



EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2026

General Certificate of Education Advanced Level

Higher 2

FURTHER MATHEMATICS

Paper 2

9649/02

14 September 2026

3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

Section A: Pure Mathematics [50 Marks]

- 1 Solve the differential equation $\frac{dy}{dx} + y \cot x = 3$, given that $y = \pi$ when $x = \frac{\pi}{2}$. Leave your answer in the form $y = f(x)$. [5]
- 2 S is the set of complex numbers z for which $|z - 5 - 4i| \leq 3$ and $\arg(z) \geq \frac{\pi}{6}$.
- (a) On a single Argand diagram, shade the region corresponding to S . [3]
- (b) The complex number w is the element of S which has the largest argument. Determine the exact value of $|w|$ and, correct to 2 decimal places, the value of $\arg(w)$. [5]
- 3 The surface S has equation $z = y \tan x$, for all values of y and $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
- (a) Find an expression for the cartesian equation of T_P , the tangent-plane to S at a general point P with coordinates $(x_0, y_0, y_0 \tan x_0)$. Leave your answer in the form $ax + by + cz = x_0 y_0$, where a , b and c are constants in terms of x_0 and y_0 . [3]
- (b) Determine the exact coordinates of the point Q such that T_Q , the tangent-plane to S at Q is parallel to the plane $x + y - z = 0$. [3]
- (c) State the cartesian equation of T_O , the tangent-plane to S at the origin O . Hence find the acute angle between the planes T_O and T_Q . [3]
- 4 (a) Use de Moivre's theorem to prove that $\tan 7\theta = \frac{t^7 - 21t^5 + 35t^3 - 7t}{7t^6 - 35t^4 + 21t^2 - 1}$, where $t = \tan \theta$. [5]
- (b) Deduce, in trigonometric form, the roots of the equation $z^3 - 21z^2 + 35z - 7 = 0$. [4]
- (c) Use the roots in part (b) to find the exact value of $\cot^2\left(\frac{\pi}{7}\right) + \cot^2\left(\frac{2\pi}{7}\right) + \cot^2\left(\frac{3\pi}{7}\right)$. [4]

- 5 The surface P has equation $z = f(x, y)$, where $f(x, y) = k(x^2 + y^2) + (y - x)^2 - 4y$, and k is a real constant.
- (a) Find f_x , f_y , f_{xx} , f_{yy} and f_{xy} in terms of x , y and k . [3]
- (b) Hence determine the values of k for which P has no stationary points. [3]
- (c) For each range of values of k where P has stationary points, determine the number of turning points and the nature of each of these turning points. [5]

Another surface R has equation $z = g(x, y) = (\sin x)(\cos y) - 2\sin(xy)$.

- (d) Determine the quadratic approximation to R , $z = Q(x, y)$ at the point where $x = \frac{\pi}{4}$ and $y = 0$. Leave all coefficients in exact form. [4]

Section B: Probability and Statistics [50 Marks]

- 6 A World Cup Referee Coin has one face coloured black and the other face coloured yellow.

Kay Long got hold of one such coin. He tossed the coin 6 times and noted the number of times he obtained black. Repeating this 80 times, he collected the following data:

No. of black	0	1	2	3	4	5	6
Frequency	5	6	19	25	17	7	1

He decided to use a goodness-of-fit test to determine if the number of black in 6 tosses follows a particular probability distribution.

- (a) State the hypotheses for the test and define clearly any symbols used. [2]
- (b) Estimate the parameter needed for the test. [2]
- (c) Carry out the goodness-of-fit test. [4]

- 7 The Tween Tyrants is a collection of 5 different figurines sold individually in blind boxes. A buyer would only know which figurine they have bought after paying for the box and opening it.

Xing has 4 different figurines in her collection and is missing only the last one.

- (a) State the assumptions under which the number of boxes Xing has to buy till she gets the last figurine may be well modelled by a geometric distribution. [1]

It is now given that the above assumptions hold. In addition, it is assumed that each figurine is equally likely to be found in any box.

- (b) Find the probability that Xing has to buy at least 5 boxes to get the last figurine. [2]

Yun has 3 different figurines in her collection and is missing 2 more.

