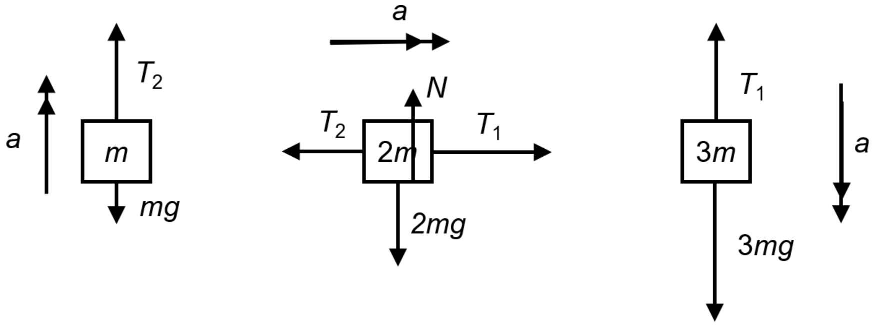
PHYSICS

9814

SUGGESTED SOLUTIONS

October/November 2024

1 **(a)**



From FBD drawn, and applying $F_{net} = ma$ (N2L),

$T_2 - mg = ma$ --- (1)

$T_1 - T_2 = 2ma$ --- (2)

$3mg - T_1 = 3ma$ --- (3)

Taking (2) + (3),

$3mg - T_2 = 5ma$ --- (4)

Taking (4) + (1),

$2mg = 6ma$

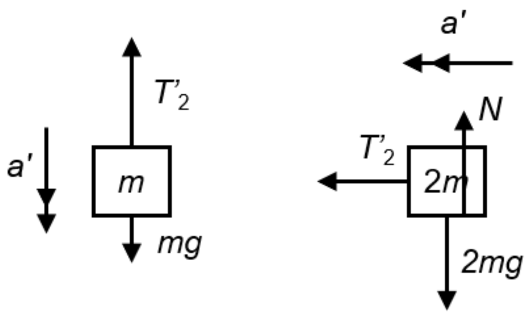
$a = \frac{1}{3}g$

Using $s_y = u_y t + \frac{1}{2}a_y t^2$

$h = \frac{1}{2}\left(\frac{1}{3}g\right)t_1^2$

$t_1 = \sqrt{\frac{6h}{g}}$

(b) **(i)**



Take rightwards as positive.

Determine the velocity of 2m when 3m just comes to rest:

$v = at_1$

$v = (\frac{g}{3})(\sqrt{\frac{6h}{g}})$

$v = \sqrt{\frac{2hg}{3}}$

When the string gets slack, 2m and m continue with velocity v (to the right) with a force of mg (to the left) causing the two masses to decelerate.

Hence considering 2m and m

$0 = v - at$

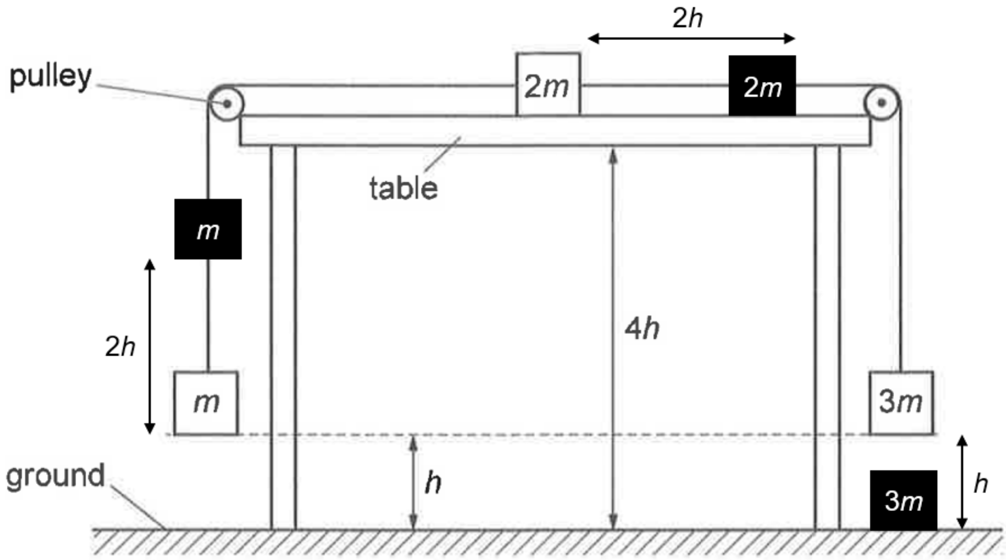
$0 = \sqrt{\frac{2hg}{3}} - (\frac{mg}{3m})(t_2 - t_1)$

$\frac{g(t_2 - t_1)}{3} = \sqrt{\frac{2hg}{3}}$

$(t_2 - t_1) = \sqrt{\frac{2hg}{3}} \times \sqrt{\frac{9}{g^2}} = \sqrt{\frac{6h}{g}}$

$t_2 = \sqrt{\frac{6h}{g}} + \sqrt{\frac{6h}{g}} = 2\sqrt{\frac{6h}{g}} = 4.9\sqrt{\frac{h}{g}}$

(ii)



(c) The student is incorrect because the acceleration of the mass *m* is constant and does not have a proportional relationship with the displacement from equilibrium position.

