RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 3: Equations and Inequalities

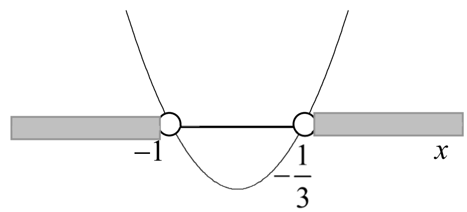
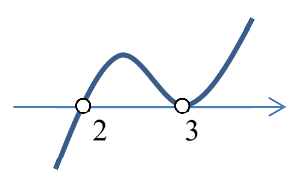
Section A (Basic Questions)

1 Without using a calculator, solve the following inequalities.

(a) $3x^2+4x+1>0$

(b) $2x^2+2x+1\le 0$

(c) $x^2-9<(x-3)(x^2-4x+9)$

Solution:	
(a)	$3x^2+4x+1>0$ For $3x^2+4x+1=0, x=\dfrac{-4\pm\sqrt{4^2-4(3)(1)}}{2(3)}=\dfrac{-4\pm\sqrt{4}}{6}=-\dfrac{1}{3}$ or -1 For $3x^2+4x+1>0$, $x<-1$ or $x>-\dfrac{1}{3}$  **OR:** $3x^2+4x+1>0$ $(3x+1)(x+1)>0$ $x<-1$ or $x>-\dfrac{1}{3}$
(b)	$2x^2+2x+1\le 0$ $\Leftrightarrow 2\left[\left(x+\dfrac{1}{2}\right)^2\right]+\dfrac{1}{2}\le 0$ But $2\left[\left(x+\dfrac{1}{2}\right)^2\right]+\dfrac{1}{2}\ge\dfrac{1}{2}>0\ \ \forall x\in\mathbb{R}$ So there is no real solution for the inequality.
(c)	$x^2-9<(x-3)(x^2-4x+9)$ $\Leftrightarrow (x-3)(x+3)<(x-3)(x^2-4x+9)$ $\Leftrightarrow (x-3)\left[x^2-4x+9-(x+3)\right]>0$ $\Leftrightarrow (x-3)\left[x^2-5x+6\right]>0$ $\Leftrightarrow (x-3)^2(x-2)>0$  Since $(x-3)^2>0\ \ \forall x\in\mathbb{R}$ and $x\ne 3$, $(x-2)>0\Rightarrow x>2$ $\therefore$ Solution is $2<x<3$ or $x>3$

2 Without using a calculator, solve the following inequalities.

(a) $\frac{x-5}{1-x} \geq 1$

(b) $\frac{3-x}{x+2} \leq \frac{1}{x}$ [4]

Solution:	
(a)	$\frac{x-5}{1-x} \geq 1 \quad , x \neq 1$ $\Leftrightarrow \frac{x-5}{1-x} - 1 \geq 0$ $\Leftrightarrow \frac{2x-6}{1-x} \geq 0$ $\Leftrightarrow (2x-6)(1-x) \geq 0$ $\Leftrightarrow 1 < x \leq 3$
(b)	$\frac{3-x}{x+2} \leq \frac{1}{x}, \quad x \neq -2, x \neq 0$ $\Leftrightarrow \frac{3-x}{x+2} - \frac{1}{x} \leq 0$ $\Leftrightarrow \frac{(3-x)x-(x+2)}{x(x+2)} \leq 0$ $\Leftrightarrow \frac{-x^2+2x-2}{x(x+2)} \leq 0$ $\Leftrightarrow \left(-x^2+2x-2\right)x\left(x+2\right) \leq 0$ $\Leftrightarrow \left(x^2-2x+2\right)x\left(x+2\right) \geq 0$ $\Leftrightarrow \left[\left(x-1\right)^2+1\right]x\left(x+2\right) \geq 0$ Since $\left(x-1\right)^2+1 > 0 \;\; \forall\, x \in \mathbb{R}$, $x\left(x+2\right) \geq 0$ $x < -2 \quad \text{or} \quad x > 0$

3 Without using a calculator, solve the following inequalities.

(i) $|2x-3|<7$

(ii) $|2+6x|>1$

Solution:

(i)

$|2x-3|<7$

$\Leftrightarrow -7<2x-3<7$

$\Leftrightarrow -4<2x<10$

$\Leftrightarrow -2<x<5$

(ii)

$|2+6x|>1$

$\Leftrightarrow 2+6x>1$ or $2+6x<-1$

$\Leftrightarrow x>-\dfrac{1}{6}$ or $x<-\dfrac{1}{2}$

OR:

Squaring both sides,

$(2+6x)^2>1^2$

$(2+6x)^2-1^2>0$

$(6x+3)(6x+1)>0$

$x<-\dfrac{1}{2}$ or $x>-\dfrac{1}{6}$

Note: the method of squaring both sides can only be used when both sides of the inequalities are positive. This method is also longer than using the identity:
$|\mathrm{f}(x)|>a \iff \mathrm{f}(x)<-a$ or $\mathrm{f}(x)>a$, where $a>0$.

