Class	Register Number	Name

南洋女子中學校

Nanyang Girls' High School

End-of-Year Examination 2024
Secondary 3

INTEGRATED MATHEMATICS 1

2 hours

Thursday 10 October **0845 – 1045**

Additional Material: NIL

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class in the spaces at the top of this page.
2. Write in dark blue or black ink.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, glue or correction tape/ fluid.
5. Write your answers and working in the space provided.
6. Answer **all** questions.
7. Omission of essential working will result in loss of marks.
8. The use of an approved scientific calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place. For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
10. The number of marks is given in brackets [ ] at the end of each question or part question.

For examiners' use	
Q1	/ 4
Q2	/ 10
Q3	/ 6
Q4	/ 8
Q5	/ 9
Q6	/ 8
Q7	/ 8
Q8	/ 9
Q9	/ 11
Q10	/ 7
Total	**/ 80**

This document consists of **19** printed pages and **1** blank page.

Mathematical Formulae

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mensuration

Area of triangle $ABC = \frac{1}{2}ab \sin C$

Arc length = $r\theta$, where θ is in radians

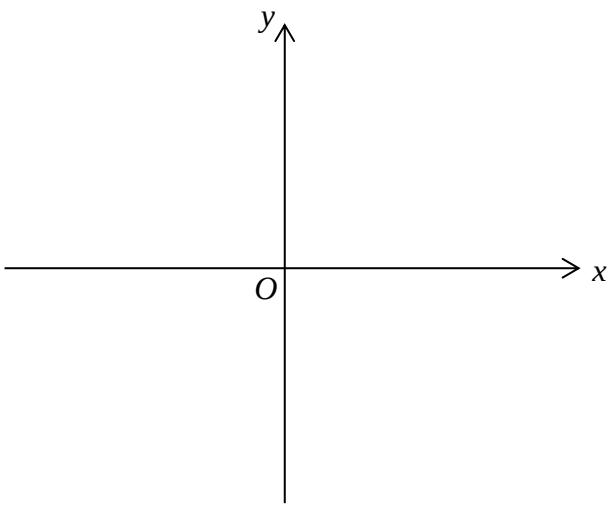
Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

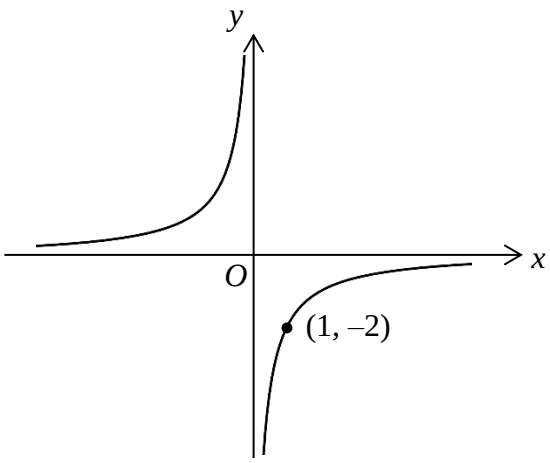
$$a^2 = b^2 + c^2 - 2bc \cos A$$

1 **(a)** On the diagram below, sketch the graph of $y = (6 - x)(x + 2)$, indicating the intercept(s) and turning point clearly. [2]



(b) The diagram below shows the graph of $y = kx^n$.

Write down the value of n and the corresponding value of k. [2]



2 The diagram shows a circle *ABCDE*, with centre *O*. *PDR* and *RA* are tangents to the circle. Angle *BAD* = 54°, angle *ODC* = 62° and angle *ADO* = 28°.

