EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2025

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP

INDEX NO.

MATHEMATICS

Paper 1

9758/01

01 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

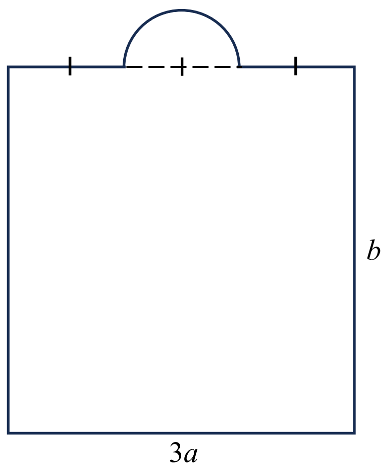
1 **(a)** Solve the inequality

$$x+1>\frac{1}{x-1}$$

giving your answer in exact form. [3]

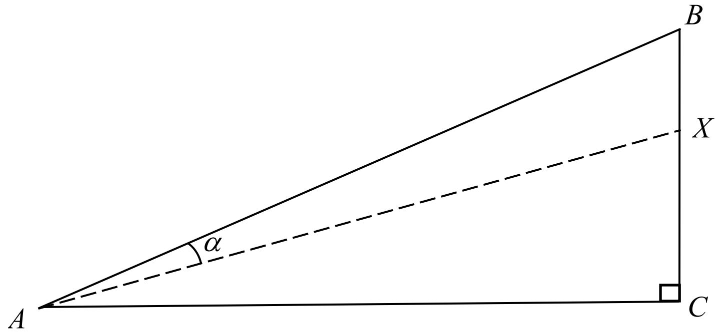
(b) Hence solve, in exact form, $\mathrm{e}^{x}+1>\dfrac{1}{\mathrm{e}^{x}-1}$. [2]

2 Pete has 180 cm of wire. He bends it to form the outline of his brand logo, which consists of a semicircle centered on top of a rectangle as shown in the diagram below. The width and length of the rectangle are $3a$ cm and b cm respectively. The diameter of the semicircle is one-third the width of the rectangle.



Find the maximum possible area enclosed by the wire, showing that it is a maximum value. Give your answer correct to 2 decimal places. [6]

3 In the right-angled triangle ABC, angle C is a right angle. $AB = 13$ cm, $BC = 5$ cm and X is a point on BC such that angle BAX is α radians.



(a) Find the exact value of $\cos\angle BAC$. [1]

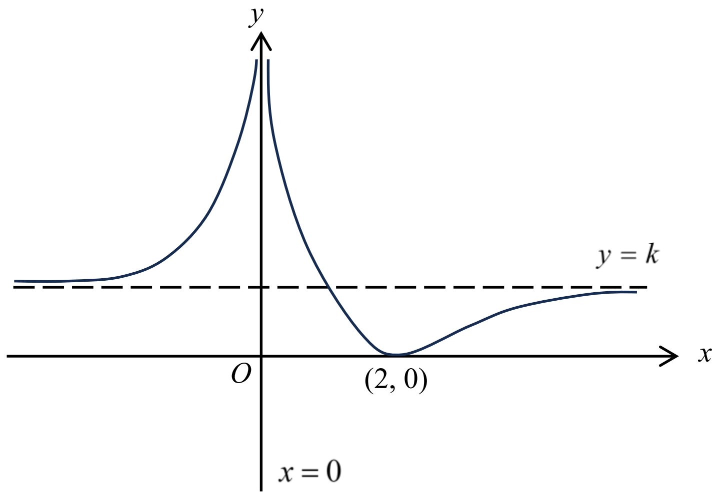
(b) By considering triangle AXC, or otherwise, show that $AX = \dfrac{156}{12\cos\alpha + 5\sin\alpha}$. [3]

(c) Given that α is a sufficiently small angle, show that

$$AX \approx 13 + p\alpha + q\alpha^2,$$

where p and q are constants to be determined exactly. [4]

4 The diagram below shows the graph of $y = \mathrm{f}(2x)$. The graph has a turning point at $(2, 0)$, and asymptotes with equations $x = 0$ and $y = k$.



(a) State a single transformation that will transform the graph of $y = \mathrm{f}(2x)$ onto the graph of $y = \mathrm{f}(2x+4)$. Hence sketch the graph of $y = \mathrm{f}(2x+4)$. [3]

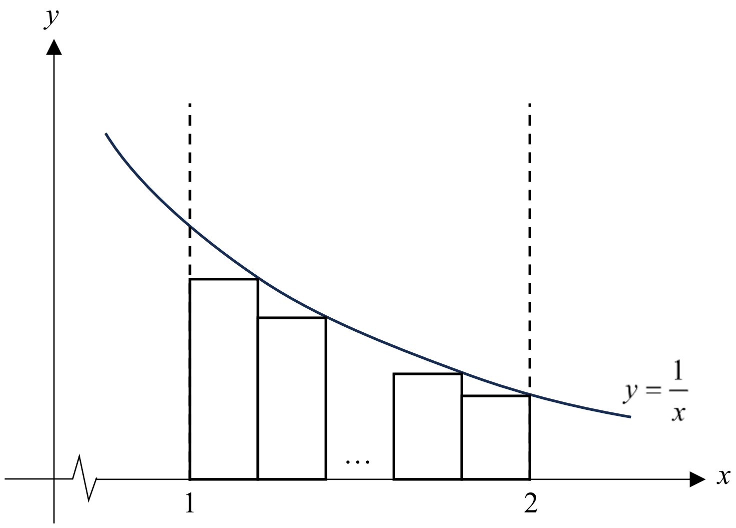
On separate clearly labelled diagrams, sketch the graphs of

(b) $y = \dfrac{1}{\mathrm{f}(2x)}$, [3]

(c) $y = -\mathrm{f}(|x|)$. [3]

5 The diagram shows the curve with equation $y = \dfrac{1}{x}$.

n rectangles of equal width are drawn under the curve between $x = 1$ and $x = 2$.



