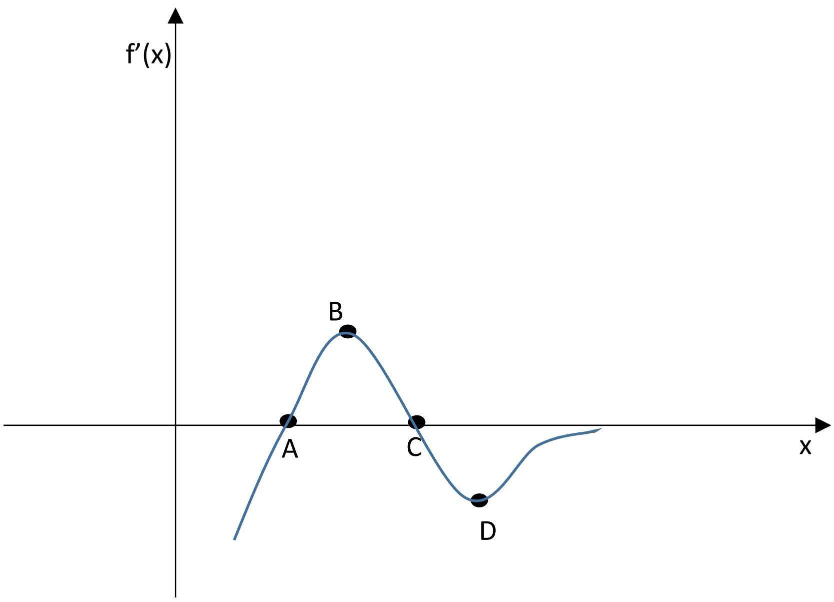
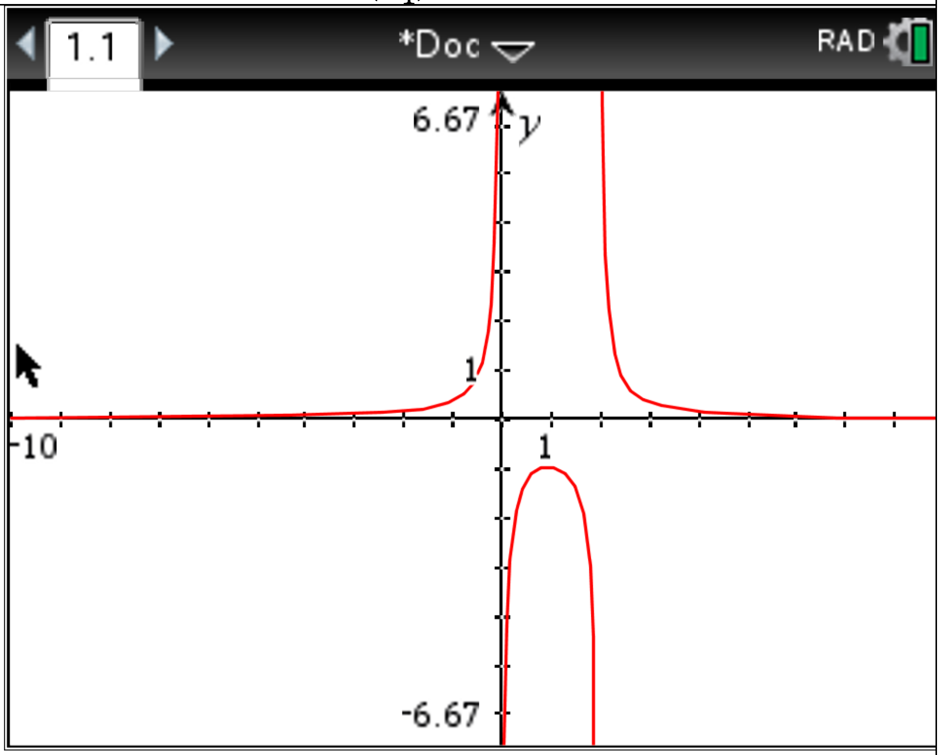
			Y5 HL Mathematics Final Examination 2018
1	[Maximum mark: 5]		
		Well done except fpr some careless mistakes	$\left(\frac{1}{\sqrt{x}}-1\right)^4(1-x)^2$ $=\left[\left(\frac{1}{\sqrt{x}}\right)^4+4\left(\frac{1}{\sqrt{x}}\right)^3(-1)+\binom{4}{2}\left(\frac{1}{\sqrt{x}}\right)^2(-1)^2+\binom{4}{3}\left(\frac{1}{\sqrt{x}}\right)^1(-1)^3 + (-1)^4\right]\times[1-2x+x^2]$ $=\cdots 1+\binom{4}{2}\left(\frac{1}{\sqrt{x}}\right)^2(-2x)+\left(\frac{1}{\sqrt{x}}\right)^4(x^2)+\cdots$ $=\cdots 1-\frac{2\times 4!}{2!\,2!}+1+\cdots$ $=1-12+1$ $=-10$
2	[Maximum mark: 5]		
	(i)	Very well done	$\frac{1}{\sqrt{4r-3}+\sqrt{4r+1}}$ $=\frac{\sqrt{4r-3}-\sqrt{4r+1}}{\left(\sqrt{4r-3}+\sqrt{4r+1}\right)\left(\sqrt{4r-3}-\sqrt{4r+1}\right)}$ $=\frac{\sqrt{4r-3}-\sqrt{4r+1}}{4r-3-(4r+1)}$ $=\frac{\sqrt{4r+1}-\sqrt{4r-3}}{4}$
	(ii)	Most tried to force an AP out of the series which is wrong	$=\frac{1}{4}\sum_{1}^{100}\left(\sqrt{4r+1}-\sqrt{4r-3}\right)$ $=\frac{1}{4}\left[\left(\sqrt{5}-\sqrt{1}\right)+\left(\sqrt{9}-\sqrt{5}\right)+\left(\sqrt{13}-\sqrt{9}\right) + \left(\sqrt{17}-\sqrt{13}\right)\ldots.+\left(\sqrt{401}-\sqrt{397}\right)\right]$ $=\frac{1}{4}\left[-1+\sqrt{401}\right]$
3	[Maximum mark: 5]		
		Many did not realise that $1-tan\theta>0$ and can be multiplied for simple solving. Most did not remember $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$ as the necessary condition	$-\frac{\pi}{4}<\theta<\frac{\pi}{4}\Rightarrow -1<tan\theta<1$ $S_\infty=\frac{1}{1-tan\theta}$ $S_\infty<\frac{3+\sqrt{3}}{2}$ $\frac{1}{1-tan\theta}<\frac{3+\sqrt{3}}{2}$ $2<\left(3+\sqrt{3}\right)(1-tan\theta)$ [since $tan\theta<1$] $2-3+3tan\theta-\sqrt{3}+\sqrt{3}tan\theta<0$ $-1+3tan\theta-\sqrt{3}+\sqrt{3}tan\theta<0$ $\left(3+\sqrt{3}\right)tan\theta<1+\sqrt{3}$

			$tan\theta < \frac{1+\sqrt{3}}{3+\sqrt{3}}$ $tan\theta < \frac{(1+\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$ $tan\theta < \frac{3+2\sqrt{3}-3}{6}$ $tan\theta < \frac{\sqrt{3}}{3}$ $\theta < \frac{\pi}{6}$ Hence $-\frac{\pi}{4} < \theta < \frac{\pi}{6}$
4	[Maximum mark: 6]		
