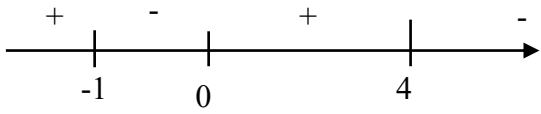
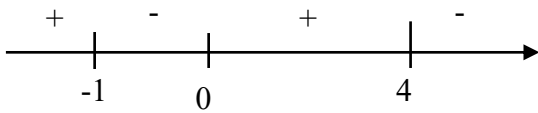


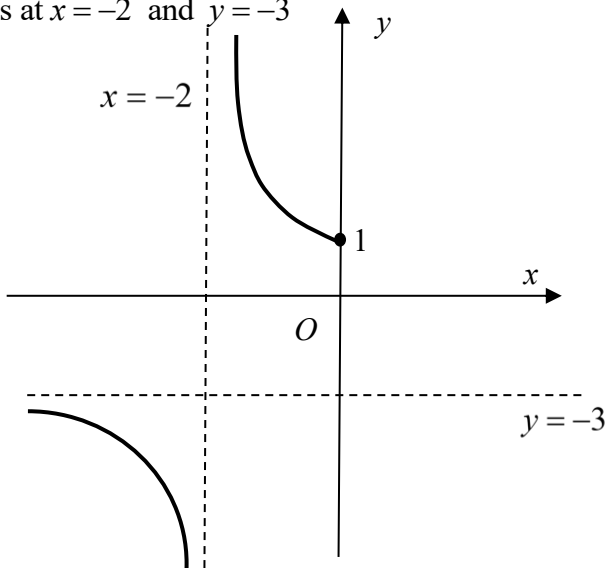
2026 ACJC H2 Math Preliminary Examination Paper 1 Solutions

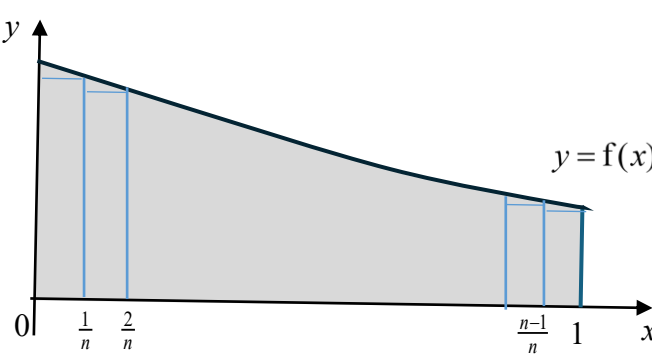
Qn	Solution	Remarks
1	$\frac{x(5x+9)}{x^2+2x+5} > x \geq 3x-6$ Consider $\frac{x(5x+9)}{x^2+2x+5} > x$ $\frac{x(5x+9)}{x^2+2x+5} - x > 0$ $\frac{5x^2+9x-(x^3+2x^2+5x)}{x^2+2x+5} > 0$ $\frac{-x^3+3x^2+4x}{x^2+2x+5} > 0$ Since $x^2+2x+5 = (x+1)^2+4 > 0$ for all x $-x^3+3x^2+4x > 0$ $-x(x^2-3x-4) > 0$ $-x(x+1)(x-4) > 0$ Using sign test:  $x < -1$ or $0 < x < 4$ Consider $x \geq 3x-6$ $x \leq 3$ Intersection of both results: $x < -1 \text{ or } 0 < x \leq 3$	Please note the following when solving inequalities: Any common term involving x cannot be cancelled on both sides of inequality We should try to factorise and not expand the terms. The denominator should not just be ignored or stating the quadratic denominator has non-real roots is not enough. Instead, it is necessary to show that the denominator is positive for all x. $-x^3+3x^2+4x > 0$ becomes $-x(x+1)(x-4) > 0$ or $x(x+1)(x-4) < 0$ There are 3 critical points when doing sign test including $x = 0$. Since question requires you to use an algebraic method, either the sign test or sketch of the curve to arrive at answer should be shown. Intersection of the answers to the two inequalities should be

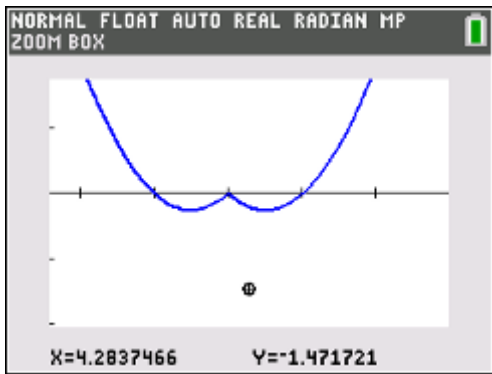
	Alternative method: $\frac{x(5x+9)}{x^2+2x+5} > x \geq 3x-6$ Consider $\frac{x(5x+9)}{x^2+2x+5} > x$, $\frac{x(5x+9)}{x^2+2x+5} > x$ Since $x^2+2x+5 = (x+1)^2+4 > 0$ for all x $x(5x+9) > x(x^2+2x+5)$ $x(5x+9) - x(x^2+2x+5) > 0$ $x(5x+9 - (x^2+2x+5)) > 0$ $x(-x^2+3x+4) > 0$ $-x(x^2-3x-4) > 0$ $-x^3+3x^2+4x > 0$ $-x(x+1)(x-4) > 0$ Using sign test:  $x < -1$ or $0 < x < 4$ Consider $x \geq 3x-6$ $x \leq 3$ Intersection of both results: $x < -1$ or $0 < x \leq 3$	considered to arrive at final answer.
--	---	--