Let S_n be the **total** area of the n rectangles.

(a) Let $n = 2$. By considering S_2, show that $\dfrac{7}{12} < \ln 2$. State your reasoning clearly. [3]

(b) By considering S_n for a suitable value of n, find a rational number q such that $\dfrac{7}{12} < q < \ln 2$. [2]

(c) Express S_n in the form

$$\sum_{r=1}^{n} \mathrm{g}(r),$$

where the function g is to be determined. [1]

(d) By considering another suitable set of n rectangles, show with the aid of a diagram that

$$\ln 2 < S_n + \frac{1}{2n}$$

where n is any given positive integer. [3]

6 There are three distinct points A, B and P. The point P lies on the circle with centre O and diameter AB. Relative to the origin O, the position vectors of points A and P are $\mathbf{a}$ and $\mathbf{p}$ respectively.

(a) By expressing $\overrightarrow{AP}$ and $\overrightarrow{BP}$ in terms of $\mathbf{a}$ and $\mathbf{p}$, show that $\overrightarrow{AP}.\overrightarrow{BP}=0$. [3]

The point C with position vector $\mathbf{c}$ lies on AB produced such that $AC : AB$ is $\lambda : 1$.

(b) Find $\mathbf{c}$ in terms of λ and $\mathbf{a}$. [2]

(c) It is given that PC is a tangent to the circle and that angle AOP is $120°$. Using a suitable scalar product, find the value of λ. [4]

7 **Do not use a calculator in answering this question.**

An arithmetic sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 9$ and $u_3 = b$, where b is a constant.

(a) Find u_2 in terms of b. [2]

A geometric sequence $v_1, v_2, v_3, \ldots$ has $v_1 = 9$ and $v_3 = b$.

(b) Find the possible values of v_2 in terms of b. [2]

It is now given that $u_2 - v_2 = 8$, and the geometric sequence $v_1, v_2, v_3, \ldots$ is convergent.

(c) Find the value of b.
Hence find u_2 and v_2. [5]

8 **Do not use a calculator in answering this question.**

The complex number w is such that $w^2 = 2\mathrm{i}$ and $0 \le \arg(w) \le \dfrac{\pi}{2}$.

(a) Find w. [3]

(b) Given that one of the roots of the equation $z^3 - z^2 + (1-\mathrm{i})z + s = 0$ is w, find the other roots of the equation and the value of s. [5]

(c) Hence, find the roots of the equation $-\mathrm{i}v^3 + v^2 + (1+\mathrm{i})v + s = 0$. [2]

9 **(a)** **(i)** State the coordinates of the turning points of the graph of $y = x^3 - 3x^2 - 9x + 5$. [1]

(ii) The function f is defined by

$$\mathrm{f}(x) = x^3 - 3x^2 - 9x + 5,\ x \in \mathbb{R},\ x \leq a$$

where a is a constant. Find the largest possible value of a such that f^{-1} exists, and state the domain of f^{-1} in this case. [2]

(b) The function g is defined by

$$\mathrm{g}(x) = \frac{1-2x}{x+2},\ x \in \mathbb{R},\ x \neq -2.$$

(i) Find $\mathrm{g}^2(x)$. [2]

(ii) Hence find the possible expressions of $\mathrm{g}^n(x)$, where n is a positive integer. [2]

(c) The function h is defined by

$$\mathrm{h}(x) = x^2,\ x \in \mathbb{R},\ x \leq 0.$$

(i) Explain why the composite function gh exists. [2]

(ii) Find the range of gh. [2]

10 **(a)** Find $\int_0^4 \mathrm{e}^{|x-1|}\,\mathrm{d}x$, leaving your answer in exact form. [4]

(b) Find $\int \sin^2 3x + \tan^2 3x\ \mathrm{d}x$. [3]

(c) Use the substitution $x = u^6$, where $u > 0$, to find $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}}\,\mathrm{d}x$. [5]

11 A cylindrical container of height h metres is originally completely filled with water. The water is flowing out through an opening at the base of the container into a pipe. The rate of flow of water through the opening at time t seconds is proportional to the square root of the height of the water in the container, y metres. Since the container is cylindrical, the amount of water in the container is also proportional to the height of water, so the differential equation relating y and t can be written as

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -k\sqrt{y}$$

where k is a positive constant.

(a) Solve this differential equation to find $\sqrt{y}$ in terms of t, h and k. [4]

(b) Hence find the time taken for the container to empty, in terms of h and k. [2]

It is found that the water was draining too fast. To regulate the flow of water, a mechanical device is placed at the opening. This device slows the flow of water into the pipe by covering all or part of the opening, where the area covered varies with time. It is proposed to model this situation using the revised differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -k\left(\frac{1+\cos t}{2}\right)\sqrt{y},$$

where k is the same constant as before.

(c) With reference to the possible values of $\cos t$, explain how the revised differential equation corresponds to a model with slower flow of water. [1]

(d) Given again that the container is originally completely filled with water, solve the revised differential equation to find $\sqrt{y}$ in terms of t, h and k. [3]

(e) In the case where $h = 10$ and $k = 0.1$, find the ratio of the time taken in the revised model to the time taken in the original model for the container to empty. Give your answer to 3 significant figures. [2]

END OF PAPER