		Implicit differentiation is ok but some had misread question as $\frac{dy}{dx} = 1$	$4x^3 + xy^2 = 5xy$ $12x^2 + x\left(2y\frac{dy}{dx}\right) + y^2 = 5x\frac{dy}{dx} + 5y$ $\frac{dy}{dx}(2xy - 5x) = 5y - y^2 - 12x^2$ $\frac{dy}{dx} = \frac{5y - y^2 - 12x^2}{2xy - 5x}$ At P, $y = x$ Subst $y = x$ into curve : $4x^3 + x^3 = 5x^2$ $x^2(x-1) = 0$ $x = 0\ (N.A.)\ or\ x = 1$ Subst $x = 1\ and\ y = 1$ into $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{8}{3}$ Equation of tangent to the curve at P is $y - 1 = \frac{8}{3}(x-1)$ $y = \frac{8}{3}x - \frac{5}{3}$ or $3y = 8x - 5$
5	[Maximum mark: 9]		
	(a)	About half got this correct. Some tried to fit singles in between couples. Qn did not say singles cannot be together	C_1, C_2, C_3, C_4, M, W No of ways $= 6! \times 2^4$ $= 720 \times 16$ $= 11520$

	(b)	Fairly well done. Some wrongly took last term as ar^{k+1}	Let P_n be the statement: $S_n = \frac{a(r^n-1)}{r-1}$ P_1: $S_1 = \frac{a(r^1-1)}{r-1} = a$ Hence P_1 is true. Assume P_k is true. $S_k = \frac{a(r^k-1)}{r-1}$ P_{k+1}: $S_{k+1} = S_k + u_{k+1}$ $S_{k+1} = \frac{a(r^k-1)}{r-1} + ar^k$ $= \frac{ar^k - a + ar^k(r-1)}{r-1}$ $= \frac{a\left(r^{k+1}-1\right)}{r-1}$ $\therefore P_{k+1}$ is true. P_1 and P_k are true $\Rightarrow P_{k+1}$ is true, $\therefore$ By Mathematical Induction, P_n is true for all $n \in Z^+$
6	[Maximum mark: 8]		
	(i)	Mostly well done except a handful who went with memory work and got the wrong formula $(\alpha+\beta+\gamma)^2$	$2x^3 + x^2 - 5x + 3 = 0$ $\alpha+\beta+\gamma = -\frac{1}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma = -\frac{5}{2}$ $\alpha\beta\gamma = -\frac{3}{2}$ $(\alpha+\beta+\gamma)^2 = (\alpha+\beta)^2 + 2(\alpha+\beta)\gamma + \gamma^2$ $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2 + 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $\left(-\frac{1}{2}\right)^2 = \alpha^2+\beta^2+\gamma^2 + 2\left(-\frac{5}{2}\right)$ $\alpha^2+\beta^2+\gamma^2 = \frac{21}{4}$
