Topic 17 – **Electromagnetic Induction**

Guiding Questions

- If an electric current produces a magnetic field, can a magnetic field produce an electric current? What is required for this?
- Can we achieve perpetual motion by invoking the magic of electromagnetism?

Content

- Magnetic flux
- Laws of electromagnetic induction

Learning Outcomes

Candidates should be able to

a. define magnetic flux as the product of an area and the component of the magnetic flux density perpendicular to that area

b. recall and solve problems using $\Phi = BA$

c. define magnetic flux linkage

d. infer from appropriate experiments on electromagnetic induction:
   i. that a changing magnetic flux can induce an e.m.f.
   ii. that the direction of the induced e.m.f. opposes the change producing it
   iii. the factors affecting the magnitude of the induced e.m.f.

e. recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law

f. explain simple applications of electromagnetic induction

Introduction

In the previous topic on Electromagnetism, we learned that when a current flows through a conductor in a magnetic field, it experiences a force. This phenomenon, governed by Fleming's Left Hand Rule, can cause the conductor to move. But what if, instead of supplying a current, we apply a force to move the conductor through the magnetic field? Could this generate a current?

The answer lies in the principle of Electromagnetic Induction, which forms the foundation of how electricity is generated in power plants and countless other systems. To understand this process, we must first explore the concept of magnetic flux and the fundamental laws of electromagnetic induction.

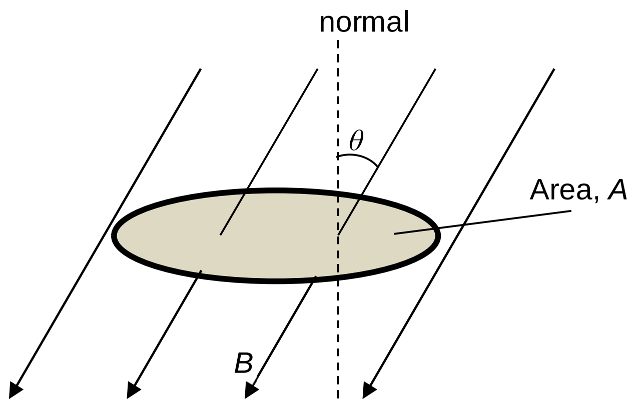
17.1 Magnetic Flux

Magnetic field lines visually represent the strength and direction of a magnetic field. However, in certain situations, it is important to quantify how much of the magnetic field penetrates a given surface. This is where the concept of magnetic flux comes in. *Magnetic flux* measures the total number of magnetic field lines passing through a specific area A and helps us analyse the interaction between magnetic fields and surfaces.

Magnetic Flux is defined as the product of an area and the component of the magnetic flux density perpendicular to that area.

For a uniform magnetic flux density B at an angle θ to the normal of an area A, the magnetic flux Φ is given by

$$\Phi = B_{\perp}A = BA\cos\theta$$



The SI unit of magnetic flux is the *weber* (symbol: Wb).

17.1.1 Magnetic Flux Linkage

If the area A is bounded by a coil and the coil has N turns, then the total magnetic flux passing through the coil, also known as *magnetic flux linkage* through the coil, is

$$N\Phi = NBA\cos\theta$$

Magnetic Flux Linkage is the product of the magnetic flux passing through the coil and the number of turns on the coil.

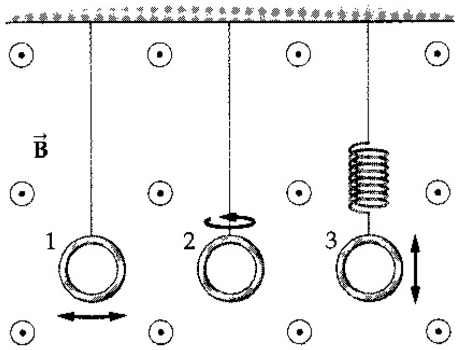
From the equation, magnetic flux linkage through a coil depends on

a) the number of turns in the coil, N
b) the magnitude of magnetic flux density, B
c) the surface area of the coil, A
d) the orientation of the coil, θ, with respect to the direction of B. (Note: θ is the angle between the magnetic flux density and the normal to the surface of the coil,)

A change in any of these variables results in a change of magnetic flux linkage.

