Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA303.1.E1 – Coordinate Geometry Exercise

1. Additional Mathematics 360 (2nd Edition) Exercise 7.3
   - 1.1 Question 14
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2. Additional Mathematics 360 (2nd Edition) Exercise 7.4
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3. Additional Mathematics 360 (2nd Edition) Exercise 8.1
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1.1 PQRS is a parallelogram with vertices at P (1,3), Q (3,5), R (6,2) and S. Find:
(i) the coordinates of S.

Equation of $\overline{PR}$:

Gradient of $\overline{PR} = \frac{2-3}{6-1}$
$= -\frac{1}{5}$

$y-3 = -\frac{1}{5}(x-1)$

$y = -\frac{1}{5}x + \frac{16}{5}$

Equation of $\overline{PQ}$

Gradient of $\overline{PQ} = \frac{5-3}{3-1}$
$= 1$

$y-1 = x-3$

$y = x-2$

?

Equation of $\overline{QS}$: $y = -\frac{1}{3}x + c$ —— ①

Sub (3,5) into ①

$5 = -\frac{1}{5} \times 3 + c$

$c = \frac{28}{5}$

$y = -\frac{1}{5}x + \frac{28}{5}$

Equation of $\overline{RS}$: $y = x + c$ —— ②

Sub (6,2) into ②

$2 = 6 + c$

$c = 4$

$y = x+4$

To find S, line RS and QS will intercept.

$\therefore -\frac{1}{5}x + \frac{28}{5} = x+4$

$x = \frac{4}{3}$

Sub $x = \frac{4}{3}$ into $y = x+4$

$y = \frac{4}{3} + 4$

$y = \frac{16}{3}$

$S : (\frac{4}{3}, \frac{16}{3})$

poor approach.. fail to understand the geometry and made false assumptions.

(ii) the coordinates of point T such that $PT = QT$ and T lies on the line $y = x$

Let T be (n, n)

$\sqrt{(n-1)^2+(n-3)^2} = \sqrt{(n-3)^2+(n-5)^2}$

$(n-1)^2+(n-3)^2 = (n-3)^2+(n-5)^2$

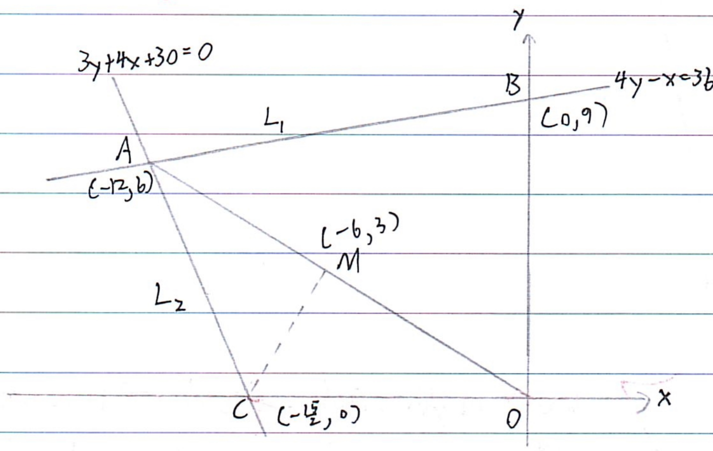
$n^2-2n+1 = n^2-10n+25$

$8n = 24$

$n = 3$

T: $(3, 3)$

1.2



The diagram shows the line L_1 and L_2 intersecting at the point A. The equation of L_1 is $4y-x=36$ and the equation of L_2 is $3y+4x+30=0$. The point M is the midpoint of OA. Line L_1 intersects the y-axis at the point B and Line L_2 intersects the x-axis at the point C.

(i) Show that CM is perpendicular to OA.