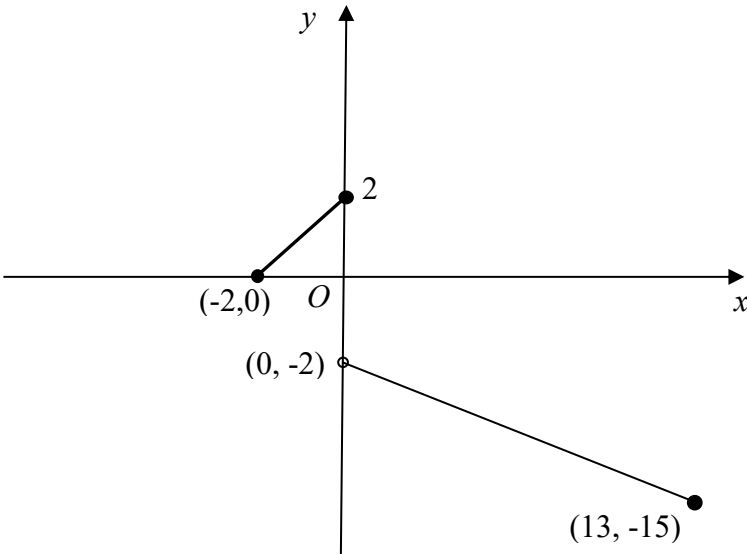
2(a)	$\ln(2 + \sin x)$ $\approx \ln(2 + x) \quad \text{since } x \text{ is small, } \sin x \approx x$ $= \ln \left(2 \left(1 + \frac{x}{2} \right) \right)$ $= \ln 2 + \ln \left(1 + \frac{x}{2} \right)$ $\approx \ln 2 + \frac{x}{2} - \frac{\left(\frac{x}{2} \right)^2}{2}$ $= \ln 2 + \frac{x}{2} - \frac{x^2}{8}$	Read question carefully, using standard series from MF27 is expected. Do NOT use the wrong method of repeated differentiation. Students need to revise the laws of logarithms. $\ln(AB) = \ln A + \ln B$
2(b)	Equation of the tangent is $y = \ln 2 + \frac{x}{2}$.	Equation of tangent is a line, not curve, which can be inferred from answer in part (a).
3(a)	Length of equilateral triangle base $= l - 2 \left(\frac{x}{\tan \frac{\pi}{6}} \right)$ $= l - 2x\sqrt{3} \quad (\text{shown})$	Students should not complicate the proof by splitting into many triangles and using different angles.
3(b)	$V = \frac{1}{2} (l - 2\sqrt{3}x)^2 \left(\sin \frac{\pi}{3} \right) x$ $= \frac{1}{2} (l - 2\sqrt{3}x)^2 \left(\frac{\sqrt{3}}{2} \right) x$ $= \frac{\sqrt{3}}{4} (l - 2\sqrt{3}x)^2 x$ $\frac{dV}{dx} = \frac{\sqrt{3}}{4} (l - 2\sqrt{3}x)^2 + \frac{\sqrt{3}}{4} [2(l - 2\sqrt{3}x)](-2\sqrt{3})x$ $= \frac{\sqrt{3}}{4} (l - 2\sqrt{3}x)(l - 2\sqrt{3}x - 4\sqrt{3}x)$ $= \frac{\sqrt{3}}{4} (l - 2\sqrt{3}x)(l - 6\sqrt{3}x)$	Students need to revise on area of triangle $= \frac{1}{2} ab \sin C.$ l is a fixed length, x is the variable, hence to maximise volume of box, you need to find $\frac{dV}{dx}$. Then set $\frac{dV}{dx} = 0$ to solve for x.

	When $\frac{dV}{dx} = 0$ $\Rightarrow \frac{\sqrt{3}}{4}(l - 2\sqrt{3}x)(l - 6\sqrt{3}x)$ $\Rightarrow x = \frac{l}{6\sqrt{3}} \quad \text{or} \quad \frac{l}{2\sqrt{3}} \quad (\text{rejected as length of base} = 0)$	Students must learn to read the diagram carefully and reject one of the solutions.
4(a)	$3xy + 1 = (x + y)^2$ $3\left(x \frac{dy}{dx} + y\right) = 2(x + y)\left(1 + \frac{dy}{dx}\right)$ $3x \frac{dy}{dx} + 3y = 2x + 2y + 2(x + y) \frac{dy}{dx}$ $\frac{dy}{dx}(x - 2y) = 2x - y$ $\frac{dy}{dx} = \frac{2x - y}{x - 2y} \quad (\text{shown})$	
4(b)	Tangent is parallel to the x-axis $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow y = 2x$ Solving simultaneously with equation of C, $3xy + 1 = (x + y)^2$ $3x(2x) + 1 = (x + 2x)^2$ $3x^2 = 1$ $x = \pm \frac{1}{\sqrt{3}}$	Tangent // to x-axis $\Rightarrow \frac{dy}{dx} = 0$, useful to draw a diagram to help visualize that the tangent line is a horizontal line. Note that the gradient is 0, not undefined. When solving $\Rightarrow \frac{dy}{dx} = 0$ $\Rightarrow \frac{2x - y}{x - 2y} = 0$ For fraction = 0, numerator = 0. Do not set denominator = 0 too!
4(c)	When $x = 0$, $\frac{dy}{dx} = \frac{1}{2}$ Acute angle between the tangents to C at $(0, 1)$ and the x-axis $= \tan^{-1} \frac{1}{2}$ or 0.464 rad (to 3 s.f.)	Recall angle of inclination. Let $\frac{dy}{dx} = \frac{1}{2} = m$, Hence angle of inclination $= \tan^{-1} m$