	(ii)	Both methods were equally popular with the usual careless mistakes	$\{y-(\alpha-1)\}\{y-(\beta-1)\}\{y-(\gamma-1)\}$ $= \{(y+1)-\alpha\}\{(y+1)-\beta\}\{(y+1)-\gamma\}$ Subst. $x = y+1$ into $2x^3 + x^2 - 5x + 3 = 0$ $2(y+1)^3 + (y+1)^2 - 5(y+1) + 3 = 0$ $2(y^3+3y^2+3y+1) + y^2 + 2y + 1 - 5y - 5 + 3 = 0$ $2y^3 + 7y^2 + 3y + 1 = 0$ Or $2x^3 + 7x^2 + 3x + 1 = 0$ Alternatively, $\alpha - 1 + \beta - 1 + \gamma - 1 = -\frac{1}{2} - 3 = -\frac{7}{2}$ $(\alpha-1)(\beta-1) + (\alpha-1)(\gamma-1) + (\beta-1)(\gamma-1)$ $= (\alpha\beta+\alpha\gamma+\beta\gamma) - 2(\alpha+\beta+\gamma) + 3$ $= -\frac{5}{2} - 2\left(-\frac{1}{2}\right) + 3$

			$=\frac{3}{2}$ $(\alpha-1)(\beta-1)(\gamma-1)$ $=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$ $=-\frac{3}{2}-\left(-\frac{5}{2}\right)+\left(-\frac{1}{2}\right)-1$ $=-\frac{1}{2}$ Equation is $x^3-\left(-\frac{7}{2}\right)x^2+\frac{3}{2}x-\left(-\frac{1}{2}\right)=0$ $2x^3+7x^2+3x+1=0$
7	[Maximum mark: 6]		
	(i)		$sin\frac{\pi}{12}=sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ $=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ $=\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}-\frac{1}{2}\times\frac{\sqrt{2}}{2}$ $=\frac{\sqrt{6}-\sqrt{2}}{4}$ Hence $\arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)=\frac{\pi}{12}$
	(ii)	Most not done using complementary angles	$x=\cos\frac{5\pi}{12}=\sin\frac{\pi}{12}$ $=\frac{\sqrt{6}-\sqrt{2}}{4}$
8	[Maximum mark: 6]		
		Fairly well done	 A, B, C, D

			Asymptote at y = 0 Shape of graph
9	[Maximum mark: 18]		
	(a)	(i) Many did not present answer as a single translation	$g(x) = x(x-2) = x^2 - 2x$ $= (x-1)^2 - 1$ x^2 is translated $\binom{1}{0}$ to $(x-1)^2$ and then translated $\binom{0}{-1}$ to $(x-1)^2 - 1$ Transformation is translation $\binom{1}{-1}$
		(ii) A few did not know how to do a reciprocal graph	 $A'(-1.24, 0.25)$, $B'(3.24, 0.25)$, $C'(-0.414, 1)$, $D'(2.41, 1)$, $E'(1, -1)$
	(b)	(i) Not many got the range of k correct	$h: x \to (x-1)^2 + 2,\ x \in R,\ x \geq 1$ $k: x \to -2x - 3, x \geq -2$ $R_k =]-\infty, 1]$ which is not a subset of the domain of h(x) for hk(x) to exist.
	(b)	(ii) Most could solve this	$hk(x) = h(ax+b)$ $4x^2 + 16x + 18 = (ax+b-1)^2 + 2$ $4x^2 + 16x + 18 = (ax+b)^2 - 2(ax+b) + 1 + 2$ $4x^2 + 16x + 18 = (ax)^2 + 2abx + b^2 - 2ax - 2b + 3$ Comparing the constant terms: $b^2 - 2b + 3 = 18$ $b^2 - 2b - 15 = 0$ $(b-5)(b+3) = 0$ $b = 5, -3$ (N.A.)