To find A, $L_1 = L_2$

$3y+4x+30=0$

$3y = -4x-30$

$y = -\frac{4}{3}x - 10$ ——— ①

$4y - x = 36$

$4y = x+36$

$y = \frac{1}{4}x+9$ ——— ②

① = ②

$-\frac{4}{3}x - 10 = \frac{1}{4}x + 9$

$-\frac{19}{12}x = 19$

$x = -12$

$y = 6$

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$A: (-12, 6)$

Equation of $\overline{OA}$:

Gradient of $\overline{OA} = \frac{6-0}{-12-0}$

$= -\frac{1}{2}$

$y - 0 = -2(x-0)$

$y = -2x$

To find L, ① $=0$

$-\frac{4}{3}x - 10 = 0$

$x = -\frac{15}{2}$

$L: (-\frac{15}{2}, 0)$

To find M,

$(x, y) = (\frac{-12-0}{2}, \frac{6-0}{2})$

$M = (-6, 3)$

Gradient of $\overline{LM} = \frac{3-0}{-6-(-\frac{15}{2})}$

$= 2$

As the gradient of $\overline{LM}$ is the negative reciprocal of the gradient of $\overline{OA}$, and $2 \times (-\frac{1}{2}) = -1$, they are perpendicular lines.

(ii) Find the ratio of the area of the triangle OAB to the area of the triangle OAL.

To find B, sub $x=0$ into ②

$\therefore y = 9$

Area of $\triangle OAB = \left|\frac{1}{2} \times 9 \times (-12)\right|$ units^2

$= 54$ units^2

Area of $\triangle OAL = \left|\frac{1}{2} \times (-\frac{15}{2}) \times 6\right|$ units^2

$= 22.5$ units^2

$OAB : OAL$

$= 54 : 22.5$

$= \therefore 12 : 5$

Area of $\triangle OAB = \frac{1}{2} \times 9 \times (-12)$ units^2

$= 54$ units^2

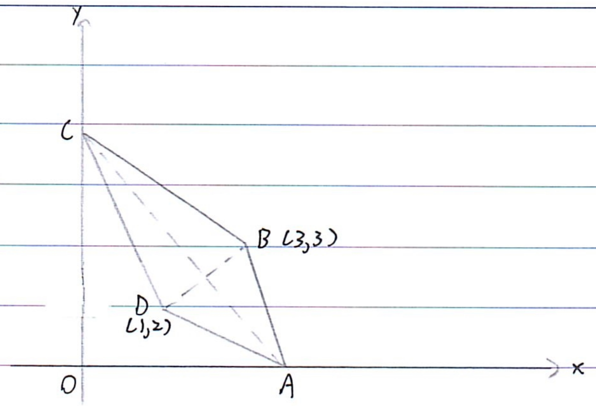
this is a negative value.

Area of $\triangle OAL = \frac{1}{2} \times (-\frac{15}{2}) \times 6$ units^2

please do not introduce unnecessary notations.

$= 22.5$ units^2

2.1



The diagram shows a kite ABCD in which B is the point (3,3) and D is the point (1,2). The point A lies on the x-axis and the point C lies on the y-axis. The diagonal AC bisects the diagonal BD at right angles.

(i) Find the equation of AC.

~~Let A be (n,0) and C be (0,m)~~

Gradient of $\overline{AC} = -1\left(\frac{3-2}{3-1}\right)$ ✗ wrong concept

$= -\frac{1}{2}$

Midpoint of $\overline{BD} = \left(\frac{3-1}{2}, \frac{3-2}{2}\right)$

$= (1, \frac{1}{2})$

Equation of $\overline{AC}$: $y = -\frac{1}{2}x + c$ —— ①

Sub $(1, \frac{1}{2})$ into ①

$\frac{1}{2} = -\frac{1}{2} + c$

$c = 1$

Equation of $\overline{AC}$: $y = -\frac{1}{2}x + 1$ —— ②

(ii) Find the coordinates of A and C

Let A be (n,0) and C be (0,m)

Sub (n,0) into ②

$0 = \frac{1}{2}n + 1$

$n = -2$

A : (−2, 0)

Sub (0,m) into ②

$m = \frac{1}{2}(0) + 1$

$m = 1$

C : (0, 1)

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(iii) Hence find the area of kite ABCD

