Atomic Structure

1. **(a)** Explain what is meant by *binding energy* of a nucleus. Use the following values of masses to calculate the binding energy per nucleon in a $^{56}_{26}Fe$ nucleus. Give your answer in MeV per nucleon.

neutron mass = 1.0087 *u*;
proton mass = 1.0073 *u*;
mass of $^{56}_{26}Fe$ nucleus = 55.9207 *u*

[6]

(b) An approximate formula for the radius *R* of a nucleus of nucleon number (mass number) *A* is

$$R \approx (1.2 \times 10^{-15})\, A^{1/3}$$

Where *R* is expressed in metres.

(i) Use this formula to calculate the approximate radius of a $^{56}_{26}Fe$ nucleus.

(ii) Assuming the nucleus to be a uniform sphere, estimate the density of nuclear matter in the iron nucleus.

(iii) Sketch a graph to show how the density of nuclear matter depends on the nucleon number A of the nucleus.

[6]

(c) The mass of 1 m^3 of iron is 8×10^3 kg. There is a general trend for the densities of solid elements to increase with increasing nucleon number. Comment on these two pieces of information in relation to your answers in (b) (ii) and (iii) and suggest an explanation for any difference between the behaviour of the density of solid elements and the density of nuclear matter.

[3]

2. **(a)** The binding energy *B* of a nucleus may be expressed as

$$B = 931.4\,(1.008665\,N + 1.007825\,Z - M)$$

In this equation, *B* is expressed in MeV and *M* is the atomic mass, expressed in u.

(i) What do the symbols *N* and *Z* represent? Why are they multiplied by the numbers 1.008665 and 1.007825 respectively?

(ii) Show how the number 931.4 arises.

(b) Figure 2.1 lists the atomic mass *M* and the binding energy *B* of some nuclides.

Nuclide	*M* / u	*B* / MeV
$^{15}_{7}N$	15.00011	115.5
$^{16}_{8}O$	15.99492	127.6
$^{18}_{8}O$	17.99916	139.8
$^{19}_{9}F$	18.99841	147.8
$^{23}_{11}Na$	22.98777	186.6
$^{24}_{12}Mg$	23.98505	198.3
$^{40}_{18}Ar$	39.96238	343.8
$^{40}_{20}Ca$	39.96259	342.0
$^{44}_{20}Ca$	43.95549	380.9
$^{45}_{21}Sc$	44.95592	387.8
$^{46}_{22}Ti$	45.95263	398.2

Fig. 2.1

(i) In nuclear physics, nuclei with equal nucleon numbers but different proton and neutron numbers are called *isobars*. Identify two isobars in Fig.2.1. [1]

(ii) Although isobars must have the same nucleon number, their atomic masses can differ because of a difference in binding energy. Confirm that, for the two isobars you have identified in (i), the heavier nuclide of the pair has a *smaller* binding energy than the lighter one. [1]

(iii) Would it be possible for the heavier nuclide of a pair of isobars to have a greater binding energy than the lighter one? Explain your answer. [2]

(iv) Suppose that a single proton were removed from each of the following nuclei: $^{16}{}_{8}O$, $^{19}{}_{9}F$, $^{24}{}_{12}Mg$, $^{45}{}_{21}Sc$, $^{46}{}_{22}Ti$. Use information from Fig. 2.1 to calculate the energy required in each case. What trend, if any, can you detect in the values of energy? [4]

[speed of light	$c = 2.9979 \times 10^{8}$ m s^{-1};
elementary charge	$e = 1.6022 \times 10^{-19}$ C;
atomic mass unit	$1\ u = 1.6605 \times 10^{-27}$ kg;
electron mass	$m_e = 0.000549\ u$;
neutron mass	$m_n = 1.008665\ u$;
proton mass	$m_p = 1.007276\ u$.]

Radioactivity

3. The radionuclide ${}^{60}_{27}\text{Co}$ is used in radiotherapy. It has a half-life of 5.27 years and upon disintegration, emits two γ rays, one of energy 1.17 MeV and the other of energy 1.33 MeV.