(a) Giving reasons for each answer, find

(i) angle *BOD*, [1]

(ii) angle *BED*, [1]

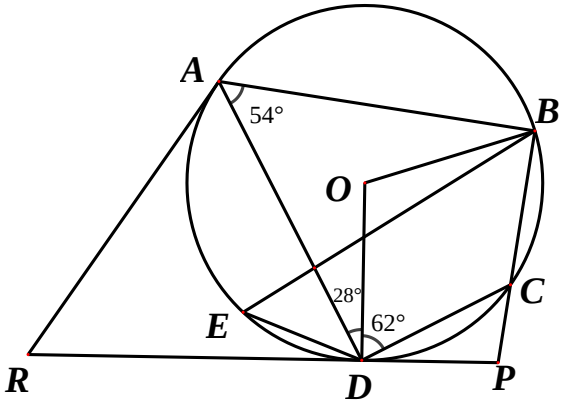
(iii) angle *BCD*, [1]

(iv) angle *RPC*, [2]

(v) angle *ARD*, [2]

(vi) angle *ABD*. [1]

(b) Explain clearly if *OARD* is a cyclic quadrilateral. [2]

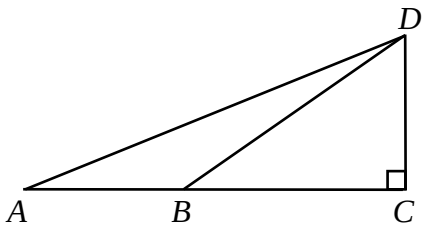


Continuation of working space for Question 2.

3 **(a)** Given that $\tan \angle ABD = -m$, find, in terms of m, the value of

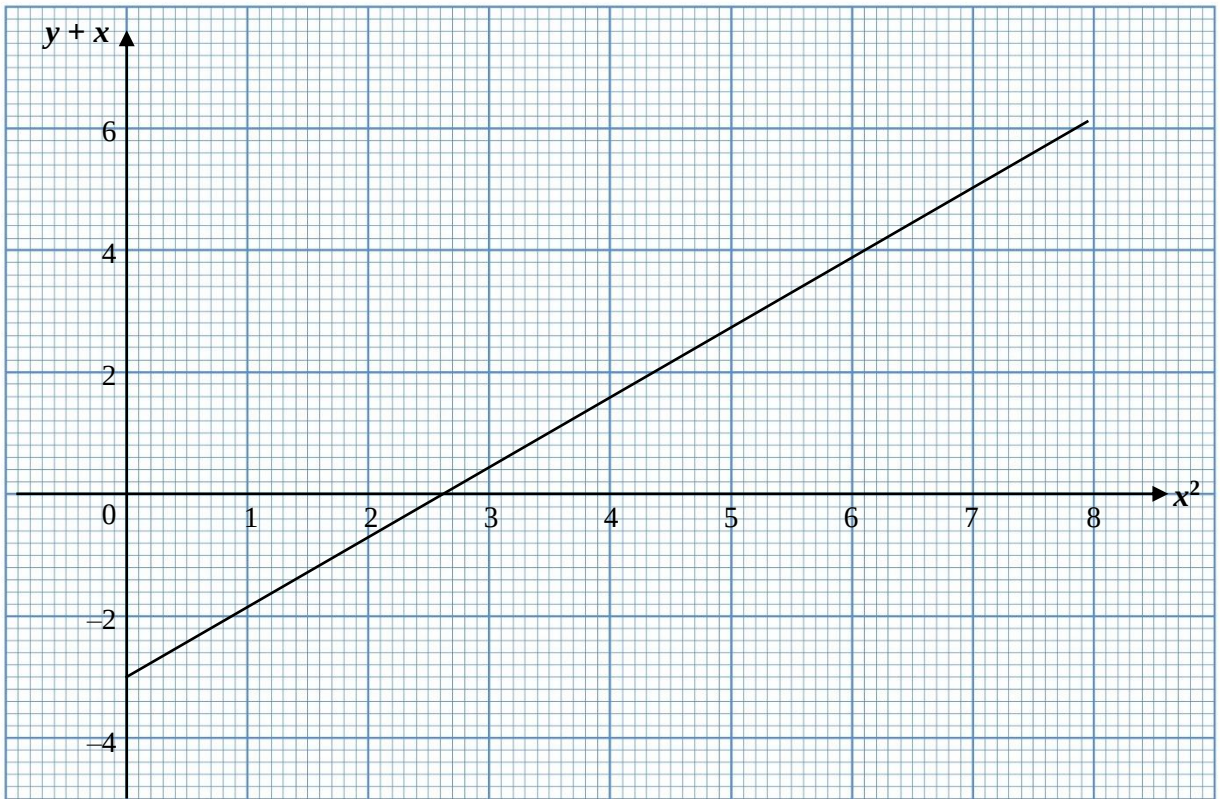
(i) $\sin \angle ABD$, [2]