To find length $BD = \sqrt{(3-2)^2+(3-1)^2}$

$= \sqrt{5}$

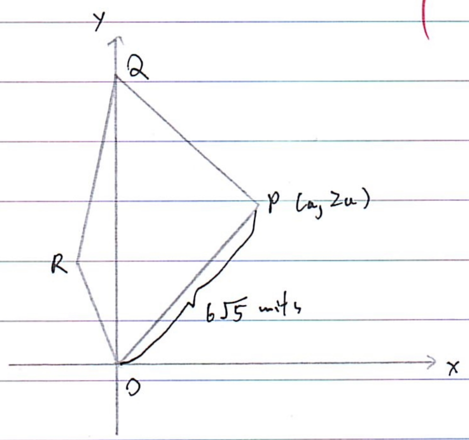
To find length $AC = \sqrt{(0-1)^2+(-2-0)^2}$

$= \sqrt{5}$

Area of $ABCD = (\sqrt{5} \cdot \sqrt{5})$ units^2

$= 5$ units^2

2.2



The diagram shows a quadrilateral OPQR. The coordinates of point P are $(a, 2a)$, where $a>0$. The length of OP is $6\sqrt{5}$ units.

(i) show that $a=6$

$\sqrt{(a-0)^2+(2a-0)^2} = 6\sqrt{5}$

$a^2 + 4a^2 = 180$

$a^2 = 36, a>0$

$a = 6$ (shown) what about -6?

PQ is perpendicular to OP and Q lies on the y-axis. Find

(ii) the equation of PQ

Equation of $\overline{PO}$: $y = 2x$

Gradient of $\overline{PQ} = -1 \div (2)$

$= -\frac{1}{2}$

$y - 12 = -\frac{1}{2}(x-6)$

Equation of $\overline{PQ}$:

$y = -\frac{1}{2}x + 15$

(iii) the coordinates of Q.

Let Q be $(0, n)$

Sub $(0, n)$ into $y = -\frac{1}{2}x + 15$

$n = -\frac{1}{2}(0) + 15$

$n = 15$

$Q: (0, 15)$

The point R lies on the perpendicular bisector of PQ. Given that the line OR is parallel to the line $11x + 2y - 4 = 0$, find

(iv) the coordinates of R

$11x + 2y - 4 = 0$

$2y = -11x + 4$

$y = -\frac{11}{2}x + 2$

Since OR is parallel, the gradient is the same.

$y = -\frac{11}{2} + c$ —— ①

Sub $(0, 0)$ into —— ①

$c = 0$

$y = -\frac{11}{2}x$

Mid point of $\overline{PQ} = \left(\frac{6+0}{2}, \frac{15+12}{2}\right)$

$= (3, 13.5)$

Gradient of perpendicular bisector = gradient of $\overline{PO} = 2$

∴ Equation: $y = 2x + c$ —— ②

Sub $(3, 13.5)$ into ②

$13.5 = 2 \times 3 + c$

$c = \frac{15}{2}$

Equation: $y = 2x + \frac{15}{2}$

R is the intersection of $y = 2x + \frac{15}{2}$ and $y = -\frac{11}{2}x$

∴ $2x + \frac{15}{2} = -\frac{11}{2}x$

$x = -1$

$y = \frac{11}{2}$

$R: (-1, \frac{11}{2})$

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(v) the area of quadrilateral OPQR.

Area of OPQR $= \left| 15 \times 6 \times \frac{1}{2} + (-1) \times 15 \times \frac{1}{2} \right|$ units^2

$= 52.5$ units^2

(flawed)

is $|a-b| = |a| + |b|$?

Stick to basics, please.