This nuclide is prepared by bombarding a suitable target (a stable nuclide) with an appropriate projectile particle. Possible projectiles are neutrons ${}^{1}_{0}\text{n}$ and deuterons ${}^{2}_{1}\text{d}$.

(a) Write down three reactions involving the target nuclei ${}^{63}_{29}\text{Cu}$, ${}^{62}_{28}\text{Ni}$ and ${}^{59}_{27}\text{Co}$ with one of the projectiles by which ${}^{60}_{27}\text{Co}$ might be produced.

[3]

(b) In radiotherapy treatment, it is necessary to determine the amount of energy absorbed from the radiation. Calculate the energy flux (the amount of energy passing normally through unit area per unit time) at a distance 1.50 m from a point source containing 950 μg of ${}^{60}_{27}\text{Co}$. Assume that the source radiates equally in all directions.

[8]

4. **(a)** A rubidium nucleus ${}^{87}_{37}\text{Rb}$ may emit a single β particle and decay to form a stable strontium nucleus. The decay constant of this reaction is 4.50 x 10^{-19} s^{-1}.

(i) Write down a nuclear equation representing the decay of the rubidium nucleus.

[1]

(ii) Explain, in terms of probability, what is meant by the *decay constant* of a radioactive nucleus.

[1]

(iii) Explain why the radioactive decay of a sample containing a large number of ${}^{87}_{37}\text{Rb}$ nuclei should obey an exponential law. Write down the relation between the number n_{Rb} of ${}^{87}_{37}\text{Rb}$ nuclei at time t, the number n_o at the start of the decay, and the decay constant λ.

[3]

(iv) Hence, show that at time t, the total number n_{sr} of stable strontium nuclei formed as a result of the decay is given by

$$n_{\text{sr}} = n_{\text{Rb}}\,(e^{\lambda t} - 1)$$

[3]

(b) The decay of this rubidium nuclide is used as a method of dating rocks. Values of n_{sr} and n_{Rb} are obtained by analysis of rock samples of fixed mass. For rocks of a particular type from a certain location, the results tabulated in Fig. 4.1 were obtained.

sample	1	2	3	4	5
n_{sr}	7.16×10^{18}	7.50×10^{18}	7.72×10^{18}	8.18×10^{18}	8.50×10^{18}
n_{Rb}	3.10×10^{18}	9.50×10^{18}	14.0×10^{18}	22.9×10^{18}	29.1×10^{18}

Fig. 4.1

i) Explain how, by plotting a graph of n_{sr}, against n_{Rb}, you could investigate whether these data are consistent with these different rock samples being formed at the same time.

[2]

ii) Draw this graph. Suggest an explanation for the existence of a non-zero intercept.

[6]

iii) Use your graph to estimate the age of this type of rock in this location. Give your answer in Ma, where 1 Ma = 1 million years = 3.16×10^{13} s

[3]

5. A radionuclide A decays to nuclide B. Nuclide B is unstable and decays into a stable nuclide C. The half-lives of A and B are approximately equal.

At time $t = 0$, the number of nuclei of A is N_o and the numbers of nuclei of both B and C are zero. Sketch labelled graphs showing the variation with time of the numbers of nuclei of A, B and C. Explain the form of each graph.

[6]

6. The abundance of Uranium-238 in naturally occurring uranium minerals on Earth is 99.28%. This means that there are 99.28 atoms of Uranium-238 for every 100 atoms of all uranium isotopes. The abundance of Uranium-235 is 0.72%. Assuming an equal amount of each isotope was present at the time of the formation of the Earth's crust, estimate the age of the Earth.

Decay constants:

Uranium-238, 15.5×10^{-11}, year^{-1};
Uranium-235, 98.5×10^{-11}, year^{-1}.