(ii) $\cos \angle CBD$. [1]



(b) When a graph of $\frac{y}{x}$ is plotted against y, the resulting line has a gradient of 3 and an intercept on the $\frac{y}{x}$ - axis of -1. Express y in terms of x. [3]

4 It is known that x and y are related by an equation of the form $y = hx^2 - x + k$, where h and k are constants. A straight-line graph is obtained when $y + x$ is plotted against x^2, for $x \geq 0$.



(i) Use the graph to estimate

(a) the value of h and of k, [3]

(b) the value of y when $x = 2$. [2]

(ii) Explain how the above graph may be used to solve the equation $x^2(h+1) = 1-k$. Hence, find the value of x that satisfies this equation. [3]

Continuation of working space for Question 4.

5 **(a)** $ABCD$ is a quadrilateral with coordinates $A(3, 4)$, $B(b, b + 1)$, $C(6, 1)$ and $D(-3, -2)$.

(i) Find the value of b, if the area of $ABCD$ is 30 square units. [2]

(ii) Find the coordinates of M, if $ACMD$ is a rectangle. [2]

(iii) Given that K is a point on AC produced such that $AK : CK$ is 3 : 2, find the coordinates of K. [2]

5 **(b)** The horizontal and vertical distances between two points P and N are 9 units and 5 units respectively, where P is on the left of and lower than N.

Write down the gradient of the line PN and hence find the acute angle the line PN makes with the x–axis. [3]

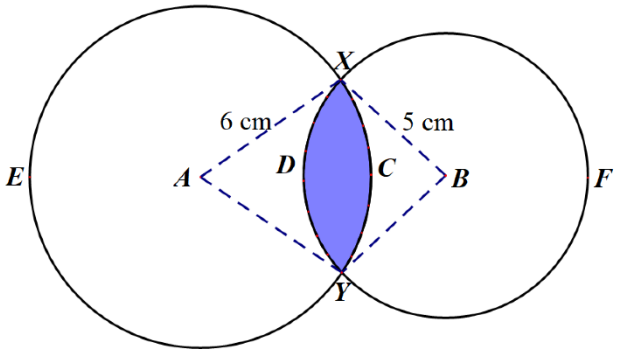
6 The equation of a circle, C_1, is $x^2 + y^2 + 2x - 6y - 15 = 0$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

(b) Determine if the point (3, 7) lies inside, outside or on the circle. [2]

(c) A tangent to the circle C_1 at T has the equation $4y + 3x + 16 = 0$. A is a point on the circle. Find the equation of the line TA if TA is perpendicular to this tangent. [3]

7 In the diagram, *XCYE* is a circle with centre *A* and radius 6 cm. *XDYF* is a circle with centre *B* and radius 5 cm.