2 (a) The minimum amount of water that can be added to obtain 0.0°C temperature will be a situation whereby all water added just freeze to ice at 0.0°C. We assume the vaccine can be kept in solid ice as long as it is at 0.0°C.

By principle of conservation of energy,

$$\begin{pmatrix}\text{Heat gain}\\ \text{by ice to reach } 0.0^\circ\text{C}\end{pmatrix} = \begin{pmatrix}\text{Heat loss}\\ \text{by water to drop to } 0.0^\circ\text{C}\end{pmatrix} + \begin{pmatrix}\text{Heat loss}\\ \text{by water to freeze}\end{pmatrix}$$

$$m_{ice}c_{ice}\Delta\theta_{ice} = m_{water}c_{water}\Delta\theta_{water} + m_{water}l_f$$

$$(0.120)(2.09\times10^3)[0-(-4.2)] = m_{water}\{(4180)[23.0-0.0]+224000\}$$

$$m_{water} = 0.00329\text{ kg} = 3.29\text{ g}$$

(b) The maximum amount of water that can be added to obtain 0.0°C temperature will be a situation whereby all ice melted and the water reach 0.0°C.

By principle of conservation of energy,

$$\begin{pmatrix}\text{Heat gain}\\ \text{by ice to reach } 0.0^\circ\text{C}\end{pmatrix} + \begin{pmatrix}\text{Heat gain}\\ \text{by ice to melt}\end{pmatrix} = \begin{pmatrix}\text{Heat loss}\\ \text{by water to drop to } 0.0^\circ\text{C}\end{pmatrix}$$

$$m_{ice}c_{ice}\Delta\theta_{ice} + m_{ice}l_f = m_{water}c_{water}\Delta\theta_{water}$$

$$(0.120)(2.09\times10^3)[0-(-4.2)] + (0.120)(224000) = m_{water}(4180)(23.0-0.0)$$

$$m_{water} = 0.291\text{kg} = 291\text{g}$$

3 (a) (i) We are considering the situation of just before the ball-ball collision (shortly after the big ball has rebounded off the ground)

The big ball ($5M$) is moving upwards with speed V (it hits the ground at speed V and rebound elastically)

The small ball (M) is moving downwards with speed V.

In the lab frame:

Taking upwards as positive,

Lab - frame velocities :

$$u_{5M,\text{lab}} = +V \quad , \quad u_{M,\text{lab}} = -V$$

$$v_{ZMF} = \frac{(5M)(V)+(M)(-V)}{5M+M} = \frac{2}{3}V$$

By conservation of energy :

Loss in GPE = Gain in KE

$$mgh = \frac{1}{2}mV^2$$

$$V = \sqrt{2gh}$$

$$\therefore v_{ZMF} = \frac{2}{3}V = \frac{2}{3}\sqrt{2gh} \text{ (shown)}$$

(ii) $v_{ball,ZMF} = v_{ball,lab} - v_{ZMF}$

Before collision (in ZMF) :

$$u_{5M,\text{ZMF}} = V - \frac{2}{3}V = \frac{1}{3}V \text{ (upwards)}$$

$$u_{M,\text{ZMF}} = -V - \frac{2}{3}V = -\frac{5}{3}V \text{ (downwards)}$$

Because collision is elastic and ZMF has zero total momentum, the velocities in the ZMF simple reverse sign on collision :

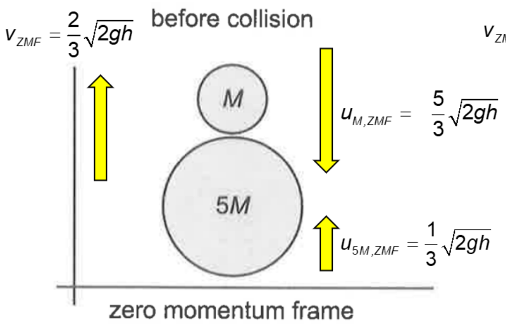
After collision (in ZMF) :