[4]

Answers

1. a) Binding energy per nucleon = 8.84 MeV
b) i) 4.6×10^{-15} m
ii) 2.3×10^{17} kgm^{-3}

2. b) iv) 12.10 MeV (for ${}^{16}_{8}O$); 8.00 MeV (${}^{19}_{9}F$); 11.70 MeV (${}^{24}_{12}Mg$); 6.90 MeV (${}^{45}_{21}Sc$); 10.40 MeV (${}^{48}_{22}Ti$)

3. Energy flux = 5.62×10^{-4} Wm^{-2}

4. b) iii) 362 Ma

6. 5.9×10^{9} years

Suggested Solutions to H3 Tutorial on Nuclear Physics

Atomic structure

1. **(a)** Nuclear binding energy is the energy equivalent of the mass defect of a nucleus. It is the energy required to separate all the nucleons of a nucleus to infinity.

The Fe nucleus has 26 protons (atomic number Z) and 30 neutrons (mass number A – atomic number Z). The total mass of 26 protons and 30 neutrons is:

$$26 \times 1.0073u + 30 \times 1.0087u = 56.4508u$$

mass defect $\Delta m = 56.4508u - 55.9207u$
$= 0.5301u$
$= 0.5301 \times 1.66 \times 10^{-27}$
$= 8.7797 \times 10^{-28}\text{kg}$

binding energy $E = \Delta mc^2$
$= (8.7997 \times 10^{-28})(3.00 \times 10^8)^2$
$= 7.9197 \times 10^{-11}\text{J}$
$= \dfrac{7.9197 \times 10^{-11}}{1.6 \times 10^{-19} \times 10^6}$
$= 494.98$ MeV

binding energy per nucleon $= \dfrac{494.98}{56}$
$= 8.84$ MeV

(b) **(i)** $R = (1.2 \times 10^{-15})(56)^{1/3}$
$= 4.591 \times 10^{-15}$
$= 4.6 \times 10^{-15}$ m (2 s.f.)

(ii) From b) i)

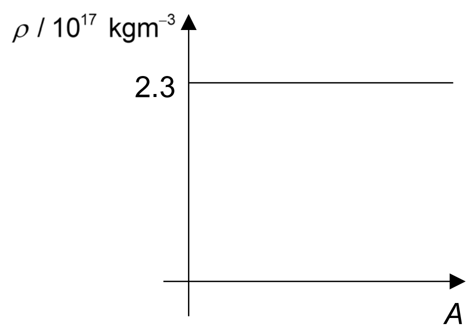
$$R = (1.2 \times 10^{-15})A^{1/3}$$

Density = Mass/Volume

$$\rho = \frac{m}{V} = \frac{Au}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3 A}$$

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

(iii) From the equation of density of nuclear matter in b ii), it is independent of mass number A. Thus the graph of density ρ against A is:



(c) From (b)(ii), the mass of 1 m^3 of nuclear matter is 2.3×10^{17} kg m^{-3}, however the mass of 1 m^3 of iron is 8×10^3 kg m^{-3} which is much smaller in magnitude. This implies nuclear matter is much denser than iron. Nuclear matter consists of protons and neutrons that are held together strongly by the short range nuclear force. Iron however is not purely nuclear matter but is made up of valence electrons, delocalised electrons and ions and held together by long range electrostatic forces of attraction/repulsion resulting in empty space between the nuclei.

From (b)(iii), the density of nuclear matter is independent of the nucleon number. However solid elements increase in density with increasing nucleon number. This implies there must be more nuclei per unit volume for solid elements with increasing nucleon number due to the variation in the electrostatic force between the ions of higher nucleon number and the valence shell electrons.

2. **(a)** **(i)** N represents the number of neutrons in the nucleus.

Z represents the number of protons in the nucleus (also known as the atomic number)

N is multiplied by 1.008665 because the mass of a neutron is 1.008665u.

Z is multiplied by 1.007825 because the mass of a proton (1.007276u) plus electron (0.000549u) is 1.007825u.

Therefore, 1.008665N + 1.007825Z is the total mass of the neutrons, protons and electrons before they combine to form the atom. If we subtract the atomic mass M from it, we get the mass defect, which multiplied by 931.4 MeV, gives the binding energy of the nucleus.

(ii) 931.4 MeV is the amount of energy released when 1 u of mass is converted to energy.

1 u is $1.6605 \text{x} 10^{-27}$ kg.