		There is no need to find the equation of tangent here. Please leave angle in radian form for calculus questions.
5(a)	$f: x \mapsto \frac{kx+2}{2-x}, \quad x \in \mathbb{R}, x \neq 2.$ If $(-7, 4)$ lies on the curve $y = f^{-1}(x)$, then $(4, -7)$ lies on the curve $y = f(x)$, $f(4) = -7$ $\frac{4k+2}{2-4} = -7$ $k = 3$ (shown)	Finding the inverse of f is not the expected method. Take note of the property of all inverse functions: If $f(a) = b$, then $f^{-1}(b) = a$
5(b)	For $y = f(x) = \frac{3x+2}{2-x} = -3 + \frac{8}{2-x}$ When $x = 0, y = 1$. Asymptotes at $x = 2$ and $y = -3$. Graph of $y = g(x) = f(x)$: Asymptotes at $x = -2$ and $y = -3$ 	Reading the question well should help. $g: x \mapsto f(x),$ for $x \in \mathbb{R}, x \leq 0$ and $x \neq -2$ Graph of function should be sketched for the domain of function only.

(c)	$g: x \mapsto f(x)$, for $x \leq 0$ and $x \neq -2$. For $x \leq 0$, replace x by $-x$. $y = f(x) = \frac{3 x +2}{2- x } = \frac{3(-x)+2}{2-(-x)} = \frac{-3x+2}{2+x}$ $2y + xy = -3x + 2$ $x = \frac{2-2y}{3+y}$ $g^{-1}(x) = \frac{2-2x}{3+x}$ $D_{g^{-1}} = R_g = (-\infty, -3) \cup [1, \infty)$ or $x < -3$ or $x \geq 1$	To find inverse of g, figure out rule of function g first. From above part, g is the reflection of f in the y axis. So $g(x) = f(-x)$ for $x \in \mathbb{R}, x \leq 0$ and $x \neq -2$ Range of g can be read from the graph in part (b). Use of notation : $\cup$ is union $\cap$ is intersection
6(a)	$h = \frac{1}{n}$	
6(b)	Total area of the n rectangles = $\frac{1}{n}f\left(\frac{1}{n}\right) + \frac{1}{n}f\left(\frac{2}{n}\right) + \dots + \frac{1}{n}f\left(\frac{n}{n}\right)$ $= \sum_{r=1}^n hf(rh) < A$ 	Rectangles drawn under the curve with width $\frac{1}{n}$ or h labelled. The rectangles should be of equal width. Explanation that relates $\sum_{r=1}^n hf(rh)$ to total area of rectangles (not strips) before concluding that it is less than area of A

6(c)	$\lim_{n \rightarrow \infty} \sum_{r=1}^n hf(rh)$ $= \int_0^1 \frac{e^{-2x}}{e^{-2x} + 1} dx$ $= \left[-\frac{1}{2} \ln e^{-2x} + 1 \right]_0^1$ $= \frac{1}{2} [\ln(2) - \ln(1 + e^{-2})]$	As the number of rectangles used in part (b) increases the total area of rectangles tends to area of A. The limit as n tends to infinity is the exact area of A which is obtained using definite integration
7(a)	$f: x \mapsto (x-4)^2 - x-4 , 0 \leq x \leq \lambda$, where λ is a constant Using GC: Greatest value of λ is $\frac{7}{2}$ or 3.5. 	For f^{-1} to exist, f must be one to one function. Note that domain of f is given as $0 \leq x \leq \lambda$. Use GC to obtain graph of f and find greatest value of λ. Use GC “2nd calc” “minimum” to find the x coordinate of the first minimum point as the domain given is $0 \leq x \leq \lambda$. You may need to use Zoom box to enlarge the graph first.

7(b)	$g(x) = \begin{cases} x+2 & \text{for } -2 \leq x \leq 0, \\ -x-2 & \text{for } 0 < x \leq 13. \end{cases}$ 	When sketching such graphs, take note which point is included and which is not. Note that $g(0)$ is 2 and not -2. Your graph should show that. There should be an unshaded circle on $(0, -2)$ to indicate that this point is not part of the graph of $g(x)$.
7(c)	Using GC: $R_f = \left[-\frac{1}{4}, 12\right]$ or $[-0.25, 12]$ $D_g = [-2, 13]$ gf exists since $R_f \subseteq D_g$	R_f and D_g should be stated and the term subset should be used (not R_f inside D_g) Use correct notation for subset.
7(d)	$g\left(-\frac{1}{4}\right) = \frac{7}{4}$ or 1.75 $g(12) = -14$ Range of function $gf(x) = [-14, -2) \cup [1.75, 2]$ or $[-14, -2) \cup \left[\frac{7}{4}, 2\right]$ $-14 \leq y < -2$ or $\frac{7}{4} \leq y \leq 2$	Restrict the x coordinates of graph of g in part (b) above, to the values in the range of f. $R_f = \left[-\frac{1}{4}, 12\right]$ Read the possible values of y from graph of g in part (b) above