			Also, comparing coefficients of x: $2ab - 2a = 16$ $2a(b-1) = 16$ $a = 2$
	(c)	Many did not reject the positive answer and most did not get the domain correct	Let $y = \frac{4}{(2x-3)^2+1}$ $y(2x-3)^2 = 4 - y$ $2x - 3 = \pm\sqrt{\frac{4-y}{y}}$ $x = \frac{1}{2}\left[3 \pm \sqrt{\frac{4-y}{y}}\right]$ $p^{-1}(x) = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{4-x}{x}}$ since $R_{p^{-1}} = D_p$ $p^{-1}(x) = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{4-x}{x}}$, $0 < x \leq 4$
10	[Maximum mark: 16]		
	(a)	(i) careless	Vector perpendicular to both lines, $\mathbf{n} = \begin{pmatrix} \mathbf{3} \\ \mathbf{2} \\ \mathbf{-2} \end{pmatrix} \times \begin{pmatrix} \mathbf{-3} \\ \mathbf{4} \\ \mathbf{-1} \end{pmatrix} = \mathbf{3}\begin{pmatrix} \mathbf{2} \\ \mathbf{3} \\ \mathbf{6} \end{pmatrix}$
		(ii) Some took AB as the shortest distance	Let A be the point (-3, -4, 6) and B be the point (4, -7, -3). $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -7 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ -9 \end{pmatrix}$ Shortest distance between the skew lines $= \frac{\left\|\begin{pmatrix} 7 \\ -3 \\ -9 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}\right\|}{\sqrt{2^2 + 3^2 + 6^2}}$ $= \frac{\|7 \times 2 + (-3) \times 3 + (-9) \times 6\|}{7}$ $= 7$ Or Plane containing π_1 $\boldsymbol{r_1}.\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix}.\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = -18$ Plane containing π_1 $\boldsymbol{r_1}.\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ -3 \end{pmatrix}.\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = -31$ Shortest distance between the skew lines = perpendicular distance between parallel planes

			$= \frac{\lvert 18-(-31)\rvert}{\left\lVert\begin{pmatrix}2\\3\\6\end{pmatrix}\right\rVert} = \frac{49}{\sqrt{2^2+3^2+6^2}} = \frac{49}{\sqrt{49}} = 7$ units
	(b)	(i) Good	$\boldsymbol{p} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + t\begin{pmatrix}1\\-2\\3\end{pmatrix}$ At $t=0$, $\boldsymbol{p} = \begin{pmatrix}1\\2\\-1\end{pmatrix}$ $\therefore P(1,2,-1)$