- (c) State the most probable number of boxes Yun has to buy to get a figurine not already in her collection. [1]

- (d) The probability that Yun has to buy at most n boxes to get a figurine not already in her collection is more than 0.99. Find the least value of n . [3]

Xing has spent more than \$100 on buying the boxes in her attempt to get the last figurine. Wan has just got his 4th different figurine in the last box he bought.

- (e) Between Xing and Wan, who is more likely to complete their collection with the next box they buy? Explain your answer. [2]

- 8 During the Perseid Meteor Shower this year, an average rate of 12 shooting stars per hour was spotted. It is assumed that the number of shooting stars spotted follows a Poisson distribution.

Ryusei spent 2 hours spotting shooting stars one night during the meteor shower.

- (a) Find the probability that more than 24 shooting stars were spotted. [2]

- (b) Find the most probable number(s) of shooting stars spotted. [1]

It is given that T minutes is the waiting time for the next shooting star.

- (c) State the probability density function of T and use it to find the probability that Ryusei has to wait more than 10 minutes for the next shooting star. [3]

- (d) Find the probability that Ryusei has to wait more than 10 minutes to spot a total of 2 shooting stars. [3]

- 9 Cyberspell, a new mobile game company, has just installed a load balancer to direct incoming network traffic to their two servers.

According to the industry standard, the mean response time for a load balancer is 250 ms. They want to test if their load balancer is meeting that standard. If it is too slow, the system is lagging; if it is too fast, it could be skipping some security checks.

They did a quick check by collecting a sample of 20 response time with the following summary:

$$\sum x = 5300 \text{ and } \sum x^2 = 1412100,$$

where x ms is a response time of their load balancer.

- (a) Construct a 95% confidence interval for the mean response time of the load balancer. [3]
- (b) State two assumptions necessary for constructing the confidence interval. [2]

Instead of conducting a 5% significance test to check if the mean response time of the load balancer is 250 ms, they were able to arrive at a conclusion using the confidence interval found in (a).

- (c) State the conclusion and explain briefly how this is arrived at. [2]

The load balancer has been set to split the traffic evenly between the two servers. To check this, they collected a random sample of 2000 user sessions and found 1060 of them being routed to Server A and the rest to Server B.

- (d) Construct a 95% confidence interval for the proportion of traffic directed to Server A by the load balancer. [3]

Under the assumption that the population proportion of traffic directed to Server A by the load balancer is 0.5, the random variable denoting the proportion of user sessions routed to Server A in a random sample of 2000 user sessions will follow a normal distribution approximately.

- (e) State the standard deviation of this normal distribution and hence explain whether it will be appropriate to use the confidence interval found in (d) to make a conclusion for a 2-tail hypothesis test on population proportion at 5% significance, similar to what was done in part (c). [2]

- 10** A new 6-week physical therapy intervention to reduce chronic lower back pain was introduced at a hospital.

Yeo Sng conducted a preliminary study to determine the effectiveness of this intervention. He recruited a random sample of 10 patients and asked them to indicate their pain level on a 100-point scale, where 0 is “no pain” and 100 is “worst pain imaginable”. This was done immediately before and immediately after the intervention. The pain levels of the patients are as follows.

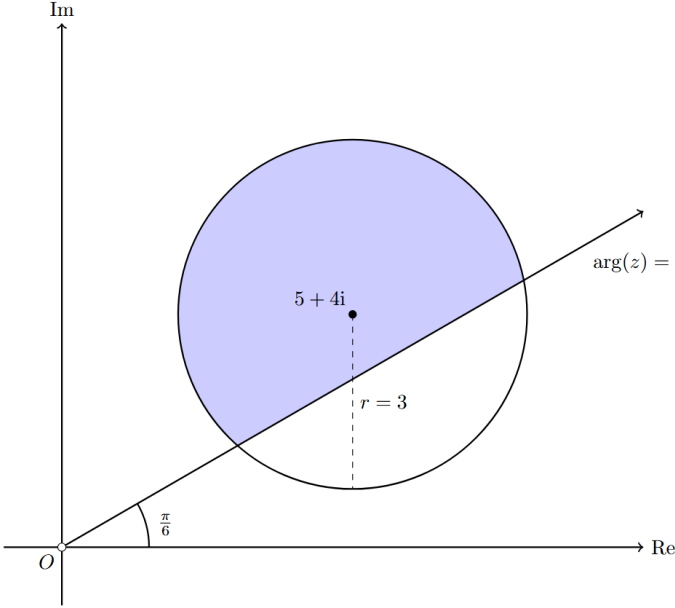
Patient	1	2	3	4	5	6	7	8	9	10
Before	88	78	57	92	57	70	88	68	72	60
After	73	50	88	84	45	70	81	42	47	54