7(e)	$gf(x) = -3$ $g(f(x)) = -3$ From graph of $g(x)$, $g(1) = -3$ $\therefore f(x) = 1$ $f(x) = (x-4)^2 - x-4 , \quad 0 \leq x \leq \frac{7}{2}$ $= (x-4)^2 - (4-x) \quad \text{since } x-4 < 0 \text{ for } 0 \leq x \leq \frac{7}{2}$ $= x^2 - 7x + 12$ $f(x) = 1$ $x^2 - 7x + 12 = 1$ $x^2 - 7x + 11 = 0$ $x = \frac{7 \pm \sqrt{49 - 4(11)}}{2} = \frac{7 \pm \sqrt{5}}{2}$ $\therefore x = \frac{7 - \sqrt{5}}{2}$ since $x \leq \frac{7}{2}$ (in domain of f)	Understanding that gf function means $g(f(x)) = -3$ ie g is acting on something to give -3. So, from part(b) above find the value of x for which $g(x) = -3$ ie $x = 1$. Domain of f has been found in part (a). Used it to simplify f. To simplify modulus function $x-4$ always look at what is inside the modulus, ie $x-4$. Since $x-4 < 0$ for $0 \leq x \leq \frac{7}{2}$ $x-4 = -(x-4)$ or $4-x$
8(a)	Area of triangle OBC $= \frac{1}{2} \mathbf{b} \times \mathbf{c}$ $= \frac{1}{2} \mathbf{b} \times (2\mathbf{a} - 2\mathbf{b})$ $= \mathbf{b} \times \mathbf{a} - \mathbf{b} \times \mathbf{b}$ $= \mathbf{b} \times \mathbf{a}$ $= \mathbf{a} \mathbf{b} \sin \frac{\pi}{6}$ $= (2)(3) \left(\frac{1}{2} \right)$ $= 3$	Errors in area of OBC due to using dot product, missing $\frac{1}{2}$, missing modulus, or using $\frac{1}{2} \mathbf{a} \times \mathbf{b}$. Note that: $\mathbf{a} \times \mathbf{b} \neq \mathbf{a} \mathbf{b} \cos \theta$ $\mathbf{b} \times \mathbf{b} = \mathbf{0} \neq \mathbf{b} ^2$ $\mathbf{a} - \mathbf{b} \neq \mathbf{a} - \mathbf{b}$

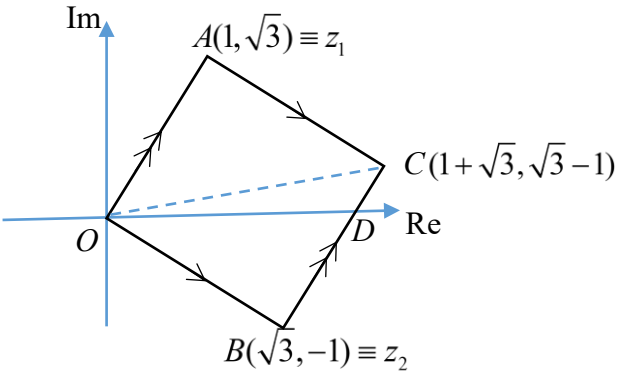
8(b)	$l_{OA} : \mathbf{r} = \lambda \mathbf{a}$ $l_{BC} : \mathbf{r} = \mathbf{b} + \mu(\mathbf{c} - \mathbf{b}) = \mathbf{b} + \mu(2\mathbf{a} - 3\mathbf{b})$ Equate the two lines: $\lambda \mathbf{a} = \mathbf{b} + \mu(2\mathbf{a} - 3\mathbf{b})$ We get $\lambda = 2\mu$ $0 = 1 - 3\mu$ $\Rightarrow \mu = \frac{1}{3}, \lambda = \frac{2}{3}$ $\overrightarrow{OX} = \frac{2}{3}\mathbf{a}$ (shown)	Some equated $\overrightarrow{OA} = \overrightarrow{BC}$ without considering <i>lines</i>. Some wrote l_{BC} as $\mathbf{r} = \mu(2\mathbf{a} - 3\mathbf{b})$ forgetting $\mathbf{b}$ or $\mathbf{c}$; or used the same λ for both lines. Students who used ratio theorem were usually stuck, and wrongly “divided by vectors”.
8(c)	$\overrightarrow{AX} = -\frac{1}{3}\mathbf{a}$ shortest distance from point X to plane p $= \overrightarrow{AX} \cdot \hat{\mathbf{n}}$ $= \left -\frac{1}{3}\mathbf{a} \cdot \mathbf{n} \right$ (since $\hat{\mathbf{n}} = \mathbf{n}$, and point A lies on plane p, hence $\mathbf{a} \cdot \mathbf{n} = d$) $= \frac{1}{3} d$, where $k = 3$	Poor presentation of vectors and mixing up of vectors ($\mathbf{n}$) and scalars ($\mathbf{n} , d$) led to invalid claims: $\mathbf{a} \cdot \mathbf{n} = d$ $\mathbf{n} = 1 \not\Rightarrow \mathbf{n} = 1$ or $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{a} \cdot \mathbf{n} = d \not\Rightarrow \mathbf{a} = \frac{d}{\mathbf{n}}$ $\mathbf{a} \cdot \mathbf{n} = d \not\Rightarrow \mathbf{a} = d$ Errors for length of projection formula include using $\overrightarrow{OX}$, using cross product, or omitting the modulus (leading to $\frac{1}{3}d$ instead). Those who tried finding the foot of perpendicular had difficulty with the simplification.