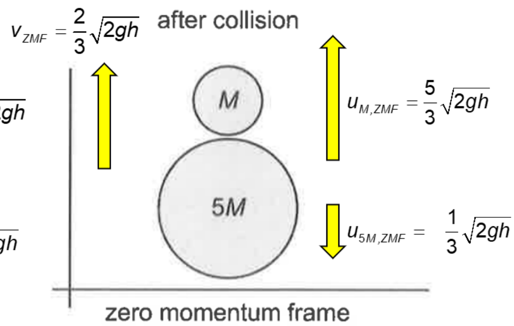
$$v_{5M,\text{ZMF}} = -\frac{1}{3}V \text{ (downwards)}$$

$$\Rightarrow v_{5M,\text{lab}} = v_{5M,\text{ZMF}} + v_{ZMF} = -\frac{1}{3}V + \frac{2}{3}V = \frac{1}{3}V \text{ (upwards)}$$

$$v_{M,\text{ZMF}} = \frac{5}{3}V \text{ (upwards)}$$

$$\Rightarrow v_{M,\text{lab}} = v_{M,\text{ZMF}} + v_{ZMF} = \frac{5}{3}V + \frac{2}{3}V = \frac{7}{3}V \text{ (upwards)}$$





(b) (i) Height to which ball 5*M* rebounds. Its upward speed after the ball - ball collision

$$v_{5M,ZMF} = -\frac{1}{3}V$$

$$v_{5M,lab} = -\frac{1}{3}V + \frac{2}{3}V = \frac{1}{3}V$$

Using Principle of conservation of energy,

Loss in KE = Gain in GPE

$$\frac{1}{2}(5M)\left(\frac{1}{3}V\right)^2 = (5M)(g)(H_{5M})$$

$$H_{5M} = \frac{1}{18}\frac{V^2}{g} = \frac{1}{18}\frac{(\sqrt{2gh})^2}{g} = \frac{1}{9}h$$

(ii) Height to which ball *M* rebounds. Its upward speed after the ball - ball collision

$$v_{M,ZMF} = \frac{5}{3}V$$

$$v_{M,lab} = \frac{5}{3}V + \frac{2}{3}V = \frac{7}{3}V$$

Using Principle of conservation of energy,

Loss in KE = Gain in GPE

$$\frac{1}{2}(M)\left(\frac{7}{3}V\right)^2 = (M)(g)(H_M)$$

$$H_M = \frac{49}{18}\frac{V^2}{g} = \frac{49}{18}\frac{(\sqrt{2gh})^2}{g} = \frac{49}{9}h$$

4(a) Ampère's law states that the line integral of $\boldsymbol{B}.d\boldsymbol{s}$ around any closed path (Ampèrian loop) is equal to $\mu_o I$, where I is the **total current** enclosed by the closed path:

$$\oint \boldsymbol{B}.d\boldsymbol{s} = \mu_0 I \ .$$

4(b)(i) Using right-hand drip, direction is clockwise

4(b)(ii) For each turn, the line integral $\oint \boldsymbol{B} \cdot d\boldsymbol{s} = \mu_o I$

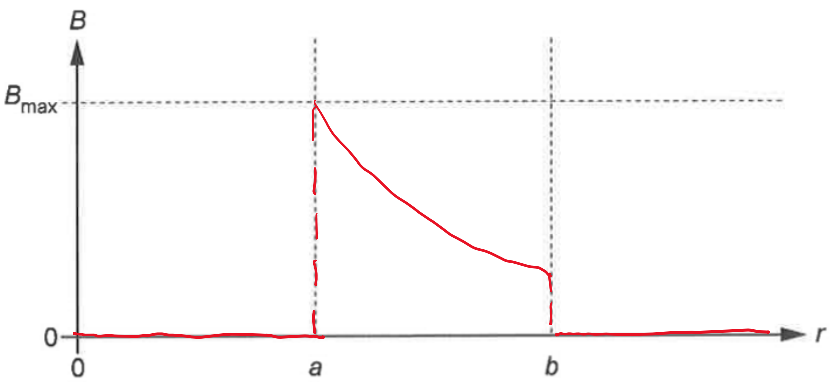
Since there are N turns, the

$$\oint \boldsymbol{B} \cdot d\boldsymbol{s} = \mu_o NI$$

$$B\oint ds = B(2\pi r) = \mu_o NI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

4(b)(iii)



From part (ii), we derived that inside the toroid ($a<r<b$), the magnetic flux density is:

$B=\mu_0 NI/2\pi r$

This shows that B is inversely proportional to r (i.e., $B \propto 1/r$) in the region $a<r<b$.