- (a) Explain why a Wilcoxon signed rank test is preferred over a paired sample t -test. [2]
- (b) Using a Wilcoxon signed rank test at the 5% significance level, test whether the intervention reduces the pain level. [5]
- (c) State an assumption necessary for using this test. [1]

Yao Suan conducted a separate study using a 5-point scale for the pain level with an unknown number of patients. She found only 2 patients who felt that the intervention increased their pain levels. Using a sign test at the 5% significance level, she concluded that there is sufficient evidence that the intervention reduces the pain level.

- (d) Find the least possible number of patients in this study. [3]
- (e) Explain why a sign test is preferred over a Wilcoxon signed rank test for Yao Suan’s study. [1]

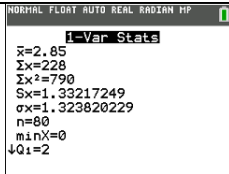
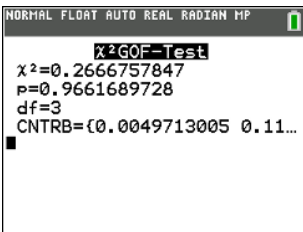
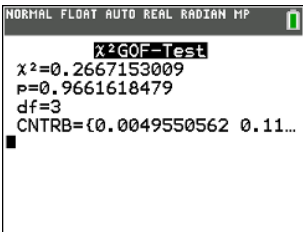
1	Solution
	$\frac{dy}{dx} + y \cot x = 3$ The integrating factor is $e^{\int \cot x \, dx} = e^{\ln(\sin x)} = \sin x$. Hence $\frac{dy}{dx} \sin x + y \cos x = 3 \sin x \Rightarrow \frac{d}{dx}(y \sin x) = 3 \sin x$ Integrating, $y \sin x = -3 \cos x + C$ At $x = \frac{\pi}{2}$, $y = \pi$, hence $\pi \sin \frac{\pi}{2} = -3 \cos \frac{\pi}{2} + C \Rightarrow C = \pi$ So $y = -3 \cot x + \pi \operatorname{cosec} x$

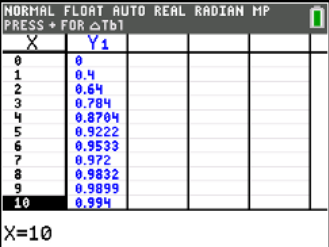
2	Solution
(a)	 The diagram shows a complex plane with a horizontal real axis (Re) and a vertical imaginary axis (Im). The origin is labeled O. A circle is centered at the point $5 + 4i$ with a radius $r = 3$. A ray originates from O and extends into the first quadrant at an angle of $\frac{\pi}{6}$ from the positive real axis. The region between this ray and the circle is shaded in light blue. The intersection of the ray and the circle is labeled $\arg(z) = \frac{\pi}{6}$.
(b)	The point representing w, the origin $(0, 0)$ and the centre of the circle $(5, 4)$ forms a right-angled triangle with the opposite side being 3 units and the hypotenuse being $\sqrt{41}$ units. Hence the angle at the origin is $\sin^{-1}\left(\frac{3}{\sqrt{41}}\right)$. Then $\arg(w) = \sin^{-1}\left(\frac{3}{\sqrt{41}}\right) + \tan^{-1}\left(\frac{4}{5}\right) = 1.16$ radians (2 d.p.). $w = \sqrt{41 - 3^2} = \sqrt{32} = 4\sqrt{2}$ units.