9(a)	$u_8 = 26$ $a + 7d = 26$ -----(1) $S_{12} = 258$ $\frac{12}{2}(2a + 11d) = 258$ $2a + 11d = 43$ -----(2) Sub (1) into (2): $2(26 - 7d) + 11d = 43$ $\therefore d = 3$ Sub $d = 3$ into (1): $a = 5$ $S_n > 1000$ $\frac{n}{2}(2(5) + (n - 1)(3)) > 1000$ $\frac{n}{2}(7 + 3n) > 1000$ Using GC, <table border="1"><thead><tr><th>n</th><th>$\frac{n}{2}(7 + 3n)$</th></tr></thead><tbody><tr><td>23</td><td>874</td></tr><tr><td>24</td><td>948</td></tr><tr><td>25</td><td>1025</td></tr></tbody></table> Minimum number of full hours is 25.	n	$\frac{n}{2}(7 + 3n)$	23	874	24	948	25	1025	Most could find a and d of this AP. But some <i>misread</i> the context that $a = d$, $d = 26$, $7d = 26$ or $a + u_{12} = 258$. In solving the final inequality, a significant number misread the last part to find the <i>minimum number of full hours before</i> “its cumulative energy stored exceeds 1000 units”. So, the inequality is $S_n > 1000$, not $S_n \leq 1000$. This gives $n > 24.7$, so the <i>minimum possible</i> n must be 25. If $S_n \leq 1000$, then a coherent reading of the question would mean that the <i>minimum possible</i> n is $n = 0$, not $n = 24$.
n	$\frac{n}{2}(7 + 3n)$									
23	874									
24	948									
25	1025									
9(b)	$u_1 = (1.02)(10000 + K) - 2500$ $u_2 = (1.02)([(1.02)(10000 + K) - 2500] + K) - 2500$ $= (1.02)^2(10000) + (1.02)^2 K + (1.02)K - (1.02)(2500) - 2500$	There were small slips in algebra, or missing out the K tokens added & 2500 tokens consumed in the 2nd month. A surprising number misread that the robot “consumed 2500								

		tokens” and added 2500 instead. For u_2, some rounded 2.0604K to 2.06K. This should not be done as 2.0604 is exact.
9 (c)	$u_n = (1.02)^n (10000) + (1.02)^n K + \dots + (1.02)^2 K + (1.02) K - 2500 \left[(1.02)^{n-1} + \dots + (1.02) + 1 \right]$ $u_n = (1.02)^n (10000) + \left[\frac{(1.02) K (1.02^n - 1)}{1.02 - 1} \right] - 2500 \left[\frac{(1)(1.02^n - 1)}{1.02 - 1} \right]$ $u_n = (1.02)^n (10000) + 51K (1.02^n - 1) - 125000 (1.02^n - 1)$ Hence $\alpha = 51$.	Students who carefully tabulated the process were generally able to solve the question. A common mistake was using $a = K$ for the first GP, instead of $a = (1.02) K$. Note that for <i>show</i> questions, the result to be shown <i>cannot</i> be assumed. Using u_1 to “backtrack” and find α leads to no credit, as the appropriate GPs were not shown nor derived.
9 (d)	$u_n = (1.02)^n (10000) + 51K (1.02^n - 1) - 125000 (1.02^n - 1)$ $= (1.02)^n (10000) + 51K (1.02)^n - 51K - 125000 (1.02)^n + 125000$ $= (1.02)^n [10000 + 51K - 125000] - 51K + 125000$ $= (1.02)^n [51K - 115000] - 51K + 125000$ Note: - Whether the wallet will run out of token in the long run depends on the coefficient of $(1.02)^n$. - If the coefficient of $(1.02)^n < 0$, when $n \rightarrow \infty$, $(1.02)^n \rightarrow -\infty$. The wallet will deplete and run out of tokens in the long run. - If the coefficient of $(1.02)^n > 0$, when $n \rightarrow \infty$, $(1.02)^n \rightarrow +\infty$. The wallet will grow and not run out of tokens in the long run. For $u_n > 0$, $51K - 115000 > 0$ $\therefore K > 2254.901961$ Hence minimum integer value of K is 2255.	Many did not make headway thinking that the question about “the long run” was asking about S_∞. Those who analysed the coefficient of 1.02^n in u_n often omitted the term $1.02^n (10\,000)$.