Outside the toroid:

- For $r<a$: The Amperian loop encloses no net current (I_{enc} = 0), so B=0.
- For $r>b$: The Amperian loop encloses equal and opposite currents (the current coming out and going in cancel), so I_{enc} = 0 and B=0.

At $r=a$ and $r=b$, there might be a discontinuity, but ideally, B is zero outside.

Therefore, the graph of B vs. r should look like:

- B=0 for $r<a$,
- B increases sharply at $r=a$ to a maximum value B_{max} at $r=a$,
- Then B decreases as $1/r$ for $a<r<b$,
- Then B drops to zero at $r=b$ and remains zero for $r>b$.

4(c) The maximum B occurs at the inner radius r=a (smallest r):

$$B_{\text{max}} = \frac{\mu_0 NI}{2\pi a}$$

The wire is insulated and wound tightly so that turns are just in contact at the inner circumference. This means that at the inner radius, the number of turns per unit length is maximized.

Specifically, at r=a, the circumference is $2\pi a$. The width of one turn (including insulation) is equal to the diameter of the insulated wire = 2.00 mm = 2.00×10^{-3} m.

Therefore, the number of turns that can fit along the inner circumference is:

N = circumference at r=a/width per turn

$$B_{\text{max}} = \frac{\mu_0 NI}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left(\frac{2\pi a}{d}\right) = \frac{\mu_0 I}{d}$$

$$I = \frac{6.00 \times 10^{-3}(2.0 \times 10^{-3})}{4\pi \times 10^{-7}} = 9.55 \text{ A}$$

5(a) Self-inductance is the ratio of the electromotive force (e.m.f.) induced in an electrical circuit/component to the rate of change of current causing it. $V = L\dfrac{dI}{dt}$

5(b) At t = 0s, I = 0 A so emf opposes the initial rate of change of current.

At t = 0s, $\dfrac{dI}{dt} = \dfrac{0.50}{4.20 \times 10^{-3}} = 119 \text{ A s}^{-1}$

$$L = \frac{E}{(\frac{dI}{dt})} = \frac{6.0}{119} = 0.050 \text{ H}$$

Alternative Method: In an RL circuit, the current as a function of time is: $I(t)=I_{max}(1-e-t/\tau)$

where $\tau=L/R_{total}$ is the time constant, and R_{total} is the total resistance (internal resistance + coil resistance).

In practice, we can use the time to reach about 63% of I_{max} to find τ.

From the graph, the time constant τ, the time taken for the current to reach $0.63\times I_{max}$ $=0.63\times0.50=0.315$ A is 4.2 ms

The maximum current I_{max} = EMF/R_{total} =0.50 A.
So, 0.50 = 6.0/R_{total} $\Longrightarrow$ R_{total} =12.0 Ω

the inductance is:

$L=\tau\times R_{total} = (4.2 \times 10^{-3})(12) = 0.050$ H

So, the inductance is 0.050 H, and the unit is **henry (H)**.

(c) From the graph, The maximum current I_{max} = EMF/R_{total} =0.50 A.
So, 0.50 = 6.0/R_{total} $\Longrightarrow$ R_{total} =12.0 Ω

The internal resistance is 1 Ω,

$R_{total} = 1+R = 12.0 \Longrightarrow R=11.0$ Ω

So, the coil resistance is 11.0 Ω.

(d) Opening the switch causes the current in the coil to collapse extremely rapidly. According to Faraday's law, this sudden drop in current induces a very large back emf across the switch gap. The high voltage in large enough to exceed the breakdown voltage of air causing ionization and a spark across the switch.

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6(a)(i) $Z = \dfrac{1}{\cos 87^\circ} = 19.1$

6(a)(ii) Possible suggestions:

- Uncertainty in the half-moon phase: The Moon's surface is rough and cratered, making it extremely difficult to judge the exact instant the lunar terminator (day-night line) is perfectly straight to signal a true half-moon.
- Extreme sensitivity near 90°: The true angle is $89.8°$. Because $\cos\theta$ changes extremely rapidly near $90°$, even a tiny $1°$ measurement error leads to a massive percentage error in the calculated distance ratio Z.
- Extended size of celestial bodies: Both the Sun and Moon subtend an angular diameter of about $0.5°$ in the sky rather than acting as point sources, making center-to-center alignment difficult to pinpoint.
- Sunlight glare: Observing the Moon and Sun at the same time in the daytime sky presents glare issues that make precise optical alignment challenging.

