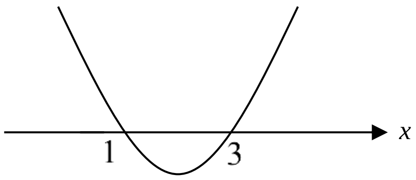
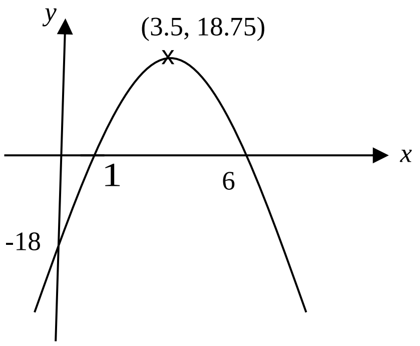
Solution to Term 2 CT Practice 4

1	$5^x + \frac{1}{25^{-0.5}} = 30\left(5^{x-1}\right)$ $5^x + 25^{0.5} = 30\left(\frac{5^x}{5}\right)$ $5^x + \sqrt{25} = 6\left(5^x\right)$ Let $y = 5^x$. $y + 5 = 6y$ $5 = 5y$ $y = 1$ $5^x = 1$ $\therefore x = 0$	
2	$(x+2)^2 - 8(x+2) + 15 \le 0$ $x^2 + 4x + 4 - 8x - 16 + 15 \le 0$ $x^2 - 4x + 3 \le 0$ $(x-3)(x-1) \le 0$  The range of values of *x* is $1 \le x \le 3$.	
3(i)	$-3x^2 + 21x - 18 = -3(x^2 - 7x + 6)$ $= -3\left[\left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 6\right]$ $= -3\left[\left(x - \frac{7}{2}\right)^2 - \frac{25}{4}\right]$ $= -3\left(x - \frac{7}{2}\right)^2 + \frac{75}{4}$ Maximum value is $\frac{75}{4}$.	

(ii)		**G1 – indicating all the intercepts** **G1 – indicating the turning point** **G1 – correct shape, smooth curve, labelling axis**
4	$\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}-\frac{2-2\sqrt{3}}{\sqrt{24}}$ $=\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}\times\frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}-\frac{2-2\sqrt{3}}{2\sqrt{6}}$ $=\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}\times\frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}-\frac{1-1\sqrt{3}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$ $=\frac{6-3\sqrt{6}-2\sqrt{6}+6}{-6}-\frac{\sqrt{6}-\sqrt{18}}{6}$ $=-2+\frac{1}{2}\sqrt{2}+\frac{3}{2}\sqrt{6}$	
5	$y=x+1$---- (1) $9x^2+y^2-3x-4=0$----(2) Sub (1) into (2): $9x^2+(x+1)^2-3x-4=0$ $10x^2-x-3=0$ $x=-\frac{1}{2}$ or $\frac{3}{5}$ When $x=-\frac{1}{2}$, $y=\frac{1}{2}$ When $x=-\frac{3}{5}$, $y=\frac{8}{5}$ The 2 coordinates are $\left(-\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{3}{5},\frac{8}{5}\right)$.	**Deduct 1 mark for not writing in coordinates form**

6	$12x-2m=(m+3)x^2$ $12x-2m=mx^2+3x^2$ $x^2(m+3)-12x+2m=0$ $b^2-4ac\ge 0$ $144-4(m+3)(2m)\ge 0$ $-8m^2-24m+144\ge 0$ $-m^2-3m+18\ge 0$ $m^2+3m-18\le 0$ $(m+6)(m-3)\le 0$ $\therefore -6\le m\le 3$	**Line meets curve means** $b^2-4ac\ge 0$ **Change the sign when you divide by negative number!**
***7**	$y=-(1+p^2)x^2+2pqx-(2q^2+1)$ Since $-(1+p^2)<0,$ the graph is an inverted U shape. b^2-4ac $=(2pq)^2-4(1+p^2)(2q^2+1)$ $=4p^2q^2-4(2q^2+1+2p^2q^2+p^2)$ $=-4p^2q^2-8q^2-4-4p^2$ $=-4(p^2q^2+2q^2+1+p^2)$ Since $p^2\ge 0$ and $q^2\ge 0$, $p^2q^2+2q^2+1+p^2\ge 1.$ $-4(p^2q^2+2q^2+1+p^2)\le -4<0,$ Hence $b^2-4ac<0.$ $\therefore y=-(1+p^2)x^2+2pqx-(2q^2+1)$ is **always negative** for all real values of *x*.	**Must mention the 2 conditions for the graph to be always negative!**

8	$\frac{1}{2}\times AM\times 2\left(\sqrt{2}+4\right)=14$ $AM\times\left(\sqrt{2}+4\right)=14$ $\therefore AM=\frac{14}{\sqrt{2}+4}$ $=\frac{14}{\sqrt{2}+4}\times\frac{\sqrt{2}-4}{\sqrt{2}-4}$ $=\frac{14\left(\sqrt{2}-4\right)}{2-16}$ $=-\left(\sqrt{2}-4\right)$ $=\left(4-\sqrt{2}\right)$ **cm**	
9	$2x-1\le x^2-4<12$ $2x-1\le x^2-4$ $-x^2+2x+3\le 0$ $(x+1)(x-3)\le 0$ $\therefore x\le -1$ or $x\ge 3$ .... (1) $x^2-4<12$ $x^2-16<0$ $(x-4)(x+4)<0$ $\therefore -4<x<4$ .... (2) Combining the 2 **inequalities**, The range of values of x is The range of values of x is $-4<x\le -1$ or $3\le x<4$.	**3 is a crowd…** **Separate them into** **2 inequalities…**

10	$\sqrt{3x-2}=2+\sqrt{x-2}$ $3x-2=4+4\sqrt{x-2}+x-2$ (squaring both sides) $\quad\quad=2+x+4\sqrt{x-2}$ $3x-2-2-x=4\sqrt{x-2}$ (isolate the surd) $2x-4=4\sqrt{x-2}$ $\left[2\left(x-2\right)\right]^2=16\left(x-2\right)$ $4\left(x^2-4x+4\right)=16x-32$ $4x^2-16x+16-16x+32=0$ $4x^2-32x+48=0$ $x^2-8x+12=0$ $\left(x-6\right)\left(x-2\right)=0$ $\therefore x=6$ or $x=2$ Check: When $x=2$, $\text{LHS}=\sqrt{3(2)-2}=2$ $\text{RHS}=2+\sqrt{2-2}=2$ When $x=6$, $\text{LHS}=\sqrt{3(6)-2}=4$ $\text{RHS}=2+\sqrt{6-2}=4$ $\therefore x=6$ or $x=2$	