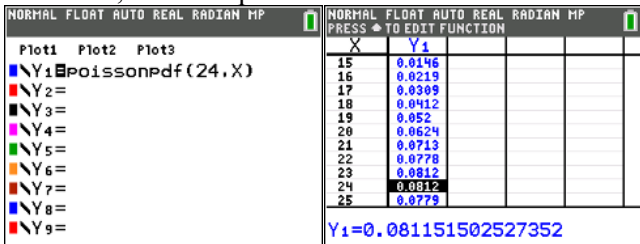
3	Solution
(a)	$\frac{\partial z}{\partial x} = y \sec^2 x \quad \frac{\partial z}{\partial y} = \tan x$ The equation is $z = y_0 \tan x_0 + (y_0 \sec^2 x_0)(x - x_0) + (\tan x_0)(y - y_0)$ $\Rightarrow (\cos^2 x_0)z - (y_0)x - (\sin x_0 \cos x_0)y = y_0 \sin x_0 \cos x_0 - x_0 y_0 - y_0 \sin x_0 \cos x_0$ $\Rightarrow (y_0)x + (\sin x_0 \cos x_0)y - (\cos^2 x_0)z = x_0 y_0$
(b)	For the planes to be parallel, then there exist some non-zero constant k such that $\begin{pmatrix} y_0 \\ \sin x_0 \cos x_0 \\ -\cos^2 x_0 \end{pmatrix} = k \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} y_0 = k & (1) \\ \sin x_0 \cos x_0 = k & (2) \\ \cos^2 x_0 = k & (3) \end{cases}$ [Note that this implies none of the unknown factors are 0.] $(2) \div (3): \tan x_0 = 1 \Rightarrow x_0 = \frac{\pi}{4}$ $\therefore y_0 = k = \cos^2 \frac{\pi}{4} = \frac{1}{2}$ Hence the coordinates of Q is $\left(\frac{\pi}{4}, \frac{1}{2}, \frac{1}{2}\right)$
(c)	At O, the equation of the tangent plane is $z = 0$. $\text{Angle required} = \cos^{-1} \frac{\begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}}{\begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \\ -1 \end{vmatrix}} = \cos^{-1} \frac{1}{\sqrt{3}} = 0.955 \text{ (3 s.f.) OR } 54.7^\circ \text{ (1 d.p.)}$

4	Solution
(a)	Using the theorem, $(\cos \theta + i \sin \theta)^7 = \cos 7\theta + i \sin 7\theta$ $(\cos \theta + i \sin \theta)^7 = \cos^7 \theta + 7i \cos^6 \theta \sin \theta + 21i^2 \cos^5 \theta \sin^2 \theta + 35i^3 \cos^4 \theta \sin^3 \theta + 35i^4 \cos^3 \theta \sin^4 \theta + 21i^5 \cos^2 \theta \sin^5 \theta + 7i^6 \cos \theta \sin^6 \theta + i^7 \sin^7 \theta$ Comparing Re and Im parts, $\cos 7\theta = \cos^7 \theta - 21 \cos^5 \theta \sin^2 \theta + 35 \cos^3 \theta \sin^4 \theta - 7 \cos \theta \sin^6 \theta$ $\sin 7\theta = 7 \cos^6 \theta \sin \theta - 35 \cos^4 \theta \sin^3 \theta + 21 \cos^2 \theta \sin^5 \theta - \sin^7 \theta$ $\tan 7\theta = \frac{7 \cos^6 \theta \sin \theta - 35 \cos^4 \theta \sin^3 \theta + 21 \cos^2 \theta \sin^5 \theta - \sin^7 \theta}{\cos^7 \theta - 21 \cos^5 \theta \sin^2 \theta + 35 \cos^3 \theta \sin^4 \theta - 7 \cos \theta \sin^6 \theta} \div \frac{\cos^7 \theta}{\cos^7 \theta}$ $= \frac{7 \tan \theta - 35 \tan^3 \theta + 21 \tan^5 \theta - \tan^7 \theta}{1 - 21 \tan^2 \theta + 35 \tan^4 \theta - 7 \tan^6 \theta}$ $= \frac{t^7 - 21t^5 + 35t^3 - 7t}{7t^6 - 35t^4 + 21t^2 - 1} \quad \text{where } t = \tan \theta$
(b)	When $\tan 7\theta = 0$, we have $t^7 - 21t^5 + 35t^3 - 7t = 0$, where $t = \tan \theta$ $t \left[(t^2)^3 - 21(t^2)^2 + 35(t^2) - 7 \right] = 0 \Rightarrow z^3 - 21z^2 + 35z - 7 = 0, \text{ where } z = t^2 \text{ \& } t \neq 0$ $\tan 7\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{7}, \frac{2\pi}{7}, \frac{3\pi}{7}, \frac{4\pi}{7}, \frac{5\pi}{7}, \frac{6\pi}{7} \text{ (reject 0 since } t \neq 0 \text{)}$ Hence the roots are $z = \tan^2\left(\frac{\pi}{7}\right), \tan^2\left(\frac{2\pi}{7}\right), \tan^2\left(\frac{3\pi}{7}\right)$. [Note: $\tan^2\left(\frac{\pi}{7}\right) = \tan^2\left(\frac{6\pi}{7}\right), \tan^2\left(\frac{2\pi}{7}\right) = \tan^2\left(\frac{5\pi}{7}\right), \tan^2\left(\frac{3\pi}{7}\right) = \tan^2\left(\frac{4\pi}{7}\right)$]
(c)	Denoting the roots in (b) as $\alpha = \tan^2\left(\frac{\pi}{7}\right), \beta = \tan^2\left(\frac{2\pi}{7}\right), \gamma = \tan^2\left(\frac{3\pi}{7}\right)$, we have $\alpha + \beta + \gamma = 21$, $\alpha\beta + \alpha\gamma + \beta\gamma = 35$, $\alpha\beta\gamma = 7$ from comparing the coefficients of the polynomial, then $\cot^2\left(\frac{\pi}{7}\right) + \cot^2\left(\frac{2\pi}{7}\right) + \cot^2\left(\frac{3\pi}{7}\right) = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{35}{7} = 5$ [Note: An alternative is considering polynomial with the reciprocals as the roots.]

