## Mathematical Formula

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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1 Solve the equation $3\left(9^x\right) = 15 - 4\left(3^x\right)$. [4]

4

$3(9^x) = 15 - 4(3^x)$

$3(3^{2x}) = 15 - 4(3^x)$

sub $3^x$ as $y$,

$3y^2 = 15 - 4y$

$3y^2 - 15 + 4y = 0$

$(3y-5)(y+3) = 0$

$y = \frac{5}{3}$ OR $y = -3$

$3^x = \frac{5}{3}$ OR $3^x = -3$ (rej)

$x \lg 3 = \lg \frac{5}{3}$

$x = 0.465$ (3 s.f.)

$3(3^{2x}) = 15 - 4(3^x)$

2 Given $\log_5 3 = k$, express the following in terms of $k$.

(i) $\log_5 15$ $\qquad \log_5(3\times 5)$ [2]

4

(ii) $\log_3 \frac{9}{5}$ $\qquad \log_3 9 - \log_3 5$ [2]

$2\log_3 3$

i) $\log_5 15 = \log_5(3\times5)$ $\qquad 2 - \frac{\log_5 5}{\log_5 3}$

$= \log_5 3 + \log_5 5$

$= k + 1$

2

ii) $\log_3 \frac{9}{5} = \log_3 3^2 - \log_3 5$

$= 2 - \frac{\log_5 5}{\log_5 3}$

$= 2 - \frac{1}{k}$

2

3 Determine if the equation $\log_3(x-4) = 2\log_3 x - \log_3 3$ has any real roots. [3]

2

$\log_3(x-4) = 2\log_3 x - \log_3 3$

$\log_3(x-4) = \log_3 x^2 - \log_3 3$

$\log_3(x-4) = \log_3\left(\frac{x^2}{3}\right)$

$x - 4 = \frac{x^2}{3}$

$3x - 12 = x^2$

$x^2 - 3x + 12 = 0$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(12)}}{2(1)}$ show value $\qquad \frac{3 \pm \sqrt{-39}}{2}$

$=$ undefined

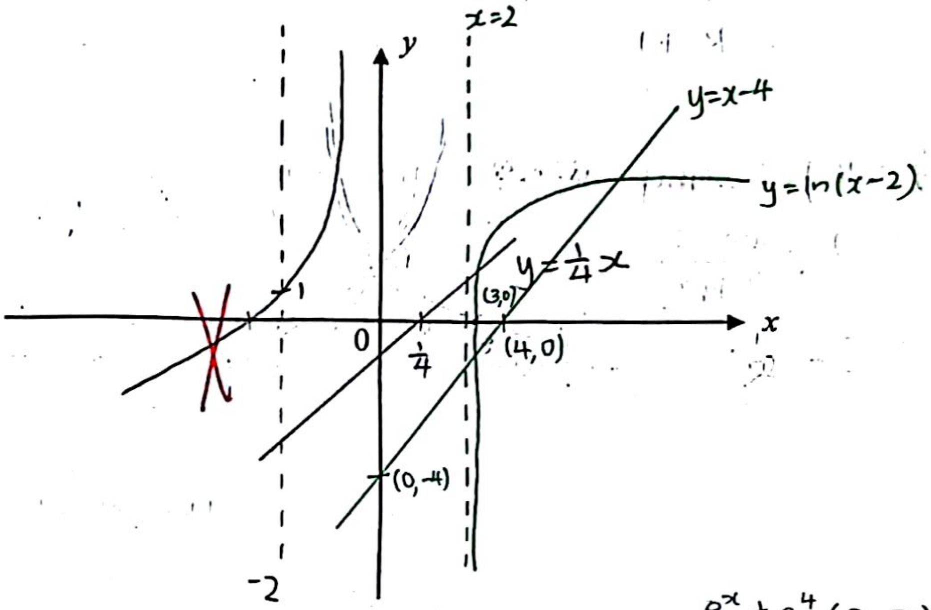
$\therefore$ hence the equation $\log_3(x-4) = 2\log_3 x - \log_3 3$ has no real roots.

4 (i) Sketch the graph of $y = \ln(x - 2)$ on the given axes, indicating clearly the intercept(s) and asymptote(s). [2]

(ii) In order to solve the equation $e^x + e^4(2 - x) = 0$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = \ln(x - 2)$. Find the equation of the straight line. Add the straight line to your diagram and find the number of solutions to the equation. [3]

x - int = 3

(i)




0

$e^x + e^4(2-x)$

$\ln y = x + 4(2-x)$

$x - 4y$

$4y = x$

$y = \frac{1}{4}x$

in ink.

$e^x + e^4(2-x) = 0$

$e^x = e^4(x-2)$

$\frac{e^x}{e^4} = x - 2$

$e^{x-4} = x - 2$

$\ln x - 2 = x - 4$

draw $y = x - 4$

there are two solutions.

