

$a^2=g^2$ — ①

$b^2=f^2$ — ②

$c=a^2+b^2+r^2$ — ③

$c=g^2+f^2-r^2$

$r^2=g^2+f^2-c$

$\therefore r=\sqrt{f^2+g^2-c}$ (shown)

P3. Given $x^2+y^2-4x+8y+42=0$, express it in the form $(x-a)^2+(y-b)^2=r^2$.

$(x-2)^2-4+(y+4)^2-16+42=0$

$(x-2)^2+(y+4)^2=-22$

There is no circle as $\sqrt{-22} \notin \mathbb{R}$

What do you observe? By using the formula $r=\sqrt{f^2+g^2-c}$, what can be deduced about the equation $x^2+y^2-4x+8y+42=0$?

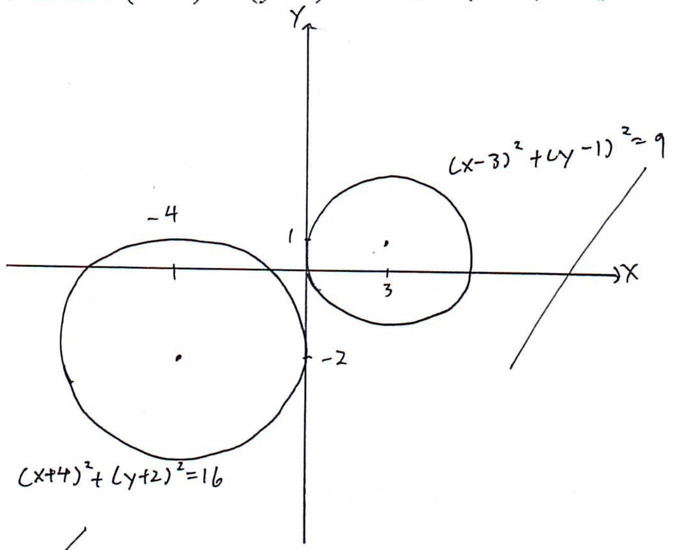
$r=\sqrt{4^2+(-2)^2-42}$ units

$=\sqrt{-22}$ units

There is no circle.

Intersection between circles

P4. Draw the two circles $(x-3)^2+(y-1)^2=9$ and $(x+4)^2+(y+2)^2=16$ on the same axes.



(a) Do the two circles intersect?

No.

(b) Find the distance between the centres of the two circles.

Distance $= \sqrt{(3+4)^2+(1+2)^2}$ units

$= \sqrt{58}$ units

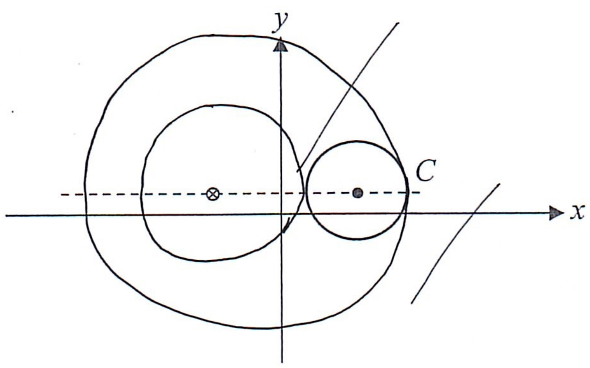
(c) Is the sum of the radii more than, less than or equal to the distance in (b)?