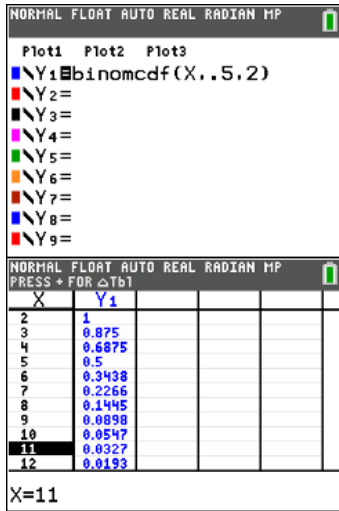
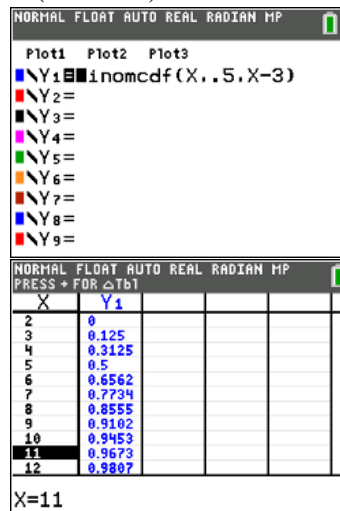
5	Solution
(a)	$f(x, y) = k(x^2 + y^2) + (y - x)^2 - 4y$ $\Rightarrow f_x = 2kx - 2(y - x) = 2(k + 1)x - 2y$ $\Rightarrow f_y = 2ky + 2(y - x) - 4 = 2(k + 1)y - 2x - 4$ $\Rightarrow f_{xx} = 2(k + 1), \quad f_{xy} = -2$ $\Rightarrow f_{yy} = 2(k + 1)$
(b)	For critical points, $f_x = 2(k + 1)x - 2y = 0 \Rightarrow y = (k + 1)x \quad (1)$ $f_y = 2(k + 1)y - 2x - 4 = 0 \Rightarrow (k + 1)y = x + 2 \quad (2)$ Sub (1) into (2): $(k + 1)^2 x = x + 2$ $\Rightarrow x = \frac{2}{k(k + 2)}$ $\Rightarrow y = \frac{2(k + 1)}{k(k + 2)}$ Since x and y are undefined, when $k = 0$ or -2, P has no stationary points.
(c)	For all other values of k, there is only 1 critical point $\left(\frac{2}{k(k + 2)}, \frac{2(k + 1)}{k(k + 2)}\right)$. $D = f_{xx}f_{yy} - (f_{xy})^2 = [2(k + 1)][2(k + 1)] - (-2)^2$ $= 4k(k + 2)$ For $D = 4k(k + 2) < 0$, we have $-2 < k < 0$ and P has a saddle point. For $D = 4k(k + 2) > 0$ and $f_{xx} = f_{yy} = 2(k + 1) > 0$, we have $k > 0$ and P has a minimum point. For $D = 4k(k + 2) > 0$ and $f_{xx} = f_{yy} = 2(k + 1) < 0$, we have $k < -2$ and P has a maximum point.
(d)	$g(x, y) = (\sin x)(\cos y) - 2\sin(xy) \Rightarrow g\left(\frac{\pi}{4}, 0\right) = \frac{1}{\sqrt{2}}$ $\Rightarrow g_x = (\cos x)(\cos y) - 2y\cos(xy) \Rightarrow g_x\left(\frac{\pi}{4}, 0\right) = \frac{1}{\sqrt{2}}$ $\Rightarrow g_y = -(\sin x)(\sin y) - 2x\cos(xy) \Rightarrow g_y\left(\frac{\pi}{4}, 0\right) = -\frac{\pi}{2}$ $\Rightarrow g_{xx} = -(\sin x)(\cos y) + 2y^2\sin(xy) \Rightarrow g_{xx}\left(\frac{\pi}{4}, 0\right) = -\frac{1}{\sqrt{2}}$ $\Rightarrow g_{yy} = -(\sin x)(\cos y) + 2x^2\sin(xy) \Rightarrow g_{yy}\left(\frac{\pi}{4}, 0\right) = -\frac{1}{\sqrt{2}}$ $\Rightarrow g_{xy} = -(\cos x)(\sin y) + 2xysin(xy) - 2\cos(xy) \Rightarrow g_{xy}\left(\frac{\pi}{4}, 0\right) = -2$ $Q(x, y) = \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)\left(x - \frac{\pi}{4}\right) + \left(-\frac{\pi}{2}\right)(y - 0) + \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right)\left(x - \frac{\pi}{4}\right)^2$ $+ (-2)\left(x - \frac{\pi}{4}\right)(y - 0) + \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right)(y - 0)^2$ $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}}\left(x - \frac{\pi}{4}\right)^2 - 2y\left(x - \frac{\pi}{4}\right) - \frac{\pi}{2}y - \frac{1}{2\sqrt{2}}y^2$

