Name: ______________________________ (   ) Class: _____

華僑中學
HWA CHONG INSTITUTION
Practice Paper A

INTEGRATED MATHEMATICS

Level **:** Secondary Three

Duration **:** 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted**.

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 6 printed pages, including this page.

Mathematical Formulae

1. **ALGEBRA**

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + \mathrm{c}^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The roots of the quadratic equation $2x^2+1=5x$ are α and β.

(i) Write down the values of $\alpha+\beta$ and $\alpha\beta$. [2]

(ii) Find an equation whose roots are given by $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$. [3]

2 **(a)** Explain why $2+4x-4x^2$ cannot be greater than 3 for all real values of *x*. [3]

(b) Find the range of values of *p* such that $4px^2-4(p+2)x+6p+5$ is **always positive** for all real values of *x*. [5]

3 **(a)** Make *p* the subject of the formula if $s=\dfrac{4(2p^2+1)}{u^2}$. [3]

(b) Simplify into a single fraction $\dfrac{x^3+8y^3}{x^2-4y^2}-\dfrac{16xy^2-4x^3}{(4y-2x)^2}$. [4]

4 **(a)** Given that $x^2-4x=y^2-4y$ and that $x\neq y$, find the value of $3\sqrt{x+y}$. [3]

(b) Using a suitable substitution, or otherwise, solve for *x* if

$$2(27^x)=45(3^x)+\frac{243}{3^x}.$$ [4]

5 It is given that f is a function such that $\mathrm{f}:x\mapsto 2-\sqrt{4-x},\ x\le k$.

(i) Write down the largest possible value of *k*. [1]

(ii) If $k=3$,

(a) find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}, [3]

(b) solve for *x* when $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [3]

6 Solve the following equations.

(a) $\log_3 x=\ln 2$, [2]

(b) $1+\log_{\sqrt{2}}(y-1)=\log_2(22-7y)$. [4]

7 A factory makes tables and chairs. The following table shows the breakdown of the amount of time taken, as well as the amount of wood and paint needed to produce each item.

	Time taken (hours)	Wood (number of blocks)	Paint (number of tins)
Table	6	4	3
Chair	3	3	2

Labour costs \$10 per hour, while wood costs \$3 per block and each tin of paint costs \$5. It is given that $\mathbf{A} = \begin{pmatrix} 6 & 4 & 3 \\ 3 & 3 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 10 \\ 3 \\ 5 \end{pmatrix}$ and $\mathbf{C} = \mathbf{AB}$.

(i) Evaluate **C** and explain what the numbers in **C** represent. [2]

(ii) If the factory makes 20 tables and 50 chairs, represent the information as a row matrix **D**. Evaluate **DC** and explain what the answer represents. [3]

8 **Answer the whole of this question on a sheet of graph paper.**

Two variables *x* and *y* are related such that $y^2 = ax^2 + bx$, where *a* and *b* are unknown constants. In an experiment, values of *y* are found for several values of *x* and the values are tabulated as shown below. It is also found that one particular value of *y* is incorrectly recorded.

x	1	2	3	4	5	6
y	2.07	3.77	5.45	6.81	8.81	10.48

(i) Using a scale of 2 cm to 1 unit for the horizontal axis and 1 cm to 1 unit for the vertical axes, plot $\dfrac{y^2}{x}$ against *x* and draw a straight line graph, and use it to identify the incorrect value of *y* recorded. [4]

(ii) Use your graph to estimate, to one decimal place, the values of *a* and *b*, and hence estimate, to two decimal places, the correct value of *y* in **(i)**. [4]

9 **(a)** Solve for x when $2(e^{3x}+2)=e^{x}(7e^{x}+5)$. [5]

(b) Given that $f(x)=8x^4+px^3-30x^2+x+q$, where p and q are unknown constants, leaves a remainder of 2 when divided by $x-2$ and a remainder of 9 when divided by $2x+3$, find the values of p and q and hence find the remainder when $f(x)$ is divided by $2x^2-x-6$. [6]

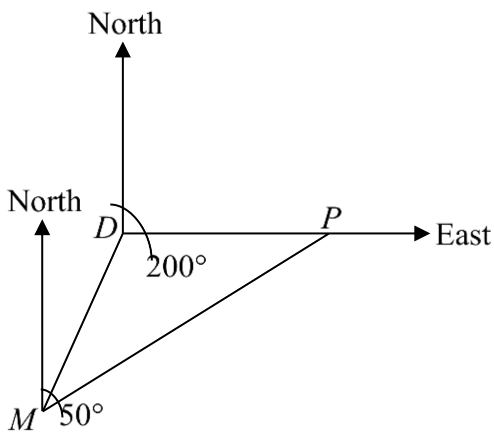
10 Two lines l_1 and l_2 are such that they are parallel, and l_2 is the perpendicular bisector of two points $A(-2,4)$ and $B(3,-6)$.

(i) Find the equation of l_2. [3]

(ii) Hence find the equation of l_1 if it passes through the point $C(5,8)$. [2]

(iii) Find the distance between the two lines. [4]

11 A man, at point *M*, is 80 metres at a bearing of $200°$ from an observation deck, *D*, which stands 25 metres tall. He begins to walk to a point *P* which is due East of *D* at a bearing of $050°$.



Ignoring the man's height, calculate

(i) the angle of elevation of the top of the deck from the man at *M*, [2]

(ii) the distance he walks when the angle of elevation of the top of the deck from his position is the greatest, and [3]

(iii) the distance *DP*. [2]

12 Sketch the graph of $y=\frac{4}{3}|3-2x|$, showing clearly any intercepts with the axes. By sketching the graph of $y=4+3x-x^2$, solve for the range of x when $12+9x-3x^2>4|3-2x|$. [5]

13 Given that $\sec x+\tan^2 x=0$, show that $\cos x=-\frac{2}{1+\sqrt{5}}$. [4]

(i) Prove that $\frac{\operatorname{cosec} x-\sec x}{\operatorname{cosec} x+1}+\frac{\operatorname{cosec} x+\sec x}{\operatorname{cosec} x-1}=2\sec^2 x+2\sec x\tan^2 x$. [3]

(ii) Hence, solve for $0°<x<360°$ the equation

$$\frac{\operatorname{cosec} x-\sec x}{1+\operatorname{cosec} x}=\frac{\operatorname{cosec} x+\sec x}{1-\operatorname{cosec} x}.$$ [4]

14 Sketch the graph of $y=\tan\frac{x}{2}$ for $0^\circ\le x\le 360^\circ$, showing clearly any asymptotes and intercepts with the axes. On the same axes, sketch the graph of $y=2|\sin x|$. Hence state the number of solutions between $0°$ and $360°$ inclusive when $\tan\frac{x}{2}=2|\sin x|$.

[4]

End of Paper