

SUMMARY: Hypothesis Testing

Six-Step Process for Hypothesis Testing (Z-test)

1. State the null and alternative hypotheses.

To test $H_0: \mu = \mu_0$ (a fixed no.)

Against H_1 : one of the 3 types $\mu \neq \mu_0$ or $\mu < \mu_0$ or $\mu > \mu_0$ depending on the question.

2. State the level of significance at which the test is conducted.

at $\alpha\%$ level of significance (usually given in question)

The values of μ_0 , σ and n must be filled in.

3. State distribution of $\bar{X}$ and Z under H_0 .

Distribution of X	Population Variance, σ^2	Under H_0 ,	Test Statistic
normal	known	$\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$	$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$
normal	unknown	Since n is large ($n \geq 30$), $\bar{X} \sim N\left(\mu_0, \frac{s^2}{n}\right)$ approx.	$Z = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \sim N(0,1)$ approx.
not normal	known	Since n is large ($n \geq 30$), $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$ approx. by Central Limit Theorem	$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ approx. by Central Limit Theorem
not normal	unknown	Since n is large ($n \geq 30$), $\bar{X} \sim N\left(\mu_0, \frac{s^2}{n}\right)$ approx. by Central Limit Theorem	$Z = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \sim N(0,1)$ approx. by Central Limit Theorem

4. State value of test statistic and the value of p .

Value of test statistic, $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \underline{\hspace{2cm}}$ or $z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$

p -value, $p = \underline{\hspace{2cm}}$

5. Compare value of p with value of $\frac{\alpha}{100}$.

If $p < \frac{\alpha}{100}$, state 'We reject H_0 at $\alpha\%$ level of significance.'

If $p > \frac{\alpha}{100}$, state 'We DO NOT reject H_0 at $\alpha\%$ level of significance.'

6. Conclusion in the context of the problem.

In the case of rejection of H_0 ,

"There is sufficient evidence at the ____% level of significance that H_1 (in the context of the question) is true."

In the case of non-rejection of H_0 ,

"There is insufficient evidence at the ____% level of significance that H_1 (in the context of the question) is true."

Example A

The mean lifetime of the fluorescent light bulbs produced by a company is μ and the standard deviation is 120 hours. If the mean lifetime of a random sample of 100 fluorescent light bulbs produced by the company is found to be 1570 hours, use a 1% level of significance to test if the mean lifetime of the fluorescent light bulbs is less than 1600 hours.

Example B

The mean lifetime of the fluorescent light bulbs produced by a company is μ and the standard deviation is 120 hours. If the mean lifetime of a random sample of 100 fluorescent light bulbs produced by the company is found to be 1623 hours, use a 1% level of significance to test if the mean lifetime of the fluorescent light bulbs is more than 1600 hours.

Solution A:

To test	$H_0 : \mu = 1600$
Against	$H_1 : \mu < 1600$
at 1% level of significance	

Solution B:

To test	$H_0 : \mu = 1600$
Against	$H_1 : \mu > 1600$
at 1% level of significance	

Step 1
State the null and alternative hypotheses.

Step 2
State the level of significance at which the test is conducted.

Under H_0 , since $n = 100$ is large, $\bar{X} \sim N\left(1600, \frac{120^2}{100}\right)$ approx. by Central Limit Theorem

$$\text{Test statistic, } Z = \frac{\bar{X} - 1600}{120 / \sqrt{100}} \sim N(0,1)$$

Step 3
State distribution of $\bar{X}$ and Z under H_0 .

$z = -2.5$ (from GC)	$z = 1.92$ (from GC)
$p\text{-value} = 0.00621$ (from GC)	$p\text{-value} = 0.0276$ (from GC)

Step 4
State value of test statistic and the value of p .

Since $p\text{-value} < 0.01$, reject H_0 .	Since $p\text{-value} > 0.01$, do not reject H_0 .
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Step 5
Compare value of p with value of $\frac{\alpha}{100}$.

There is sufficient evidence at the 1% level of significance that the mean lifetime of the fluorescent light bulbs is less than 1600 hours.	There is insufficient evidence at the 1% level of significance that the mean lifetime of the fluorescent light bulbs is more than 1600 hours.
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Step 6
Conclusion in the context of the problem.