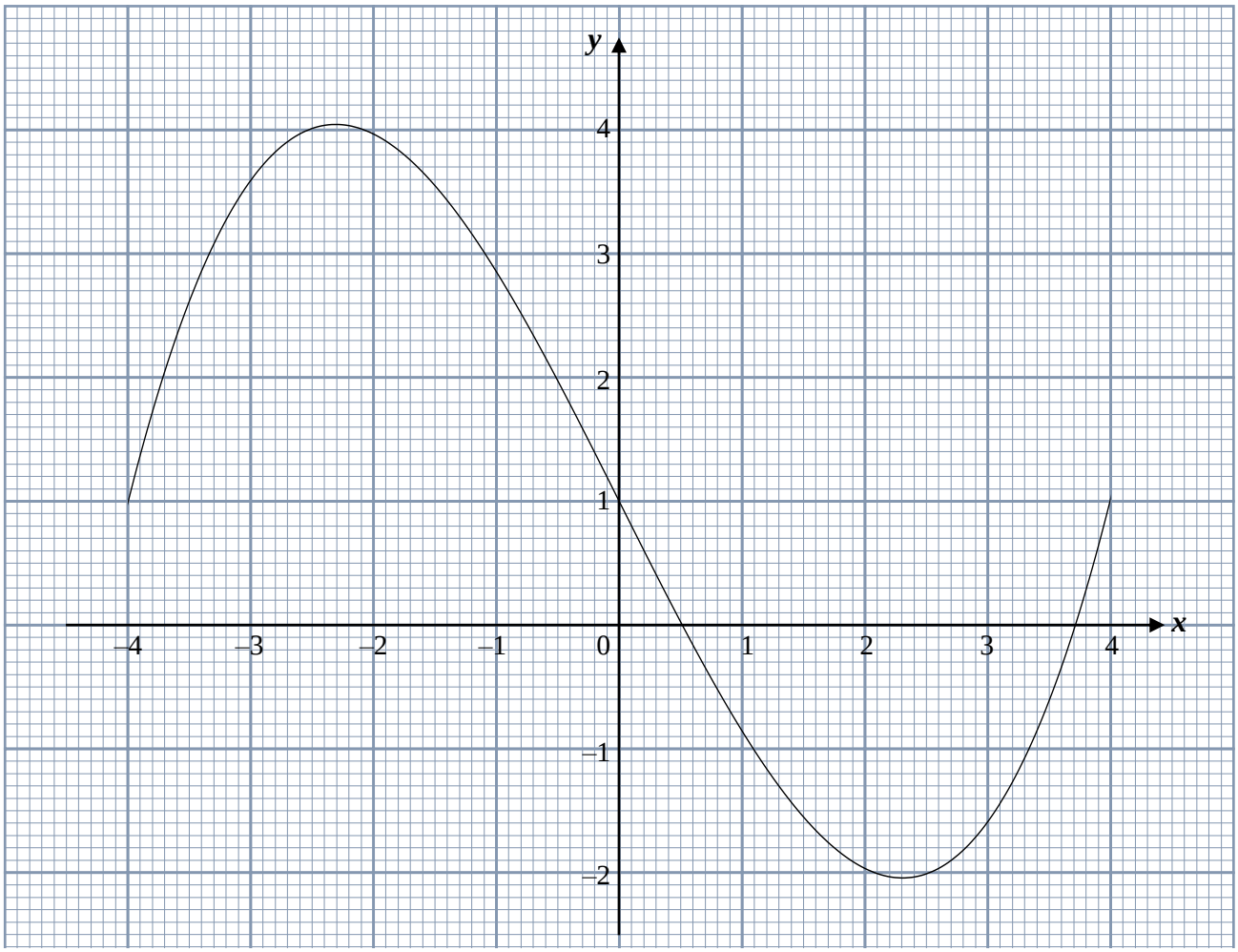
(a) Given *XY* is 6.78 cm, find angle *XAY* in radians. [2]

(b) If the perimeter of the shaded region is 14.65 cm, show that angle *XBY* is 1.49 radians. [3]

(c) Find the area of the shaded region. [3]

Continuation of working space for Question 7.

8 The diagram below shows the graph of $y = \frac{1}{8}x^3 - 2x + 1$ for $-4 \leq x \leq 4$.



(a) The equation $\frac{1}{8}x^3 - 2x = m - 1$ has only one solution. State the possible values of m. [2]

(b) By drawing a tangent on the same axes, find the gradient of the curve at $x = 1$. [3]

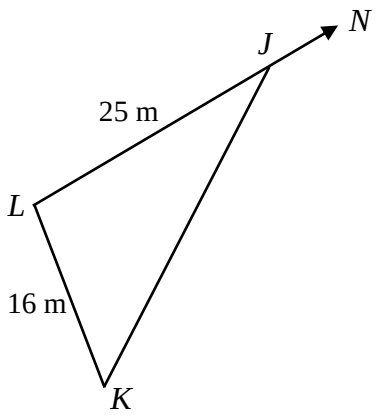
(c) The equation $x^3 - 12x - 8 = 0$ can be solved by drawing a suitable straight line on the same axes.

(i) Find the equation of this straight line. [2]

(ii) By drawing this straight line, solve the equation $x^3 - 12x - 8 = 0$. [2]

Continuation of working space for Question 8.