6(b)(i) $v = r\omega$

$$v = (3.84 \times 10^5)\left(\frac{2\pi}{27.3 \times 24}\right) = 3680 \text{ km h}^{-1}$$

6(b)(ii) $v_{\text{max}} = (1.50 \times 10^8)\left(\dfrac{2\pi}{365 \times 24}\right) + (3.84 \times 10^5)\left(\dfrac{2\pi}{27.3 \times 24}\right) = 111000 \text{ km h}^{-1}$

6(b)(iii)

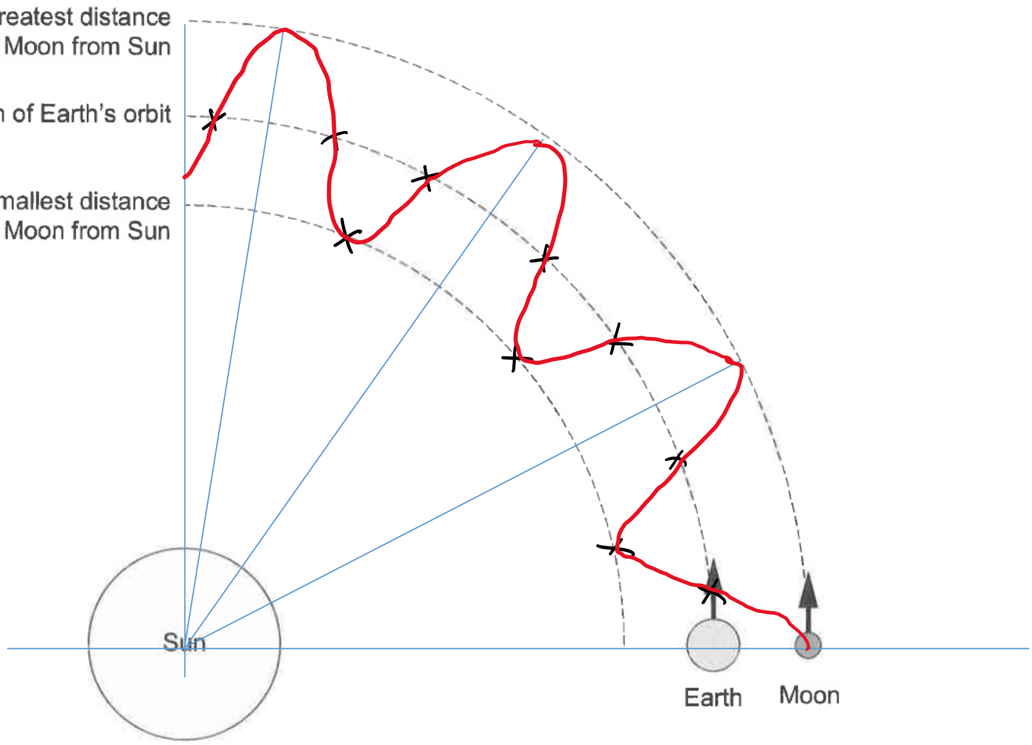

Fig. 6.2 (not to scale)

Path as shown in red. Should not have any form of retrograde. Explicit calculation should be shown that there are 3.34 cycles.

6(c)

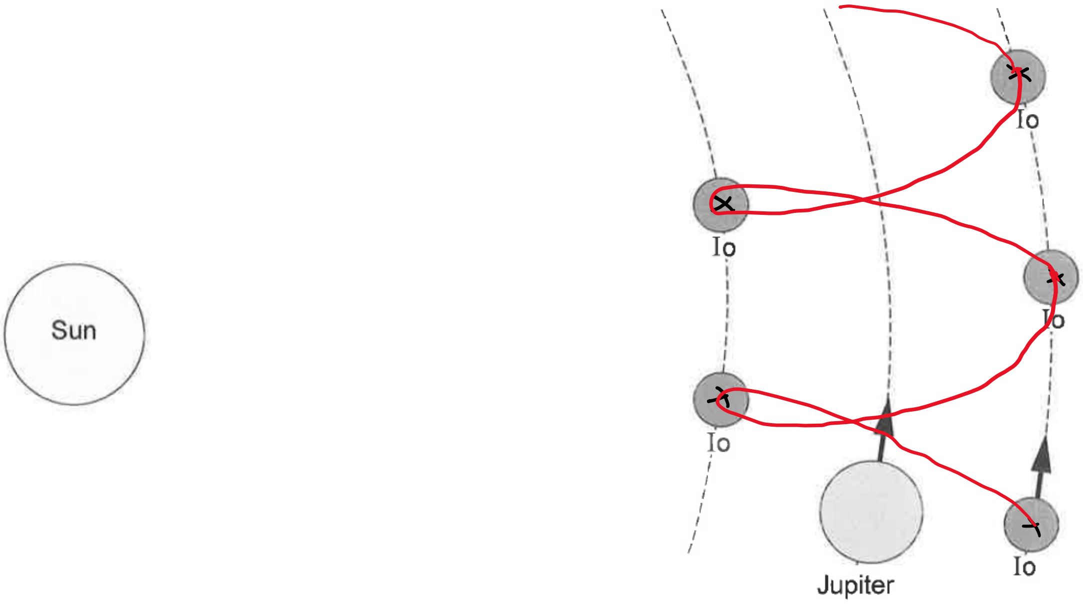

Fig. 6.3 (not to scale)

Path as shown in red. Should reflect retrograde.