6	Solution																																																				
(a)	Let X be the number of black obtained in 6 tosses. Let p be the probability of getting black in 1 toss. H_0 : X follows $B(6, p)$ H_1 : X does not follow $B(6, p)$																																																				
(b)	From GC, $\sum x = 228$. So $\hat{p} = \frac{228}{480} = 0.475$ <u>OR</u> From GC, $\bar{x} = 2.85$. So $\hat{p} = \frac{2.85}{6} = 0.475$																																																				
(c)	Under H_0 , the expected frequencies are <table border="1" data-bbox="185 647 850 857"><tr><td>No. of black</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Obs Freq</td><td>5</td><td>6</td><td>19</td><td>25</td></tr><tr><td>Exp Freq</td><td>1.6751</td><td>9.0935</td><td>20.569</td><td>24.813</td></tr><tr><td>No. of black</td><td>4</td><td>5</td><td>6</td><td></td></tr><tr><td>Obs Freq</td><td>17</td><td>7</td><td>1</td><td></td></tr><tr><td>Exp Freq</td><td>16.837</td><td>6.0935</td><td>0.91886</td><td></td></tr></table> Combining first two and last two columns, <table border="1" data-bbox="185 909 970 1016"><tr><td>No. of black</td><td>0 and 1</td><td>2</td><td>3</td><td>4</td><td>5 and 6</td></tr><tr><td>Obs Freq</td><td>11</td><td>19</td><td>25</td><td>17</td><td>8</td></tr><tr><td>Exp Freq</td><td>10.769</td><td>20.569</td><td>24.813</td><td>16.837</td><td>7.0124</td></tr></table> Under H_0 , $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(3)$ From GC, p -value = 0.966 Since p -value is much larger than 0.05, H_0 is not rejected at the 5% level of significance. There is insufficient evidence to conclude that X does not follow $B(6, 0.475)$.  using full value  using 5 s.f. <u>OR</u> Since the p -value is very large, there is extremely weak evidence to claim that the distribution is not binomial.					No. of black	0	1	2	3	Obs Freq	5	6	19	25	Exp Freq	1.6751	9.0935	20.569	24.813	No. of black	4	5	6		Obs Freq	17	7	1		Exp Freq	16.837	6.0935	0.91886		No. of black	0 and 1	2	3	4	5 and 6	Obs Freq	11	19	25	17	8	Exp Freq	10.769	20.569	24.813	16.837	7.0124
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7	Solution																								
(a)	The assumptions are: The probability of getting the last figurine remains constant across the boxes. The figurine found in each box is independent across the boxes.																								
(b)	Let X be the number of boxes Xing has to buy till she gets the last figurine. $X \sim \text{Geo}\left(\frac{1}{5}\right)$ $P(X \geq 5) = 1 - P(X \leq 4) = 0.4096 = \frac{256}{625}$																								
(c)	Most probable number is 1 (mode)																								
(d)	Let Y be the number of boxes Yun has to buy to get a figurine not already in her collection. $Y \sim \text{Geo}\left(\frac{2}{5}\right)$ $P(Y \leq n) > 0.99$ From GC, least $n = 10$  <table border="1"> <thead> <tr> <th>X</th> <th>Y1</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>0.4</td></tr> <tr><td>2</td><td>0.64</td></tr> <tr><td>3</td><td>0.784</td></tr> <tr><td>4</td><td>0.8704</td></tr> <tr><td>5</td><td>0.9222</td></tr> <tr><td>6</td><td>0.9533</td></tr> <tr><td>7</td><td>0.972</td></tr> <tr><td>8</td><td>0.9832</td></tr> <tr><td>9</td><td>0.9899</td></tr> <tr><td>10</td><td>0.994</td></tr> </tbody> </table> X=10	X	Y1	0	0	1	0.4	2	0.64	3	0.784	4	0.8704	5	0.9222	6	0.9533	7	0.972	8	0.9832	9	0.9899	10	0.994
X	Y1																								
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(e)	Both are equally likely to complete their collection. For both Xing and Wan, the number of boxes they have to buy till they get their last figurine follow the same distribution $\text{Geo}\left(\frac{1}{5}\right)$. Since a geometric distribution is memoryless, it does not matter how many boxes Xing has bought previously.																								