The amount of energy that can be converted from 1 u can be calculated using the equation:

$$E = mc^2 = \frac{1.6605 \times 10^{-27} \times c^2}{1.60 \times 10^{-13}} = 931.4\,\text{MeV}$$

(b) **(i)** ${}^{40}_{18}\text{Ar}$ and ${}^{40}_{20}\text{Ca}$

(ii) The ${}^{40}_{20}\text{Ca}$ nuclide which has an atomic mass of 39.96259u is heavier than ${}^{40}_{18}\text{Ar}$ which has a mass of 39.96238 u. ${}^{40}_{20}\text{Ca}$ has a smaller binding energy of 342.0 MeV (smaller than ${}^{40}\text{Ar}$ which has binding energy of 343.8 MeV).

(iii) It is still possible for a heavier nuclide to have a greater binding energy, if its constituents were heavier to begin with.

Neutrons are slightly heavier than protons. Hence, if a nuclide has a larger neutron-to-proton ratio, it could still end up heavier than its isobar with a lower neutron-to-proton ratio, even if its binding energy is higher.

(iv) Removing a proton from $^{16}_{8}O$ converts it into $^{15}_{7}N$. Energy required is the difference between the binding energy of $^{16}_{8}O$ and $^{15}_{7}N$

127.6 MeV – 115.5 MeV = 12.10 MeV.

Removing a proton from $^{19}_{9}F$ converts it into $^{18}_{8}O$. Energy required is the difference between the binding energy of $^{19}_{9}F$ and $^{18}_{8}O$

147.8 MeV – 139.8 MeV = 8.000 MeV.

Removing a proton from $^{24}_{12}Mg$ converts it into $^{23}_{11}Na$. Energy required is the difference between the binding energy of $^{24}_{12}Mg$ and $^{23}_{11}Na$

198.3 MeV – 186.6 MeV = 11.70 MeV.

Removing a proton from $^{45}_{21}Sc$ converts it into $^{44}_{20}Ca$. Energy required is the difference between the binding energy of $^{45}_{21}Sc$ and $^{44}_{20}Ca$

387.8 MeV – 380.9 MeV = 6.900 MeV.

Removing a proton from $^{48}_{22}Ti$ converts it into $^{45}_{21}Sc$. Energy required is the difference between the binding energy of $^{48}_{22}Ti$ and $^{45}_{21}Sc$

398.2 MeV – 387.8 MeV = 10.40 MeV.

These energies, known as proton separation energies, are greatest when the ratio of proton:neutron is 1:1 (refer to the table below)

Nuclide	No. of Protons	No. of Neutrons	Proton/ Neutron	Proton Separation Energy (MeV)
$^{16}_{8}O$	8	8	1.0	12.10
$^{24}_{12}Mg$	12	12	1.0	11.70
$^{48}_{22}Ti$	22	24	0.92	10.40
$^{19}_{9}F$	9	10	0.90	8.000
$^{45}_{21}Sc$	21	24	0.88	6.900

Radioactivity

3. (a) $^{63}_{29}\text{Cu} + ^{1}_{0}\text{n} \rightarrow ^{60}_{27}\text{Co} + ^{4}_{2}\text{He}$

$^{62}_{28}\text{Ni} + ^{2}_{1}\text{d} \rightarrow ^{60}_{27}\text{Co} + ^{4}_{2}\text{He}$

$^{59}_{27}\text{Co} + ^{1}_{0}\text{n} \rightarrow ^{60}_{27}\text{Co}$

(b)

$$N = \frac{\left(\frac{950\times10^{-6}}{1000}\right)}{60\times1.66\times10^{-27}} = 9.53815\times10^{18}$$

$$\text{Energy flux} = \frac{\text{Activity (A)}\times\text{energy per emission}}{4\pi r^2}$$

$$= \frac{\lambda N\times\text{energy per emission}}{4\pi r^2}$$

$$= \frac{\left(\frac{\ln 2}{(5.27\times365\times24\times60\times60)}\right)(9.53815\times10^{18})\left((1.17+1.33)\times10^{6}\times1.60\ \times10^{-19}\right)}{4\pi(1.5)^2}$$

$$= 5.6278\times10^{-4}$$

$$= 5.63\times10^{-4}\ \text{W m}^{-2}$$

4 (a) (i) The nuclear equation is:

$^{87}{}_{37}\text{Rb} \rightarrow {}^{87}{}_{38}\text{Sr} + {}^{0}{}_{-1}\text{e} + \bar{\nu}$

(ii) The decay constant is the probability that a radioactive nucleus will decay per unit time.