10(a) (i)	Method 1 Since a and b are real number, $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$ is also a root of the equation $z^3 + az^2 + bz - 1 = 0$. $\left(z + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(z + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = \left(z + \frac{1}{2}\right)^2 - \left(\frac{i\sqrt{3}}{2}\right)^2$ $= \left(z^2 + \frac{1}{4} + z + \frac{3}{4}\right) = z^2 + z + 1$ is a quadratic factor of $z^3 + az^2 + bz - 1$ Hence $(z^3 + az^2 + bz - 1) = (z^2 + z + 1)(z + c)$ Equate constant: $-1 = c$ Equate coefficient of z^2: $a = c + 1 = -1 + 1 = 0$ Equate coefficient of z: $b = c + 1 = 0$ Hence $z^3 - 1 = (z^2 + z + 1)(z - 1)$	Method 1 is most common used and those who used method 1 are quite successful in finding the quadratic factor and linear factor. Students who used method 2 are less successful. Common mistake include $\left(z - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$ $\left(z - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right) =$ $(z^2 - z + 1)$ Some do not know the difference between a root and a factor. Eg: $\left(z + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ is a root of $z^3 - 1 = 0$ which is incorrect. Should be $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$ is a root of the equation $z^3 - 1 = 0$ And $\left(z + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ is a factor of $z^3 - 1$
	Method 2 $z^3 = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3 = \frac{1}{8}(-1 + i\sqrt{3})^3 =$ $= \frac{1}{8} \left[(-1)^3 + 3(-1)^2(i\sqrt{3}) + 3(-1)(i\sqrt{3})^2 + (i\sqrt{3})^3 \right]$ $= \frac{1}{8} \left[-1 + 3(i\sqrt{3}) + 9 - 3i\sqrt{3} \right] = \frac{1}{8}(8) = 1$	

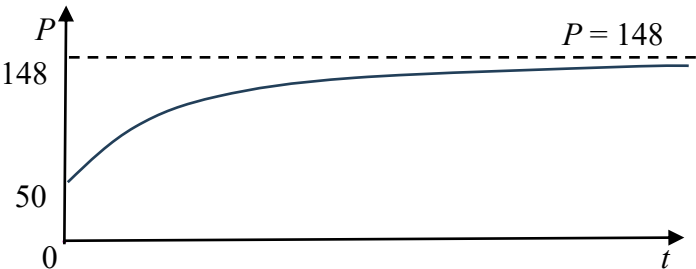
	$z^2 = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 = \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{i\sqrt{3}}{2}\right) + \left(\frac{i\sqrt{3}}{2}\right)^2$ $= \frac{1}{4} - \frac{i\sqrt{3}}{2} - \frac{3}{4} = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$ $z^3 + az^2 + bz - 1 = 0$ $1 + a\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) + b\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) - 1 = 0$ Equate real: $1 - \frac{a}{2} - \frac{b}{2} - 1 = 0 \Rightarrow a = -b \dots\dots\dots(1)$ Equate imaginary: $-\frac{a\sqrt{3}}{2} + \frac{b\sqrt{3}}{2} = 0 \Rightarrow a = b \dots\dots(2)$ From equation (1) and (2) $a = 0$ and $b = 0$ $z^3 + az^2 + bz - 1 = z^3 - 1$ If $z^3 - 1 = 0 \Rightarrow z = 1$ Hence $(z - 1)$ is a factor of $z^3 + az^2 + bz - 1$ $z^3 - 1 = (z - 1)(z^2 + \alpha z + \beta)$ Equate constant : $-1 = -1(\beta) \Rightarrow \beta = 1$ Equate coefficient of z^2 : $0 = \alpha - 1 \Rightarrow \alpha = 1$ Hence $z^3 - 1 = (z^2 + z + 1)(z - 1)$	
	<u>Method 3</u> $z^3 = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3 = \frac{1}{8}(-1 + i\sqrt{3})^3$ $= \frac{1}{8} \left[(-1)^3 + 3(-1)^2(i\sqrt{3}) + 3(-1)(i\sqrt{3})^2 + (i\sqrt{3})^3 \right]$ $= \frac{1}{8} \left[-1 + 3(i\sqrt{3}) + 9 - 3i\sqrt{3} \right] = \frac{1}{8}(8) = 1$ Hence $-\frac{1}{2} + \frac{i\sqrt{3}}{2}$ is a root of $z^3 - 1 = 0$ $z^3 + az^2 + bz - 1 = z^3 - 1$ Equate coefficient of z^2 : $a = 0$ Equate coefficient of z : $b = 0$ $z = 1$ is also a root of $z^3 - 1 = 0$ Hence $(z - 1)$ is a factor of $z^3 + az^2 + bz - 1$ $z^3 - 1 = (z - 1)(z^2 + \alpha z + \beta)$ Equate constant : $-1 = -1(\beta) \Rightarrow \beta = 1$ Equate coefficient of z^2 : $0 = \alpha - 1 \Rightarrow \alpha = 1$ Hence $z^3 - 1 = (z^2 + z + 1)(z - 1)$	

10(a) (ii)	The other roots are $z = 1$ and $z = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$	Quite a number of students only mention $z = 1$ is the other root and miss out on $z = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$.
10(a) (iii)	Since $z = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ is a root of the equation $z^3 + az^2 + bz - 1 = 0$ and using the results in part (i), $z^3 - 1 = 0$ $(z^2 + z + 1)(z - 1) = 0$ $1 + z + z^2 = 0 \text{ and } z^3 = 1$ Hence, $1 + z^5 + z^7 = 1 + (z^3)(z)^2 + (z^3)^2(z)$ $= 1 + (1)z^2 + (1)^2 z$ $= 1 + z^2 + z$ $= 0 \text{ (Shown)}$	Students who realise that $z^3 = 1$ are quite successful in solving this question
10(b) (i)		In an Argand diagram all points must be identifiable either by coordinates or modulus and argument. Many did not indicate coordinates of point C. Some diagrams are not drawn to scale. It should indicate that 1 is smaller than $\sqrt{3}$. Point C should not be drawn below the x-axis, since $\sqrt{3} > 1$.