	(b)	(ii)some switched cartesian to vector form instead of working the other way round	$x = 1+t \Rightarrow t = x-1$ $y = 2-2t \Rightarrow t = \frac{2-y}{2}$ $z = 3t-1 \Rightarrow t = \frac{z+1}{3}$ $\therefore x-1 = \frac{2-y}{2} = \frac{z+1}{3}$
	(b)	(iii)(a) Good	$2(1+t)+(2-2t)+3t-1=6$ $t=1$
		(iii) (b) Mostly well done	At $t=1$, $\boldsymbol{p} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \begin{pmatrix}1\\-2\\3\end{pmatrix} = \begin{pmatrix}2\\0\\2\end{pmatrix}$ Coordinates of P are (2, 0, 2)
		(iii) (c) Mostly well done	Distance travelled $= \left\lvert\begin{pmatrix}2\\0\\2\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix}\right\rvert = \left\lvert\begin{pmatrix}1\\-2\\3\end{pmatrix}\right\rvert$ $= \sqrt{1^2+(-2)^2+3^2} = \sqrt{14}$
11	[Maximum mark: 16]		
	(a)	Poorly done. Correct ones did not remove $\frac{1}{1+w^3}$	$(1-z)^6 - z^6 = 0$ $\left(\frac{1-z}{z}\right)^6 = 1$ $1, w, w^2, w^3, w^4$ and w^5 are solutions of the complex equation $z^6 = 1$ Let $w = \frac{1-z}{z}$, $z = \frac{1}{1+w}$ $z = \frac{1}{1+1}, \quad \frac{1}{1+w}, \quad \frac{1}{1+w^2}, \quad \frac{1}{1+w^3}\ (N.A.), \quad \frac{1}{1+w^4}, \quad \frac{1}{1+w^5}$ $\therefore z = \frac{1}{2}, \quad \frac{1}{1+w}, \quad \frac{1}{1+w^2}, \quad \frac{1}{1+w^4}, \quad \frac{1}{1+w^5}$

	(b)	(i) some had problem with the argument	$\lceil\sqrt{3}-i\rceil=\sqrt{3+1}=2$ $arg(\sqrt{3}-i)=-\frac{\pi}{6}$ Hence $\sqrt{3}-i=2cis\left(-\frac{\pi}{6}\right)$ By de Moivre's Theorem, $(\sqrt{3}-i)^n=2^n cis\left(-\frac{n\pi}{6}\right)$ $=2^n\left(cos\frac{1}{6}n\pi-isin\frac{1}{6}n\pi\right)$
		(ii) most could not give final answers	$(\sqrt{3}-i)^n=2^n\left(cos\frac{1}{6}n\pi-isin\frac{1}{6}n\pi\right)$ is real and negative $\Rightarrow sin\frac{n\pi}{6}=0$ and $cos\frac{n\pi}{6}<0$ $n=0,6,12,18,24,\ldots..$ and $n=6,18,30,42,\ldots\ldots$ Hence $n=6,18,30,42\ ,\ldots\ldots$ Or $n=6\times 1,6\times 3,6\times 5,6\times 7,\ldots.$
		(iii) numerous different errors	Subst. $\sqrt{3}-I$ into $z^9+16(1+i)z^3+a+ib=0$, $2^9\left(cos\frac{3}{2}\pi-isin\frac{3}{2}\pi\right)+16(1+i)\left[2^3\left(cos\frac{1}{2}\pi-isin\frac{1}{2}\pi\right)\right]+a+ib=0$ $512(i)+16(1+i)\times 8(-i)+a+ib=0$ $512(i)+128(-i+1)+a+ib=0$ $a+ib=-128-384i$ $\therefore a=-128\ and\ b=-384$