Example 1

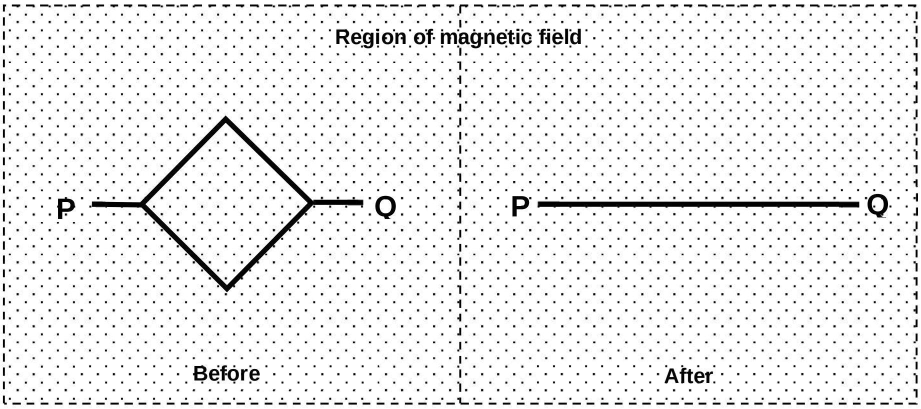
The three loops of wire shown are all in a region of uniform magnetic field. Loop 1 swings back and forth as the bob on a pendulum, loop 2 rotates about a vertical axis and loop 3 oscillates vertically on the end of a spring. Which loop(s) have a magnetic flux that changes with time?



Solution

Example 2

A coil of 10 turns and area 0.35 m^2 is placed perpendicular to a magnetic field of density 2.0 T as shown in the diagram on the left. Points P and Q are pulled apart until the coil becomes a straight line as shown in the diagram on the right. What is the change in magnetic flux linkage through the coil?



Solution

17.2 Electromagnetic Induction

Ten years after the discovery that an electric current could produce a magnetic field, independent experiments conducted independently by Joseph Henry in United States and Michael Faraday in England showed that a changing magnetic field could induce an electric current in a circuit. The results of these experiments led to the formulation of a basic and important law known as Faraday's law.

17.2.1 Faraday's Law

Faraday's Law of electromagnetic induction states that the magnitude of the induced e.m.f. is directly **proportional** to the **rate of change** of the magnetic flux linkage.

Mathematically:

$$|\varepsilon| \propto \frac{d(N\Phi)}{dt} = \frac{d(NBA\cos\theta)}{dt}$$

e.m.f. induced depends on quantities such as N number of coils, magnetic flux density B, area of coil A and angle θ between B and normal of A.

17.2.2 Lenz's Law

Lenz's Law states that the induced current is in a direction so as to produce effects which oppose the change in magnetic flux.

Lenz's law is not written separately but combined with Faraday's law to form an equation.

$$\varepsilon = \frac{-d(NBA\cos\theta)}{dt}$$

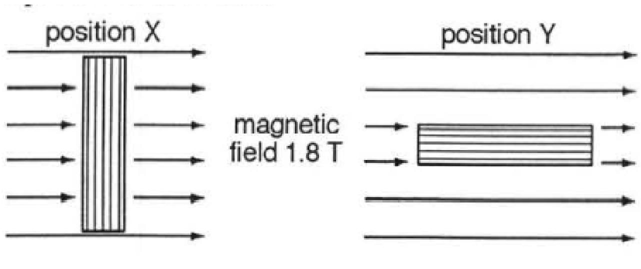
The negative sign represents direction of induced current is always such the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

Furthermore, the direction of an induced e.m.f. is that of the induced current. The key word in Lenz's law is "opposition."