10(b) (ii)	<u>Method 1</u> $iz_2 = i(\sqrt{3} - i) = i\sqrt{3} - i^2 = i\sqrt{3} + 1 = z_1.$ Hence $\angle AOB = \frac{\pi}{2}$ and $OA = z_1 = iz_2 = z_2 = OB$ $\therefore OACB$ is a square. <u>Method 2</u> $z_1 = \sqrt{3+1} = z_2$ $\arg(z_1) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$ $\arg(z_2) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ Hence $\angle AOB = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$ $\therefore OACB$ is a square. <u>Method 3</u> $z_1 = \sqrt{3+1} = z_2$ $m_1 = \text{gradient of } OA = \frac{\sqrt{3}}{1} = \sqrt{3}$ $m_2 = \text{gradient of } OB = -\frac{1}{\sqrt{3}}$ $m_1 m_2 = (\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right) = -1$ Hence OA is perpendicular to OB. $\therefore OACB$ is a square.	Some students assume that $iz_2 = z_1$ without showing any working. Note that for “show that” questions, the result must be demonstrated and not assumed. <u>Note:</u> 1) If $OA = AC = OB = BC$ then $OACB$ is a square. is not correct. It should be a rhombus. 2) Solution such as $\hat{AOB} = \tan^{-1} \sqrt{3} + \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{2}$ is not sufficient. More working needs to be shown, as this is a “show that” question. <u>Note:</u> 1) Incorrect $\vec{OA} = 1 + i\sqrt{3}$ Should be $\vec{OA} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ 2) Incorrect $A = \sqrt{3+1}$ Should be $OA = \sqrt{3+1}$
--	---	--

10(b) (iii)	$\arg(z_1) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$ $\text{Angle } COD = \frac{\pi}{3} - \hat{AOC} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ $z_1 + z_2 = (1 + \sqrt{3}) + (\sqrt{3} - 1)i$ $\arg(z_1 + z_2) = \frac{\pi}{12}$ $\tan \frac{\pi}{12} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)^2}{3 - 1} = \frac{3 + 1 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$	Some do not know the difference between "$\tan^{-1}$" and "$\tan$". Some used the wrong formula for the argument of z. Hence common errors are $\arg(z_1) = \tan\left(\frac{\sqrt{3}}{1}\right)$ $\arg(z_1) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Also note that $\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}.$ and not $\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60^\circ$ Answer should be in radians not degrees. Also note that $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ Solution such as 1) $\arg(z_1 + z_2) = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{12}$ is not sufficient. 2) $\arg(z_1 + z_2)$ $= \tan^{-1}\left(\frac{\sqrt{3} - 1}{\sqrt{3} + 1}\right) = \frac{\pi}{12}$ For a “show” question, steps need to be shown clearly.
--------------------------------------	---	---

11(a)	$\frac{dP}{dt} = P(1 - 0.2 \ln P)$ Let $y = \ln P$ $\frac{dy}{dt} = \frac{dy}{dP} \frac{dP}{dt} = \frac{1}{P} (P(1 - 0.2 \ln P)) = 1 - 0.2 \ln P$ Hence $\frac{dy}{dt} = (1 - 0.2y)$	This is a “show” question. Students need to show all the steps clearly. Some treat $\frac{dy}{dP} = \frac{1}{P}$ as a fraction and wrote $dP = P dy$ which is incorrect.
11(b)	$\int \frac{1}{(1 - 0.2y)} dy = \int dt$ $\frac{1}{-0.2} \ln 1 - 0.2y = t + C$ $\ln 1 - 0.2y = -0.2t - 0.2C$ $1 - 0.2y = Ke^{-0.2t} \text{ (where } K = \pm e^{-0.2C} \text{)}$ $y = \frac{1}{0.2} (1 - Ke^{-0.2t})$ $y = 5(1 - Ke^{-0.2t})$ When $t = 0, P = 50, y = \ln 50$ Hence $\ln 50 = 5(1 - K)$ $K = 1 - \frac{1}{5} \ln 50 = 0.217595 = 0.2176 \text{ (to 4 decimal places)}$ $y = 5(1 - Ke^{-0.2t})$	Some students assume that $1 - 0.2y$ is positive and did not include the modulus sign. Some did not leave their answer for K to 4 decimal place. They need to read the question carefully.
11(c)	$\ln P = 5(1 - Ke^{-0.2t})$ $P = e^{5(1 - Ke^{-0.2t})} \text{ where } K = 0.2176 \text{ (to 4 decimal places)}$	
11(d)	When $t \rightarrow \infty, e^{-0.2t} \rightarrow 0$ $P \rightarrow e^5 \approx 148$ In the long run the number of deer increases and tends to 148 deer.	Leaving the answer as $P \rightarrow e^5$ is not sufficient. Need to explain in <u>context</u> what happen to the deer in the long run.