9 Leo, Jacob and Kent are on level ground. Jacob stands at a point J which is north of Leo, L, at a distance 25 m away. Kent stands at a point K, on a bearing of 113° and 16 m from Leo.



(a) Calculate the area of triangle JKL. [2]

(b) Find the distance between Jacob and Kent. [2]

(c) Find the bearing of Jacob from Kent. [3]

Leo is at the foot of a building 52 m high. Kent walked in a straight line towards Jacob. He spotted a bird resting at the top of the building as he started walking towards Jacob.

(d) Find the shortest distance between Kent and Leo. [2]

(e) What is the largest angle of elevation of the bird from Kent, assuming Kent's height is negligible? [2]

Continuation of working space for Question 9.

10 Lynn is a Yoga instructor and conducts lessons at three community clubs: Anchorvale, Bishan and Chong Pang. She offers yoga lessons on weekdays and weekends. The table shows the current number of students she has at the various locations:

	Anchorvale	Bishan	Chong Pang
Weekdays	10	12	6
Weekends	15	9	11

Each of her student signs up for a 8-week block of lessons and attends one lesson per week. She charges every student a fee of \$8 for a weekday lesson and \$15 for a weekend lesson.

(i) Represent the weekday and weekend charges as a 1×2 matrix **C**. [1]

(ii) Given that $\mathbf{S}=\begin{pmatrix}10 & 12 & 6\\ 15 & 9 & 11\end{pmatrix}$, evaluate $\mathbf{T} = 8\mathbf{CS}$ and state what each of the elements of **T** represents. [3]

In order to attract more people to take up Yoga, Lynn decides to reduce the price of a weekday lesson by 25% and a weekend lesson by 20% in the next 8-week block.

(iii) Write down two matrices such that the elements of their product under matrix multiplication give the new cost of the weekday and weekend yoga lessons respectively. Evaluate the product. [3]

Continuation of working space for Question 10.

END OF PAPER

Answer key

1b	$n = -1$; $k = -2$		
2ai	108°	2aiv	98°
2aii	54°	2av	56°
2aiii	126°	2avi	62°
3ai	$\frac{m}{\sqrt{m^2+1}}$	3b	$y = \frac{x}{3x-1}$
3aii	$\frac{1}{\sqrt{m^2+1}}$		
4ia	$h \approx 1.15$ (3sf) ; $k = -3$	4ii	$x \approx 1.38$ (3sf) [rej -1.38 as $x \geq 0$]
4ib	$y = -0.4$		
5ai	$d = 7$	5aiii	$K(12, -5)$
5aii	$M(0, -5)$	5b	29.1° (1dp)
6a	Centre (–1, 3); Radius = 5 units	6c	$y = \frac{4}{3}x + 4\frac{1}{3}$
6b	The point (3, 7) is outside the circle.		
7a	1.20 (3 s.f.)	7c	11.0 cm^2 (3.s.f.)
8a	Possible values of m = 4.05 or –2.05	8ci	$y = -\frac{1}{2}x + 2$
8b	Gradient ≈ –1.63 (accept –1.4 to –1.8)	8cii	$x = -3.05, -0.7$ or 3.75 $[\pm 0.05]$
9a	184 m^2 (3sf)	9d	10.7 m (3sf)
9b	34.5 m (3sf)	9e	78.4° (1dp)
9c	334.8° (1dp)		
10i	$\mathbf{C} = \begin{pmatrix} 8 & 15 \end{pmatrix}$	10iii	$\begin{pmatrix} 8 & 15 \end{pmatrix}, \begin{pmatrix} 0.75 & 0 \\ 0 & 0.8 \end{pmatrix}$ $\begin{pmatrix} 6 & 12 \end{pmatrix}$
10ii	$\mathbf{T} = \begin{pmatrix} 2440 & 1848 & 1704 \end{pmatrix}$ The elements of **T** represent the total fees she will collect for a 8-week block of lessons at each community club respectively, with \$2440 at Anchorvale, \$1848 at Bishan and \$1704 at Chong Pang.		