Note: <u>A current is induced only if there is a complete circuit. An e.m.f. is always induced when there is a change in magnetic flux linkage.</u>

Example 3

A circular coil of diameter 2.0 cm has 3000 turns. It is rotated in a magnetic field of flux density 1.8 T from position X to position Y in 0.060 s.



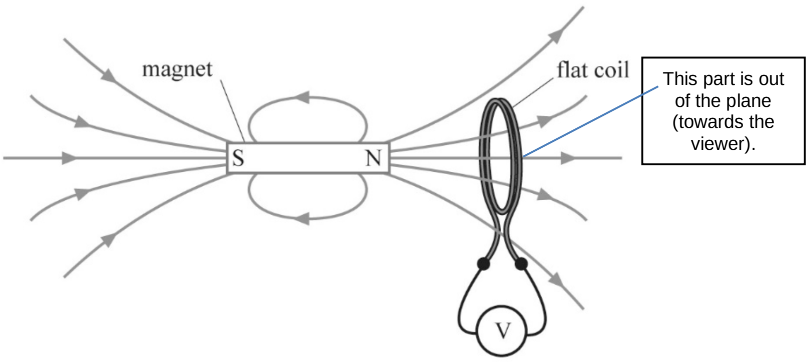
What is the average e.m.f. induced in the coil during the rotation?

A 28 V **B** 36 V **C** 113 V **D** 1800 V

Example 4

The diagram shows a magnet placed close to a flat circular coil.

a) Explain why there is no induced e.m.f. even though there is magnetic flux linking the coil.



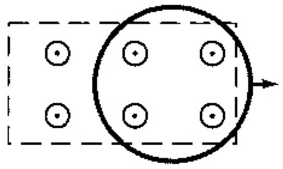
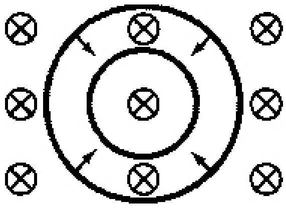
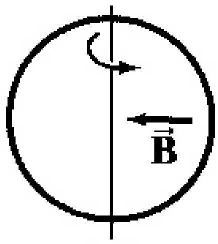
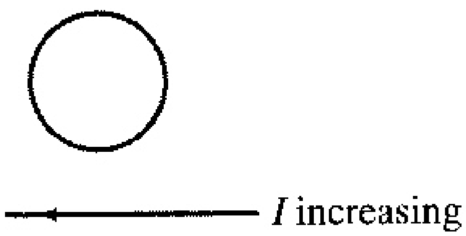
b) Explain why there is an induced e.m.f. when the magnet is pushed towards the coil.

c) Indicate direction of current in the coil by drawing arrows on the coil.

d) What if coil was pushed away from magnet?

Example 5

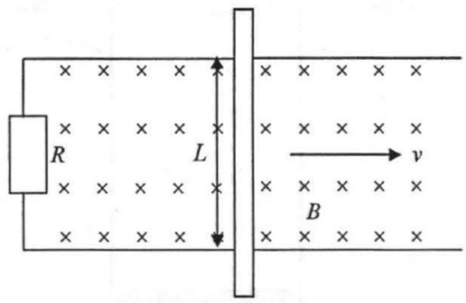
Predict the direction of the induced current in the circular coil for each of the following cases:

Action	Direction of induced current
 Pulling the coil to the right out of a magnetic field pointing out of the plane of the page.	
 Shrinking a coil in a magnetic field pointing into the plane of the page.	
 Rotating the coil by pulling the left side towards us and pushing the right side in with the magnetic field pointing right to left.	
 Increasing the current in a wire that is placed below a coil in the plane of the page.	