8	Solution
(a)	Let X be the number of shooting stars spotted in 2 hours. $X \sim \text{Po}(12 \times 2)$, i.e. $X \sim \text{Po}(24)$ $P(X > 24) = 1 - P(X \leq 24) = 0.44600 = 0.446$
(b)	From GC, the most probable numbers are 23 and 24.  [Note: the GC shows that $P(X = 24)$ is slightly greater than $P(X = 23)$ but this is a calculator issue because they are algebraically equal.]
(c)	$T \sim \exp\left(\frac{12}{60}\right)$, i.e. $T \sim \exp\left(\frac{1}{5}\right)$ The pdf of T is $f(t) = \begin{cases} \frac{1}{5}e^{-\frac{1}{5}t} & , t \geq 0, \\ 0 & , \text{otherwise.} \end{cases}$ $P(T > 10) = 1 - P(T \leq 10) = 1 - \int_0^{10} \frac{1}{5}e^{-\frac{1}{5}t} dt = 0.13534 = 0.135$
(d)	Let Y be the number of shooting stars spotted in 10 minutes. $Y \sim \text{Po}\left(12 \times \frac{10}{60}\right)$, i.e. $Y \sim \text{Po}(2)$ Required probability = $P(Y \leq 1) = 0.40601 = 0.406$ <u>Alternative</u> (using exponential distribution) $P(T_1 + T_2 > 10) = 1 - P(T_1 + T_2 \leq 10)$ $= 1 - \int_0^{10} P(T \leq 10 - t) \cdot f(t) dt$ [Think: summing up from $t = 0$ to $t = 10$ for $P(T_2 \leq 10 - t) \cdot P(T_1 = t)$.] $= 1 - \int_0^{10} \left(1 - e^{-\frac{1}{5}(10-t)}\right) \left(\frac{1}{5}e^{-\frac{1}{5}t}\right) dt = 0.406$