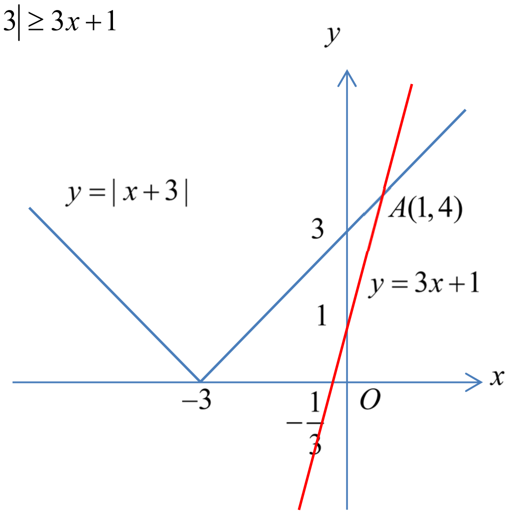
4 By sketching appropriate graphs on a single diagram, find the exact solution set to the following inequalities.

(i) $|x+3|\geq 3x+1$

(ii) $|11x|>|x-1|$

Solution:

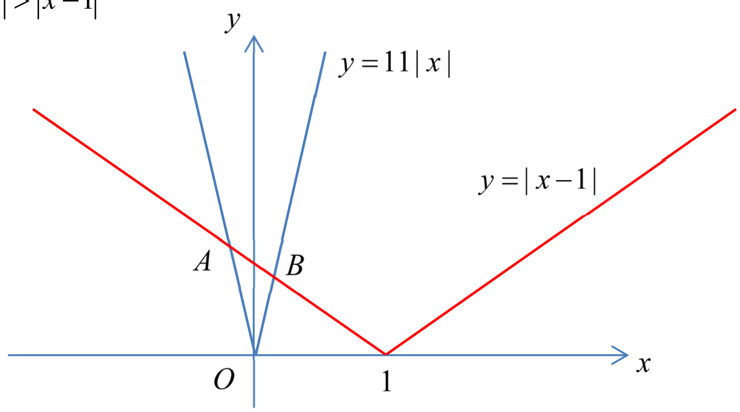
(i) $|x+3|\geq 3x+1$



At point A, $x+3=3x+1$

$\Rightarrow x=1$

For $|x+3|\geq 3x+1$, $x\in(-\infty,1]$

(ii)	$\lvert 11x\rvert > \lvert x-1\rvert$  At A, $-(x-1) = -11x \Rightarrow x = -\dfrac{1}{10}$ At B, $-(x-1) = 11x \Rightarrow x = \dfrac{1}{12}$ $\therefore$ Solution set is $\left(-\infty, -\dfrac{1}{10}\right) \cup \left(\dfrac{1}{12}, \infty\right)$

5 9740/2012/01/Q1

A cinema sells tickets at three different prices, depending on the age of the customer. The age categories are under 16 years, between 16 and 65 years, and over 65 years. Three groups of people, A, B and C, go to the cinema on the same day. The numbers in each age category for each group, together with the total cost of the tickets for each group, are given in the following table.

Group	Under 16 years	Between 16 and 65 years	Over 65 years	Total cost
A	9	6	4	\$162.03
B	7	5	3	\$128.36
C	10	4	5	\$158.50

Write down and solve the equations to find the cost of a ticket for each of the age categories. [4]

Solution:

Let \$$x$, \$$y$ and \$$z$ be the cost of a ticket for age categories "Under 16 years", "Between 16 and 65 years" and "over 65 years" respectively.

Then,

$$9x + 6y + 4z = 162.03$$
$$7x + 5y + 3z = 128.36$$
$$10x + 4y + 5z = 158.50$$

By GC, we have x =7.65, y=9.85, z =8.52.