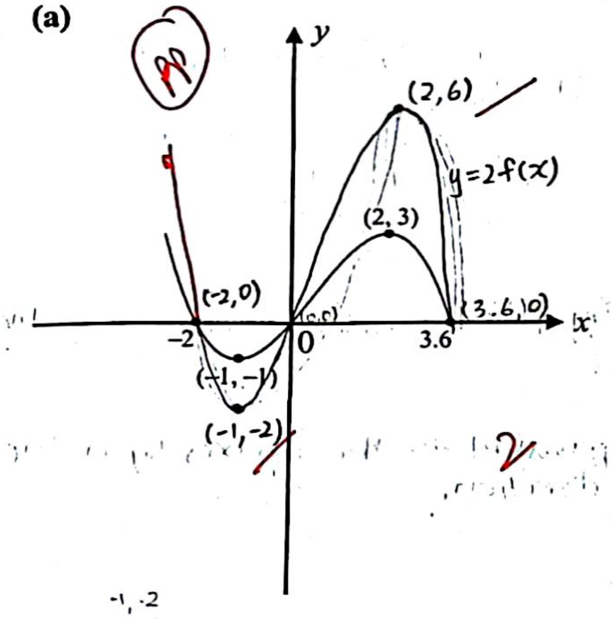
**5** The graph of $y = \mathrm{f}(x)$ is shown in the diagram below. On the same axes, sketch and label the following graphs, showing clearly the new positions of the points indicated in the diagram:

**(a)** $y = 2\mathrm{f}(x)$; → scale ×2 of y [2]

**(b)** $y = \mathrm{f}(-x)$. → reflect x-axis [2]

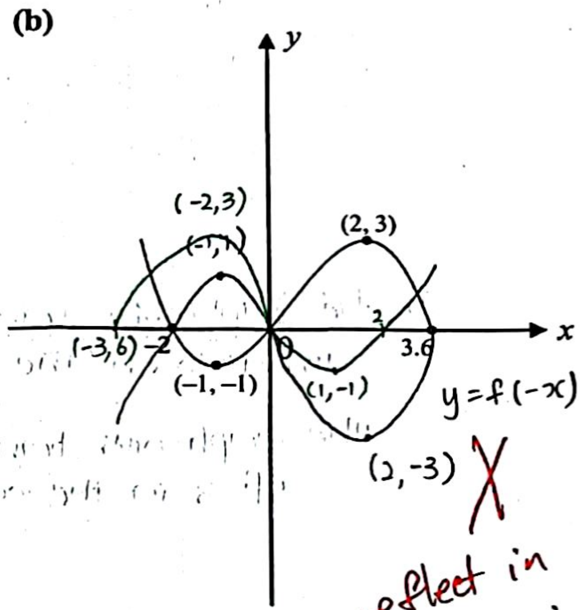
2

**(a)**




2

**(b)**




reflect in y-axis.

$x^2+3x+2$

**6** **(a)** The graph of $y = x^2 + 3x - 1$ can be obtained from the graph of $y = (x+1)(x+2)$ under a single transformation. Describe this transformation. [2]

2

**(b)** The graph of $y = g(x)$ undergoes two successive transformations:

**I** Scaling parallel to the $x$-axis by factor $\frac{1}{2}$. $y=f(ax)$

↘ multiply 2

**II** Reflection in the $x$-axis. $y=f(-x)$

The resulting graph is $y = -4x^2 + 10x + 1$. Find an expression for $g(x)$. [2]

4

a) ~~[illegible] translated by [illegible] units in the negative direction on the y-axis.~~

The graph was translated parallel to the ~~y-~~axis by ~~a factor of~~ 3 in the negative direction.

2.

b) $y=-4x^2+10x+1$

before transformation II,

$y=4x^2-10x(+1)$ $-1$ $y=4x^2-10x-1$

before transformation I, $y=g(x) = (2x)^2-5(2x)-1$

$y=2x^2-5x+1$ $= x^2-5x-1$

0

$g(x) = 2x^2-5x+1$

7 Chilled dessert is reheated to a temperature of 85°C. It subsequently cools in such a way that its temperature, $M$ °C, $t$ minutes after removal from the heating source, is given by $M = 23 + Ae^{-kt}$, where $A$ and $k$ are constants. 2 minutes after removing from the heating source, the temperature of the dessert is found to be 55°C.

4

(i) Show that $A = 62$. [1]

(ii) Find the value of $k$. [2]

(iii) The dessert should only be served when its temperature is less than 50°C. Determine, with clear working, whether the dessert is suitable to be served 3 minutes after removing from the heating source. [3]

i) $M = 23 + Ae^{-kt}$

sub $e^{-kt}$ as 1, M as 85,

$85 = 23 + A$

$A = 62$ (shown)

ii) $55 = 23 + 62e^{-2k}$

$62e^{-2k} = 32$

$e^{-2k} = \frac{16}{31}$

$-2k = \ln\left(\frac{16}{31}\right)$

$-k = \cancel{0.66139}\ 0.66140$

$k = 0.661$ (3s.f.)

calculation

1

$k = -0.33070$ (5s.f.)

$= -0.331$ (3s.f.)

iii) $M = 23 + 64e^{-3(0.66140)}$ 1.

$M = 31.799$ (5s.f.)

$M = 31.8$ (3s.f.) 1.

$M = 23 + 62e^{-3(0.33070)}$

$= \cancel{46.79}\ 45.99$ (4s.f.)

As the temperature, M, is ~~below~~ 50°C, the dessert is ~~not~~ suitable to be served 3 minutes after removing from the heating source.

2

read qn carefully.

*End of Test*