Solution to Common Test Practice 1

1 The line $x + y = 3$ meets the curve $x^2 - 2x + 2y^2 = 3$ at two distinct points.

Find the coordinates of these two points. [4]

$x + y = 3$

$y = 3 - x$ ....(1)

$x^2 - 2x + 2y^2 = 3$ ......(2)

Substitute (1) into (2)

$x^2 - 2x + 2(3 - x)^2 = 3$

$x^2 - 2x + 2(9 - 6x + x^2) = 3$

$x^2 - 2x + 18 - 12x + 2x^2 - 3 = 0$

$3x^2 - 14x + 15 = 0$

$(3x - 5)(x - 3) = 0$

$x = \frac{5}{3}$ or $x = 3$

$y = 3 - \frac{5}{3}$ or $y = 3 - 3$

$= \frac{4}{3}$ $= 0$

Points are $\left(\frac{5}{3}, \frac{4}{3}\right)$ and $(3, 0)$.

2 Find the possible values of k for which the line $y = kx - 1$ does not intersect the curve $y = x^2 + 3$. [3]

$y = kx - 1$ .... (1)

$y = x^2 + 3$ .... (2)

$x^2 + 3 = kx - 1$

$x^2 - kx + 4 = 0$

Discriminant < 0

$(-k)^2 - 4(1)(4) < 0$

$k^2 - 16 < 0$

$(k - 4)(k + 4) < 0$

$\therefore \ -4 < k < 4$

3 Using a suitable substitution, solve the equation $4^{x-1}+16^x=66$. [4]

$4^{x-1}+16^x=66$

Let $u=4^x$

$\frac{u}{4}+u^2=66$

$4u^2+u=264$

$4u^2+u-264=0$

$(4u+33)(u-8)=0$

$u=8$ or $u=\frac{-33}{4}$ (n.a.)

$4^x=8$

$2^{2x}=2^3$

$\therefore x=\frac{3}{2}$

4 Sketch the graph of $y=4x^2-24x+20$, indicating clearly the coordinates of the turning point, x and y intercepts. [4]

$y=4x^2-24x+20$

$y-$intercept is $(0,20)$.

$4x^2-24x+20=0$

$4(x^2-6x+5)=0$

$4(x-5)(x-1)=0$

$\therefore x=5$ or $x=1$

$x-$intercepts are $(5,0)$ and $(1,0)$.

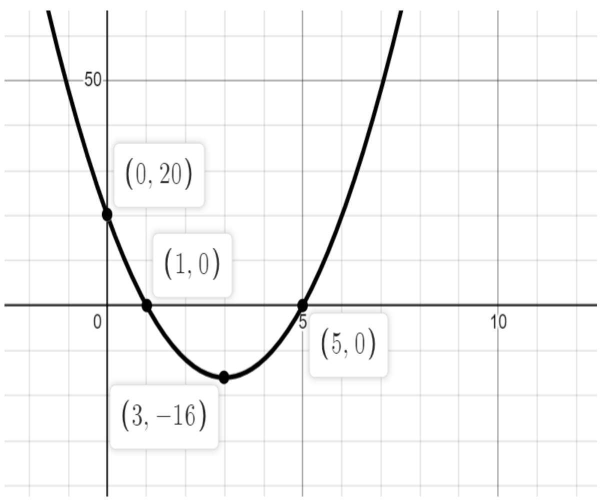
$y=4x^2-24x+20$

$=4(x^2-6x+5)$

$=4[(x-3)^2-9+5]$

$=4[(x-3)^2-4]$

Minimum point is $(3,-16)$.



Not labelling the axes and graph results in loss of 1 mark!

Not writing the values of the 3 critical points results in loss of another mark!

Curve not drawn symmetrical about the turning point results in another 1 mark gone!

5 The area of a rectangle $ABCD$ is given by $\left(1+3\sqrt{2}\right)$ m^2, and the length of side AB is $\left(2+\sqrt{8}\right)$ m.

Find, **without** using a calculator, the length of side BC in the form of $(a+b\sqrt{2})$m, where a and b are real numbers. [3]

$$
\begin{aligned}
\text{Length of } BC &= \frac{1+3\sqrt{2}}{2+\sqrt{8}} \\
&= \frac{1+3\sqrt{2}}{2+\sqrt{8}} \times \frac{2-\sqrt{8}}{2-\sqrt{8}} \\
&= \frac{\left(1+3\sqrt{2}\right)\left(2-2\sqrt{2}\right)}{4-8} \\
&= \frac{2-2\sqrt{2}+6\sqrt{2}-12}{-4} \\
&= \frac{-10+4\sqrt{2}}{-4} \\
&= \frac{5-2\sqrt{2}}{2} \\
&= \left(\frac{5}{2}-\sqrt{2}\right) \text{ m}
\end{aligned}
$$