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6(d)(i) Earth has an orbital period of 1 yr.
t is time that has passed in years.

$$\theta_v = \frac{2\pi t}{0.614} \text{ --(1)}$$

$$\theta_e = \frac{2\pi t}{1} \text{ --(2)}$$

(1) – (2) = $\theta_v - \theta_e = (\frac{2\pi t}{0.614} - \frac{2\pi t}{1}) = 2\pi$

t = 1.59 yrs

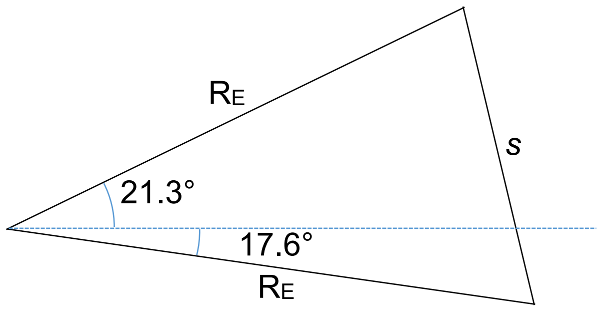
6(d)(i) Venus and Earth are orbiting about the Sun in a different plane.

6(e)(i) $d = \sqrt{r^2 - (\frac{vt_1}{2})^2} - \sqrt{r^2 - (\frac{vt_2}{2})^2}$

$$d = \left(r^2 - (\frac{vt_1}{2})^2\right)^{0.5} - \left(r^2 - (\frac{vt_2}{2})^2\right)^{0.5} = r\left[\left(1 - \frac{v^2 t_1^{\ 2}}{4r^2}\right)^{0.5} - \left(1 - \frac{v^2 t_2^{\ 2}}{4r^2}\right)^{0.5}\right]$$

$$d \approx r\left[1 - (0.5)(\frac{v^2 t_1^{\ 2}}{4r^2}) - 1 + (0.5)(\frac{v^2 t_2^{\ 2}}{4r^2})\right] = \frac{1}{8} v^2 r^{-1} (t_2^{\ 2} - t_1^{\ 2})$$

6(e)(ii)



$$s^2 = R_E^{\ 2} + R_E^{\ 2} - 2R_E^{\ 2}(\cos(21.3° + 17.6°)$$

$$s^2 = (2 \times 6370^2)(1 - \cos 38.9°)$$

$$s = 4242 \text{ km}$$

6(e)(iii) Comparing similar triangles using ratio method,

$$\frac{s}{d} = \frac{(1-0.723)\text{AU}}{0.723\text{AU}}$$

$$s = \frac{(1-0.723)\text{AU}}{0.723\text{AU}} \times 7.82 \times 10^{-5} \text{AU}$$

4242 km = 2.996×10^{-5} AU
1 AU = 1.42×10^8 km

6(e)(iv) % error $= \frac{(1.496 - 1.416) \times 10^8}{1.496 \times 10^8} \times 100\% = 5.3\%$

7(a)(i) Total energy = $\frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$

(ii) Total energy = $\frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$

(iii) There is no external force and thus no work done on the system of planet and the mass m. Hence, by COE, energy is conserved and thus total energy at each point is the same.

(iv)

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

By conservation of angular momentum,

$$mr_1v_1 = mr_2v_2$$

$$\frac{1}{2}v_1^2 - \frac{GM}{r_1} = \frac{1}{2}v_1^2\frac{r_1^2}{r_2^2} - \frac{GM}{r_2}$$

$\vdots$

$$v_1^2\left(\frac{r_2^2 - r_1^2}{r_2}\right) = 2GM\left(\frac{r_2 - r_1}{r_1}\right)$$

$$v_1^2 = 2GM\left(\frac{r_2}{r_1(r_2+r_1)}\right)$$

$$= GM\left(\frac{2}{r_1} - \frac{2}{r_2+r_1}\right)$$

$$= GM\left(\frac{2}{r_1} - \frac{1}{a}\right)$$

$$v_1 = \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{a}\right)} \qquad \text{(shown)}$$

In your own answers in exams, you must show detailed algebraic working since it is a proof question.