(iii) From (ii), for a large number n of nuclei, if we consider a time interval dt short, then the proportion of atoms decaying (dn) in this short time interval (dt) will be $-\lambda n$. That is,

$$\frac{dn}{dt} = -\lambda n$$

Integrating,

$$\int_{n_0}^{n}\frac{dn}{n} = -\int_{0}^{t}\lambda\ dt \ \rightarrow\ \ln\left(\frac{n}{n_o}\right) = -\lambda t \ \rightarrow\ n_{\text{Rb}} = n_o e^{-\lambda t}$$

It can be seen from this equation that the decay obeys the exponential law. This is more visible for a large number of nuclei given the random nature of decay.

(iv) One Sr nucleus is formed when one Rb nucleus decays. But the total number of nuclei remains constant.

$$n_{Sr} = n_o - n_{Rb}$$

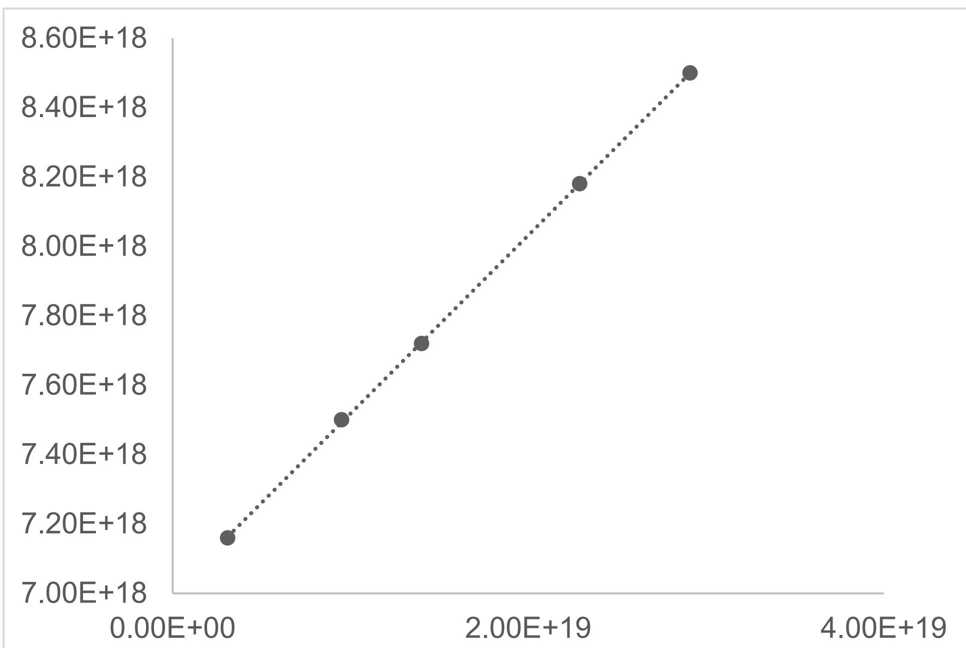
Since $n_{Rb} = n_o e^{-\lambda t}$, $\quad n_o = n_{Rb} e^{\lambda t}$

$$n_{Sr} = n_{Rb} e^{\lambda t} - n_{Rb}$$

$$n_{Sr} = n_{Rb}(e^{\lambda t} - 1)$$

(b) **(i)** Since strontium is formed from the decay of rubidium, the ratio of n_{Sr} formed to n_{Rb} remaining should be the same for rocks of the same age. However, there may exist certain amounts of strontium in the rock which are not formed from the decay of rubidium. This strontium should be subtracted from n_{Sr}. Nevertheless, the data points should all lie on the same straight line.