6 Given that $\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}+\sqrt{48}-\dfrac{4}{\sqrt{3}}$ can be expressed in the form of $a+b\sqrt{3}$, find the value of a and of b. [4]

$$
\begin{aligned}
\frac{\sqrt{3}-1}{2\sqrt{3}+3}+\sqrt{48}-\frac{4}{\sqrt{3}} &= \frac{\sqrt{3}-1}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3} + 4\sqrt{3} - \frac{4\sqrt{3}}{3} \\
&= \frac{6-5\sqrt{3}+3}{12-9} + 4\sqrt{3} - \frac{4\sqrt{3}}{3} \\
&= \frac{9-5\sqrt{3}}{3} + 4\sqrt{3} - \frac{4\sqrt{3}}{3} \\
&= 3 - \frac{5\sqrt{3}}{3} + 4\sqrt{3} - \frac{4\sqrt{3}}{3} \\
&= 3+\sqrt{3}
\end{aligned}
$$

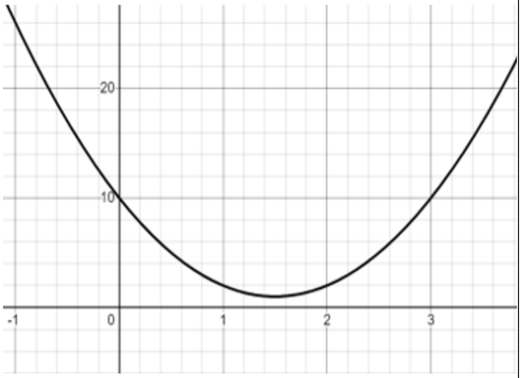
$\therefore a=3,\ b=1.$

7 Show that $4x^2 - 12x + 10$ is always positive for all real values of x . [3]

Method 1

$$y = 4x^2 - 12x + 10$$
$$= 4\left[x^2 - 3x + \frac{10}{4}\right]$$
$$= 4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{10}{4}\right]$$
$$= 4\left(x - \frac{3}{2}\right)^2 + 1 \geq 1$$

Method 2 (sketch graph)



$\therefore 4x^2 - 12x + 10$ is always positive for all real values of x.

8 Show that the roots of $x^2 + (p-2)x = 2p$ are real. [3]

$$x^2 + (p-2)x = 2p$$
$$x^2 + (p-2)x - 2p = 0$$

Discriminant $= \left(p-2\right)^2 - 4(1)(-2p)$

$$= p^2 - 4p + 4 + 8p$$
$$= p^2 + 4p + 4$$
$$= (p+2)^2 \geq 0$$

$\therefore$ the roots of $x^2 + (p-2)x = 2p$ are real.

9 Solve the equation $\sqrt{25x - 50} - \sqrt{x-2} = 12$. [3]

$$\sqrt{25x - 50} - \sqrt{x-2} = 12$$
$$\sqrt{25\left(x-2\right)} - \sqrt{x-2} = 12$$
$$5\sqrt{x-2} - \sqrt{x-2} = 12$$
$$4\sqrt{x-2} = 12$$
$$\sqrt{x-2} = 3$$
$$x - 2 = 9$$
$$\therefore x = 11$$

10 Find the solution set of each of the following inequalities:

(i) $x^2 > 3x + 18$ [2]

(ii) $(x+2)^2 < 9$ [2]

Hence, state the set of values of x which satisfy both of these inequalities. [1]

(i) $x^2 > 3x + 18$

$x^2 - 3x - 18 > 0$

$(x-6)(x+3) > 0$

$x < -3$ or $x > 6$

(ii) $(x+2)^2 < 9$

$x^2 + 4x + 4 - 9 < 0$

$x^2 + 4x - 5 < 0$

$(x+5)(x-1) < 0$

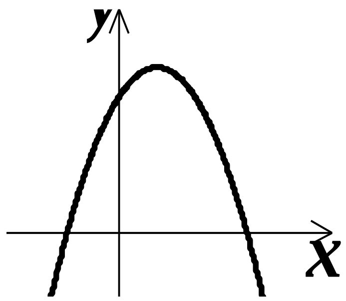
$\therefore -5 < x < 1$

To satisfy (1) and (2),

$-5 < x < -3$

can draw the 2 inequalities on the number number for the final solution!

11 The given diagram shows a quadratic curve with a y-intercept of 48 which crosses the x-axis at $x = -4$ and $x = 6$.



(i) State the equation of the line of symmetry for the curve. [1]

(ii) Find the equation of the parabola in the form $y = ax^2 + bx + c$ where $a \neq 0$. [3]

(i)	$x = \frac{-4+6}{2}$ $= 1$
(ii)	Let $y = a(x+4)(x-6)$ At $(0,48)$, $48 = a(-24)$ $\therefore a = -2$ $y = -2(x+4)(x-6)$ $= -2(x^2 - 2x - 24)$ $= -2x^2 + 4x + 48$