Sum of radius $= 3+4$

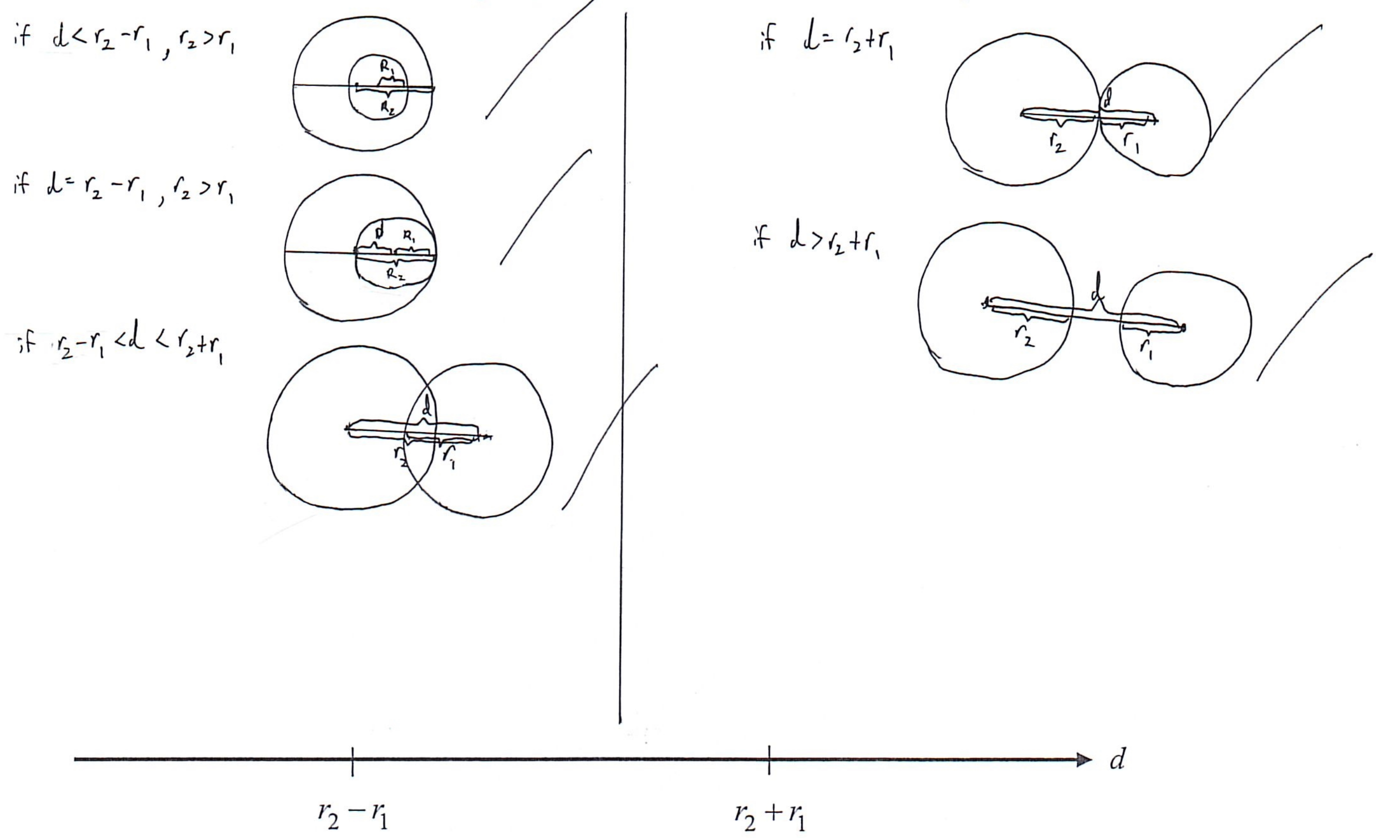
$= 7$

$\sqrt{58} > 7$

P5. Draw **two** circles, given the centre of the circle marked with ($\otimes$) such that they touches the circle C at only one point. The centre of C is represented by the black dot ($\bullet$).



P6. Given two circles $(x-a_1)^2+(y-b_1)^2=r_1^{\,2}$ and $(x-a_2)^2+(y-b_2)^2=r_2^{\,2}$, explain how you can determine whether the circles (i) do not intersect, (ii) touch at one point, or (iii) intersect at two points, without attempting to solve the two simultaneous equations.



P7. Without solving equations, determine if the two circles $x^2+y^2-4x+6y=3$ and $x^2+y^2+2x-2y=62$ intersect.

$x^2+y^2-4x+6y-3=0$ —— ①

$(x-2)^2+(y+3)^2=16$

$x^2+y^2+2x-2y-62=0$ —— ②

$(x+1)^2+(y-1)^2=64$

Center of ① : $(2,-3)$

Center of ② : $(-1,1)$

$\therefore d = \sqrt{(1+2)^2+(1+3)^2}$ units

$= 5$ units

$r_1 + r_2 = 12$

$r_2 - r_1 = 4$

$\therefore r_2 - r_1 < d < r_2 + r_1$, 2 points of intersection.