3.1 Find the equation of the tangent to the circle $x^2+y^2+2x-2y-3=0$ at the point $(1,2)$

$x^2+2x+y^2-2y-3=0$

$(x+1)^2-1+(y-1)^2-1-3=0$

$(x+1)^2+(y-1)^2=5$

Center of circle: $(-1,1)$

Gradient of line from radius:

Gradient $= \frac{1-2}{-1-1}$

$= \frac{1}{2}$

$\therefore$ Gradient of tangent $= -1 \div (\frac{1}{2})$

$= -2$

Equation of tangent: $y = -2x + c$ —— ①

Sub $(1,2)$ into ①

$2 = -2 + c$

$c = 4$

Equation: $y = -2x + 4$

3.2 A circle passes through the points $P(3,0)$ and $Q(0,5)$. Its center lies on the line $y = x+2$

(i) Find the equation of the perpendicular bisector of PQ

Midpoint of $\overline{PQ} = \left(\frac{3+0}{2}, \frac{0+5}{2}\right)$

$= \left(\frac{3}{2}, \frac{5}{2}\right)$

Gradient of perpendicular bisector $= -1 \div \left(\frac{5-0}{0-3}\right)$

$= \frac{3}{5}$

$\therefore y = \frac{3}{5}x + c$ —— ①

Sub $\left(\frac{3}{2}, \frac{5}{2}\right)$ into ①

$\frac{5}{2} = \frac{3}{5} \cdot \frac{3}{2} + c$

$c = \frac{8}{5}$

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Equation: $y = \frac{3}{5}x + \frac{8}{5}$

(ii) Hence show that the coordinates of the centre of the circle are $(-1, 1)$

$\frac{3}{5}x + \frac{8}{5} = x+2$

$-\frac{2}{5}x = \frac{2}{5}$

$x = -1$

$y = 1$ (shown)

(iii) Find the equation of the circle.

Radius $= \sqrt{(-1-3)^2+(1-0)^2}$ units

$= \sqrt{17}$

Radius $= \sqrt{(-1-3)^2+(1-0)^2}$ units

$= \sqrt{17}$ units

Equation: $(x+1)^2+(y-1)^2=17$

A second circle with the equation $x^2+y^2+ax+by-14=0$ has the same center as the first circle.

(iv) Write down the value of a and b.

$a = 2$

$b = -2$

(v) Show that the second circle lies inside the first circle.

Radius of smaller circle $= \sqrt{(1)^2+(1^2)-(-14)}$ units

$= 4$ units

What about their centres?

As $4 < \sqrt{17}$, the second circle is inside the first circle, as their center are the same (shown)

3.3 The equation of circle C_1 is $x^2+y^2-10x+4y=-20$

(i) Find the radius and coordinates of center of circle

Radius $= \sqrt{2^2+(-5)^2-20}$ units

$= 3$ units

Center of circle: $(5, -2)$

The coordinates of the center of a second circle, C_2, are $(-2, 3)$. The equation of the tangent to C_2 at a point P is $y-2x=17$.

(ii) Find the coordinates of P.

$y = 2x + 17$

Gradient of perpendicular line $= -1 \div 2$

$= -\frac{1}{2}$

Equation : $y = -\frac{1}{2}x + c$ —— ①

sub $(-2, 3)$ into ①

$3 = -\frac{1}{2}(-2) + c$

$c = 2$

$\therefore y = -\frac{1}{2}x + 2$

P is intersection of both lines.

$-\frac{1}{2}x + 2 = 2x + 17$

$\frac{5}{2}x = -15$

$x = -6$

$y = 5$

$P : (-6, 5)$

(iii) Find the equation of C_2:

Radius $= \sqrt{(-2-(-6))^2 + (3-5)^2}$ units

$= \sqrt{20}$ units

Equation: $(x+2)^2 + (y-3)^2 = 20$

(iv) Determine whether C_1 and C_2 intersect each other. Show your method clearly.

Let the radius of C_1 be R_1. units

Let the radius of C_2 be R_2. units

$R_1 + R_2 = 3 + \sqrt{20}$

$= 7.47$ ~~(3s.f.)~~

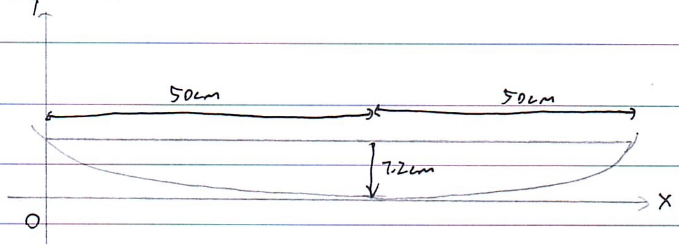
$R_2 - R_1 = 1.47$ (3s.f.)