9	Solution
(a)	Let X ms be the response time of the load balancer and μ ms be its mean. $\bar{x} = \frac{5300}{20} = 265$ $s = \sqrt{\frac{1}{20-1} \left[141210 - \frac{(5300)^2}{20} \right]} = 20$ Since $T = \frac{\bar{X} - 250}{\frac{s}{\sqrt{20}}} \sim t(19)$, a 95% confidence interval for μ is $(255.64, 274.36) = (256, 274)$
(b)	The two assumptions are: The sample collected is a random sample. The response time of the load balancer is normally distributed.
(c)	Since 250 is not within the 95% confidence interval, the observed sample mean of 265 will be in the critical region of a 5% significance test with $\mu = 250$ as H_0. So there is sufficient evidence to conclude that the mean response time of the load balancer is not 250 ms.
(d)	Let Y be the proportion of traffic directed to Server A by the load balancer. $\hat{p} = \frac{1060}{2000} = 0.53$ Since $Y \sim N\left(p, \frac{p(1-p)}{n}\right)$ approx. by CLT, a 95% confidence interval for p is $(0.50813, 0.55187)$ $= (0.508, 0.552)$
(e)	The standard deviation of the normal distribution is $\sqrt{\frac{0.5(1-0.5)}{2000}} = 0.11180$. In the confidence interval, the standard deviation used is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.53(1-0.53)}{2000}} = 0.11160$. They are not equal, so the width of the confidence interval and the acceptance region are not the same. However, the difference here is small so the attempt is still appropriate.

10	Solution																																																							
(a)	To use a t-test, we need to assume that the difference in pain levels for each patient follows a normal distribution which is difficult to ascertain here. No such assumption is needed for a Wilcoxon test. The pain level indicated is not objective and cannot be treated as an absolute measurement scale which is assumed in a t-test. A Wilcoxon test only ranks the differences and not treat them as absolute measures, so it is preferred.																																																							
(b)	Let B and A be the pain levels before and after intervention respectively. Let $D = B - A$ and m be its population median. $H_0: m = 0$ $H_1: m > 0$ <table><tr><td>Patient</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Before</td><td>88</td><td>78</td><td>57</td><td>92</td><td>57</td><td>70</td><td>88</td><td>68</td><td>72</td><td>60</td></tr><tr><td>After</td><td>73</td><td>50</td><td>88</td><td>84</td><td>45</td><td>70</td><td>81</td><td>42</td><td>47</td><td>54</td></tr><tr><td>d</td><td>15</td><td>28</td><td>-31</td><td>8</td><td>12</td><td>0</td><td>7</td><td>26</td><td>25</td><td>6</td></tr><tr><td>rank</td><td>5</td><td>8</td><td>9</td><td>3</td><td>4</td><td></td><td>2</td><td>7</td><td>6</td><td>1</td></tr></table> Discarding Patient 6, we have $p = 9$, $q = 36$ and so $t = 9$ From table, at 5% significance level, with $n = 9$, critical region is $T \leq 8$ Since $t > 8$, H_0 is not rejected at the 5% significance level. There is insufficient evidence to claim that the median is positive and that the intervention has reduced pain level.	Patient	1	2	3	4	5	6	7	8	9	10	Before	88	78	57	92	57	70	88	68	72	60	After	73	50	88	84	45	70	81	42	47	54	d	15	28	-31	8	12	0	7	26	25	6	rank	5	8	9	3	4		2	7	6	1
Patient	1	2	3	4	5	6	7	8	9	10																																														
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d	15	28	-31	8	12	0	7	26	25	6																																														
rank	5	8	9	3	4		2	7	6	1																																														
(c)	Assume that the distribution of D is symmetrical about 0.																																																							
(d)	$H_0: m = 0$ $H_1: m > 0$ Let S be the number of negative differences. Under H_0, $S \sim B(n, 0.5)$ Since $s = 2$, p-value = $P(S \leq 2)$ For H_0 rejected at 5% significance level, $P(S \leq 2) < 0.05$  Let S be the number of positive differences. Under H_0, $S \sim B(n, 0.5)$ Since $s = n - 2$, p-value = $P(S \geq n - 2)$ For H_0 rejected at 5% significance level, $P(S \geq n - 2) < 0.05$ $P(S \leq n - 3) > 0.95$  From GC, least $n = 11$																																																							
(e)	With a 5-point scale, the data is ordinal and the magnitude of the differences may not be meaningful. Hence a sign test which disregard the magnitude of the differences is preferred.																																																							