7(b)(i) By Kepler's third law,

$$\frac{T_E^2}{T_M^2} = \frac{r_E^3}{r_M^3} \Rightarrow T_M^2 = \frac{r_M^3}{r_E^3}T_E^2 = \frac{(2.28\times10^{11})^3}{(1.50\times10^{11})^3}\times1.00^2 = 3.512 \Rightarrow T_M = 1.87 \text{ years}$$

7(b)(ii) semi-major axis of the elliptical orbit, $a = \frac{2.28+1.50}{2}\times10^{11} = 1.89\times10^{11}$ m

$$\frac{T_E^2}{T_H^2} = \frac{r_E^3}{r_H^3} \Rightarrow T_H^2 = \frac{a^3}{r_E^3}T_E^2 = \frac{(1.89\times10^{11})^3}{(1.50\times10^{11})^3}\times1.00^2 = 2.000 \Rightarrow T_H = 1.414 \text{ years}$$

time of transfer $= \frac{T_H}{2} = 0.707$ years

7(b)(iii) $\theta = \frac{0.707}{1.87}\times2\pi = 2.38 \text{ rad} = 136°$

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7(b)(iv) $v_E = \dfrac{2\pi r_E}{T_E} = \dfrac{2\pi \times 1.50\times10^{11}}{365\times24\times60\times60} = 2.9885\times10^4 \text{ m s}^{-1}$

$$v_1 = \sqrt{6.67\times10^{-11}\times1.99\times10^{30}\times\left(\frac{2}{1.50\times10^{11}} - \frac{1}{1.89\times10^{11}}\right)} = 3.2672\times10^4 \text{ m s}^{-1}$$

$$\Delta v = v_1 - v_E = (3.2672 - 2.9885)\times10^4 = 2787 = 2790 \text{ m s}^{-1}$$

7(b)(v) Let the change in mass of the satellite be dm (<0). The mass of fuel ejected is then $-dm$. Let the speed change of the satellite be dv.

By conservation of momentum, since, in the frame of reference of the satellite, everything was stationary before the ejection,

$(-dm)(-u) + m_f dv = 0$ (the fuel is ejected in the negative direction)

$m_f dv = -u dm$

$$\int_{m_A}^{m} \frac{dm}{m_f} = -\int \frac{dv}{u}$$

$$\ln\frac{m_f}{m_A} = -\frac{1}{u}\int dv = -\frac{\Delta v}{u}$$

$$\frac{m_f}{m_A} = e^{-\frac{\Delta v}{u}}$$

$m_f = m_A e^{-\frac{\Delta v}{u}}$ (shown)

mass of fuel ejected $= m_A - m = m_A(1 - e^{-\frac{\Delta v}{u}})$

$= 2.00\times10^4 \times (1 - e^{-\frac{2790}{4000}}) = 10043 = 1.00\times10^4$ kg

8 (a) (i) By conservation of angular momentum,
Initial angular momentum of A + Initial angular momentum of B
= final combined angular momentum of A and B

$$870\times 6.8\times 10^{-2}+420\times 0.115=(6.8\times 10^{-2}+0.115)\omega$$

$$\omega = 587\text{rad s}^{-1}$$

(ii)

$$\tau = I\alpha$$

$$\alpha = 3000\text{ rad s}^{-2}$$

$$\omega_f = \omega_i - \alpha t$$

$$t = 0.0943\text{s}$$

(iii)

The loss in kinetic energy (KE) is equal to the heat gained by the material

$$KE_f - KE_i = mc\Delta T$$

$$\frac{1}{2}I_A\omega_A{}^2+\frac{1}{2}I_B\omega_B{}^2-\frac{1}{2}(I_A+I_B)\omega^2 = mc\Delta T$$

$$m = 0.242kg$$

You should show all substitution in exams.

(b) mass = density × volume

$$=\frac{\pi}{4}(25^2-20^2)\times 6\times 10^{-6}\times 2700$$

$$= 2.86\text{kg}$$

Moment of Inertial, $I = \frac{1}{2}m((\frac{0.25}{2})^2+(\frac{0.2}{2})^2) = 0.00367\text{kgm}^{-2}$

(ii) mass of the 3 spokes = density × volume = 0.12717 kg (show all substitution in exams)
Total mass = 2.86 + 0.12717 = 2.99 kg ≈ 3.0 kg

c (i) There must be a resultant clockwise torque on flywheel to rotate it clockwise.
Hence, $T_1 > T_2$.