(ii) The graph of n_{Sr} vs n_{Rb} is as follows:



The non-zero intercept indicates the presence of initial amounts of strontium in the rock samples.

This amount of strontium must be subtracted from the total amount, in order to obtain the actual amount of strontium formed from the radioactive decay of rubidium.

(iii) Subtracting the intercept from the graph above, we get a straight line through the origin with a gradient of

$$\text{gradient} = \frac{(8.50-7.16)\times 10^{18}}{(29.1-3.10)\times 10^{18}}$$
$$= 0.0515385$$

The equation of the graph becomes

$$n_{\text{Sr}} = 0.0515385\, n_{\text{Rb}}$$

Applying the equation

$$n_{Sr} = n_{Rb}(e^{\lambda t} - 1)$$
$$5.1538\times 10^{-2}\ n_{\text{Rb}} = n_{\text{Rb}}(e^{\lambda t} - 1)$$
$$e^{\lambda t} = 1.0515385\text{, where } \lambda = 4.50\times 10^{-19}\,\text{s}^{-1}$$
$$t = 1.11676\times 10^{17}\,\text{s}$$
$$t = \frac{1.11676\times 10^{17}}{3.16\times 10^{13}}$$
$$= 3534\ \text{Ma}$$

5.

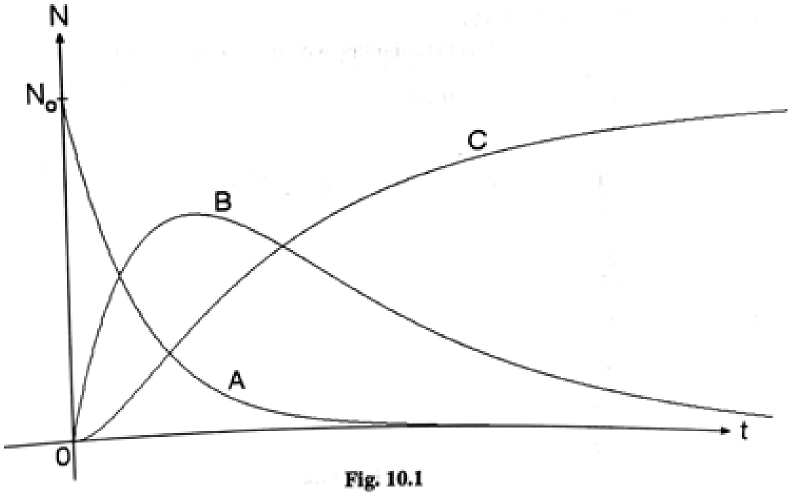


Fig. 10.1

Nuclide A undergoes nuclear decay which follows the exponential decay function of $N = N_0 e^{-\lambda t}$.

B's graph initially increases from zero as nuclide B is formed from the decay of nuclide A. However, since nuclide B is unstable, it decays to form nuclide C. The graph stops rising after awhile and decreases.

Graph C rises continuously as it is formed from B and will not decay into other products as it is stable. The initial increase of C is gradual since the number of nuclide B is small at the start. Eventually the number of nuclei of C will be equal to that of A's initial N_o, while A and B will fall to zero.

6. Let N_0 be the initial number of Uranium-235 and Uranium-238 atoms at the time of the formation of the Earth's crust

Let N_{235} and N_{238} be the respective number of nuclei at time *t* in years after the formation of the Earth's crust

$$N_{238} = N_0 \ e^{(-15.5\times10^{-11}\ t)} \ \text{... (1)}$$

$$N_{235} = N_0 \ e^{(-98.5\times10^{-11}\ t)} \ \text{... (2)}$$

Dividing equation (1) by equation (2),

$$\frac{N_{238}}{N_{235}} = \frac{N_0 \; e^{(-15.5\times10^{-11} \; t)}}{N_0 \; e^{(-98.5\times10^{-11} \; t)}}$$

$$\frac{99.28}{0.72} = e^{83\times10^{-11} t}$$

$$t = 5.94 \times 10^{9} \text{ years}$$