Distance between 2 midpoints $= \sqrt{(-6-5)^2 + (5-(-2))^2}$

$= 13.04$ (3s.f.)

As $d > R_1 + R_2$, the circles do not intersect.

3.4 Diameter of a Broken pipe. An archaeologist finds a piece of what seems to be a fragment of a circular pipe. She places the ends of a metre-long stick against the inside surface of the pipe and finds that the midpoint of the stick is 7.2cm from the pipe wall.



(i) Assuming that the cross-section of the fragment is part of a circle of radius r cm, model the cross-section using an equation in terms of r.

$r^2 = 50^2 + (r-7.2)^2$

this is not an equation of the curve

(ii) Hence find the inside diameter of the pipe, giving your answer in metres.

$r^2 = 50^2 + r^2 - 14.4r + 51.84$

$-14.4r + 51.84 = -2500$

$r = 177$ cm (3 s.f.)

$= 1.77$ m

did you read?

Very poor effort.
28/3/24

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1.1 (i) Since it is parallelogram,

PR and QS intersect at their midpoints.

Point of intersection $= \left(\frac{1+6}{2}, \frac{3+2}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

$S: \left(\frac{7}{2}\times2-3, \frac{5}{2}\times2-5\right)$

$= (4, 0)$

2.1 (i) AC is perpendicular bisector of BD.

Gradient of BD $= \frac{3-2}{3-1}$

$= \frac{1}{2}$

$\therefore$ Gradient of AC $= (-1) \div \frac{1}{2}$

$= -2$

Midpoint of BD $= \left(\frac{3+1}{2}, \frac{3+2}{2}\right)$

$= \left(2, \frac{5}{2}\right)$

$-2(x-2) = y - \frac{5}{2}$

AC: $y = -2x + \frac{13}{2}$

(ii) When $y=0$,

$x = \frac{13}{4}$

$\therefore A: \left(\frac{13}{4}, 0\right)$

When $x=0$,

$y = \frac{13}{2}$

$\therefore C: \left(0, \frac{13}{2}\right)$

(iii) Area $= \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 & \frac{13}{4} & 3 \\ 3 & \frac{13}{2} & 2 & 0 & 3 \end{vmatrix}$ units^2

$= \frac{1}{2}\left\{3\left(\frac{13}{2}\right) + \frac{13}{4}(3) - \frac{13}{2} - \frac{13}{2}\right\}$ units^2

$= 8.125$ units^2

2.2 (iv) Area of OPQR $= \frac{1}{2}\begin{vmatrix} 6 & 0 & -1 & 0 & 6 \\ 12 & 15 & \frac{11}{2} & 0 & 12 \end{vmatrix}$ units^2

$= \frac{1}{2}\{6(15)+15\}$ units^2

$= 52.5$ units^2

3.3 (iv) Let R_1 units be radius of C_1

Let R_2 units be radius of C_2.

$D = \sqrt{(-2-5)^2+(3+2)^2}$ units

$= \sqrt{74}$ units.

Since $\sqrt{74} > r_1 + r_2$, the circles do not intersect.

$R_1 + R_2$? what's r_1 & r_2?

3.4 (i) x-coordinate of centre $= \frac{50+50}{2}$

$= 50$

y-coordinate of centre $= r$

$\therefore$ Equation: $(x-50)^2+(y-r)^2=r^2$

(ii) When $x=100$, $y=7.2$

$(100-50)^2+(7.2-r)^2=r^2$

$2500+51.84-14.4r+r^2=r^2$

$r = \frac{15949}{90}$

$\therefore$ Inside diameter $= 2r$ cm

$= \frac{15949}{90} \times 2$ cm

$= 3.54$m (3 s.f.)