(ii)

$$\omega_{flywheel} = 8\omega_F$$

By conservation of rotational kinetic energy,

Now $\frac{1}{2}(0.115)648^2 = \frac{1}{2}(1.4)(8\omega_F)^2+\frac{1}{2}(0.115)\omega_F{}^2$

$$\omega_F{}^2 = 538.2\ (\text{rad s}^{-1})^2$$

$$\text{Energy of flywheel} = \frac{1}{2}(1.4)(64\omega_F{}^2) = 24\text{kJ}$$

(iii) There is friction between the axle and the belt which may need work to be done against friction and thus energy lost as thermal energy.

(iv) There will be lesser energy transferred from rod B to flywheel since there will be friction between the 2 belts when there is a twist. Hence, thermal energy is generated and thus lesser kinetic energy transferred to flywheel.
Since friction is an internal force to the whole system and thus no net external force and thus net torque applied to the system, angular momentum will be conserved.

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9(a)(i) ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$

9(a)(ii) The total mass of neptunium-238 and the beta particle (electron) is (237.99979+0.00055)u = 238.00034u, which is greater than the mass of uranium-238 (238.00019u). Hence, this reaction cannot take place spontaneously, since there must be energy input to the system for it to happen.

9(b)(i) mass difference $= (238.00019 - 233.99409 - 4.00150)u = 0.0046u$

$$\text{energy released} = \frac{0.0046 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.6 \times 10^{-19}} = 4.295 \times 10^{6}\ \text{eV} = 4.3\ \text{MeV}$$

Note: It is necessary to show a few more d.p. before rounding to the value to be shown.

9(b)(ii) 1. mass number OR energy, 2. linear momentum

9(b)(iii) Since the thorium-234 nucleus was stationary, by conservation of linear momentum,

$$234u \times v_{\text{th}} = 4u \times v_{\alpha} \Rightarrow v_{\text{th}} = \frac{4}{234} v_{\alpha}$$

By conservation of energy,

$$\frac{1}{2} m_{\text{th}} v_{\text{th}}^{\ 2} + \frac{1}{2} m_{\alpha} v_{\alpha}^{\ 2} = 4.3\ \text{MeV}$$

$$\frac{1}{2}\left(234 \times \left(\frac{4}{234}\right)^2 + 4\right) \times 1.66 \times 10^{-27} \times v_{\alpha}^{\ 2} = 4.3 \times 10^{6} \times 1.6 \times 10^{-19}$$

$$v_{\alpha} = 1.43 \times 10^{7}\ \text{m s}^{-1}$$

9(c)(i) $$\text{electrostatic energy} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9 \times 10^{9} \times \frac{90 \times 2}{(5.8 + 1.7) \times 10^{-15}} \times (1.6 \times 10^{-19})^2$$

$$= 5.530 \times 10^{-12}\ \text{J} = 34.6\ \text{MeV}$$

9(c)(ii) The interaction between the particles is due to both strong force and electrostatic force. The <u>strong force is attractive, hence resulting in a negative potential energy</u>. Hence, the <u>total energy released, being the sum of the positive electrostatic energy and the negative potential energy, is smaller than the electrostatic energy alone</u>.

9(d)(i) $$N_{\text{U}} = N_0 e^{-\lambda t}, \lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{4.47 \times 10^{9} \times 365 \times 24 \times 60 \times 60} = 4.917 \times 10^{-18}$$

At t = 0,

$$\frac{dN_{\text{Th}}}{dt} = -\frac{dN_{\text{U}}}{dt} = \lambda N_0 = 4.917 \times 10^{-18} \times \frac{2.38 \times 10^{-3}}{238 \times 1.66 \times 10^{-27}} = 2.962 \times 10^{4}\ \text{s}^{-1}$$

9(d)(ii) In radioactive equilibrium, activities are equal.

$$\lambda_{U-238}N_{U-238} = \lambda_{U-234}N_{U-234}$$

$$\frac{N_{U-234}}{N_{U-238}} = \frac{\lambda_{U-238}}{\lambda_{U-234}} = \frac{\ln 2}{(t_{1/2})_{U-238}} \times \frac{(t_{1/2})_{U-234}}{\ln 2} = \frac{(t_{1/2})_{U-234}}{(t_{1/2})_{U-238}}$$

$$\frac{m_{U-234}}{m_{U-238}} = (\frac{(t_{1/2})_{U-234}}{(t_{1/2})_{U-238}}) \times (\frac{234u}{238u})$$

$$m_{U-234} = 2.38 \times (\frac{2.48 \times 10^{5}}{4.47 \times 10^{9}}) \times (\frac{234}{238}) = 1.30 \times 10^{-4} \text{ g}$$