		The number of deer does not converge to a “point 148”.
11(e)		Need to indicate axial intercept (0,50). Draw a smooth curve approaching the asymptote which should be a dotted line and not a bold line. The curve should not intersect the asymptote or lie on the asymptote. Asymptote must be labelled The curve does not exist for $t < 0$.
11(f)	$\frac{dP}{dt} = 0.2P$ $\int \frac{1}{P} dP = \int 0.2 dt$ $\ln P = 0.2t + C$ $P = Be^{0.2t}, \text{ where } B = e^C$ When $t = 0, P = 50$ $B = 50$ Hence $P = 50e^{0.2t}$	Leaving your answer as $P = e^{0.2t + \ln 50}$ is considered incomplete
11(g)	Model used by scientist A is more appropriate because it suggest that population will not grow indefinetly but the second model suggest that population can grow without limit. This is not realistic as both space and resources are limited in real life.	By stating that the deer are increasing infinitely is not sufficient. They need to state if they are referring to model B or A.

		Explaining why Model A is appropriate is not sufficient. Students also need to discuss Model B. Simply stating that Model B “goes to infinity” is not sufficient. Must clearly state what tends to infinity, e.g. the deer population increases indefinitely.
12(a)	$x = a \operatorname{cosec} \theta, y = 2a \cot \theta$ --- (1) $y = -x$ --- (2) Sub (1) into (2): $2a \cot \theta = -a \operatorname{cosec} \theta$ $\cos \theta = -\frac{1}{2}$ $\theta = \frac{2\pi}{3}$ Sub $\theta = \frac{2\pi}{3}$ into $x = a \operatorname{cosec} \theta$, $x = a \operatorname{cosec} \left(\frac{2\pi}{3} \right) = \frac{a}{\sin \left(\frac{2\pi}{3} \right)} = \frac{a}{\sqrt{3}/2} = \frac{2a}{\sqrt{3}}$ The x-coordinate of the point of intersection is $\frac{2a}{\sqrt{3}}$.	Given a graph represented in parametric equations and another graph represented in cartesian equation, students need to be able to find the intersection point(s) of the 2 graphs by substituting the pair of parametric questions into the cartesian equation to formulate an equation <u>in terms of the parameter</u> and then solve for the value of parameter.

12 (b)	$x = a \operatorname{cosec} \theta, y = 2a \cot \theta$ $\frac{dx}{d\theta} = -a \operatorname{cosec} \theta \cot \theta, \frac{dy}{d\theta} = -2a \operatorname{cosec}^2 \theta$ $\frac{dy}{dx} = \frac{-2a \operatorname{cosec}^2 \theta}{-a \operatorname{cosec} \theta \cot \theta} = \frac{2 \operatorname{cosec} \theta}{\cot \theta}$ (shown)	Students need to remember their trigo formulae. Students who do not remember how to differentiate $\operatorname{cosec} \theta$ and $\cot \theta$, will face difficulty when trying to show the required expression.
12 (c)	Equation of normal to C at the point $P(a \operatorname{cosec} p, 2a \cot p)$, means $\theta = p$: $y - 2a \cot p = -\frac{\cot p}{2 \operatorname{cosec} p}(x - a \operatorname{cosec} p)$ When $x = 0, y - 2a \cot p = -\frac{\cot p}{2 \operatorname{cosec} p}(-a \operatorname{cosec} p)$ $\Rightarrow y = \frac{1}{2} a \cot p + 2a \cot p = \frac{5}{2} a \cot p$ $\therefore B\left(0, \frac{5}{2} a \cot p\right)$ When $y = 0, -2a \cot p = -\frac{\cot p}{2 \operatorname{cosec} p}(x - a \operatorname{cosec} p)$ $\Rightarrow x = 4a \operatorname{cosec} p + a \operatorname{cosec} p = 5a \operatorname{cosec} p$ $\therefore A(5a \operatorname{cosec} p, 0)$ $\therefore M\left(\frac{5}{2} a \operatorname{cosec} p, \frac{5}{4} a \cot p\right)$ To find the cartesian equation: $x = \frac{5}{2} a \operatorname{cosec} p, y = \frac{5}{4} a \cot p$ $1 + \cot^2 p = \operatorname{cosec}^2 p$ $1 + \left(\frac{4y}{5a}\right)^2 = \left(\frac{2x}{5a}\right)^2$	This question requires a number of steps as follows 1) Find equation of normal at $\theta = p$. 2) The normal cuts the axis. Sub $x = 0$ to find y-intercept. Sub $y = 0$ to find x-intercept. 3) Find coordinate of mid-point. 4) To find cartesian equation of curve traced by midpoint, since it involves trigo, please use trigo identity. Students need to revise on trigo identity as it is not provided in MF27.

12 (d)	Let w = Area of triangle AOB $= \frac{1}{2}(5a \operatorname{cosec} p) \left(-\frac{5}{2} a \cot p \right)$ $= -\frac{25}{4} a^2 \operatorname{cosec} p \cot p$ When $p = 2$ and $a = 1$, $\frac{dp}{dt} = 0.1$, $\frac{dw}{dt} = \frac{dp}{dt} \times \frac{dw}{dp}$ $= (0.1) \left(\frac{dw}{dp} \Big _{p=2} \right)$ $= (0.1)(9.7527)$ $= 0.975 \text{ (to 3 s.f.)}$	Note that the question stated $\frac{\pi}{2} < \theta < \pi$, which is the second quadrant where only sine is positive, cosine and tangent are negative. Hence when we use area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$ you need to make sure base and height are positive. Please learn to use the GC to find $\frac{dw}{dp} \Big _{p=2}$ without doing differentiation algebraically.
------------------------------------	--	---