Example 6

A rod of length 0.20 m moves at a steady speed of 3.0 m s^{-1} at right angles to a magnetic field of flux density 0.10 T.

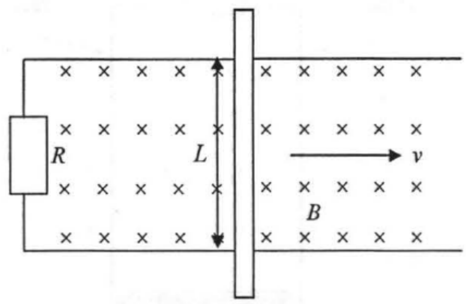
(a) Use Faraday's law to determine the e.m.f. induced across the ends of the wire. Also determine which end is the positive terminal.



(b) Determine magnitude and direction of current if R = 2.0 Ω. Indicate direction using arrows on the circuit diagram.

(c) How to apply Lenz's law to example 6 (b)?

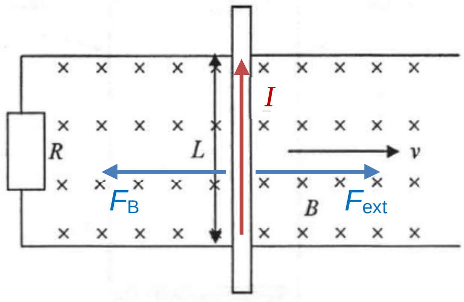
Solution:



Quiz: How to apply Fleming's Right Hand Rule (see Annex) to Example 6 (b)?

17.2.3 Exploring Energy Conservation with Lenz's Law

In Example 6, how can the rod maintain a constant speed?



Answer: The induced current dissipates energy in the resistor, according to $P = I^2R$, and $E = Pt = I^2Rt$. Hence, the rod will slow down, as kinetic energy converts into electrical energy.

If a constant velocity is desired, then an external force is required. Consider the induced current:

$$I_{ind} = \frac{\varepsilon_{ind}}{R} = \frac{BLv}{R}$$

A magnetic force F_B is produced to the left due to the induced current, according to Fleming's LHR

$$F_B = BI_{ind}L$$

Note that the magnetic force F_B is always opposite direction to v, which causes the rod to slow down. Hence an external force F_{ext} is required, whose magnitude is same as F_B but in opposite direction, that is, to the right, to maintain constant velocity.

Then, work done by F_{ext} would convert into electrical energy, so $F_{ext}v = P = I_{ind}^2R$.

To verify this relation, we simply replace F_B with $BI_{ind}L$, and I_{ind} with $\frac{BLv}{R}$.

The rate of work done by external force F_{ext} is given by

$$F_{ext}v = BI_{ind}Lv = B\left(\frac{BLv}{R}\right)Lv = \frac{B^2L^2v^2}{R}$$

The electrical power dissipated in resistor is given by

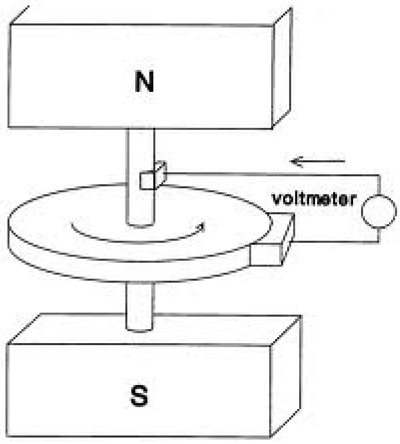
$$I_{ind}^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

So, we verify work done by F_{ext} converts into electrical energy, and the electrical power = rate of doing work, $P = \frac{B^2L^2v^2}{R}$.

Lastly, Lenz' law is a consequence of the *principle of conservation of energy*. We find the direction of the induced current through Lenz's law, which then induces a force opposite to velocity, which then converts kinetic energy into electrical energy, hence energy is conserved.

Example 7

A Disc Rotating Perpendicularly in a Uniform Magnetic Field

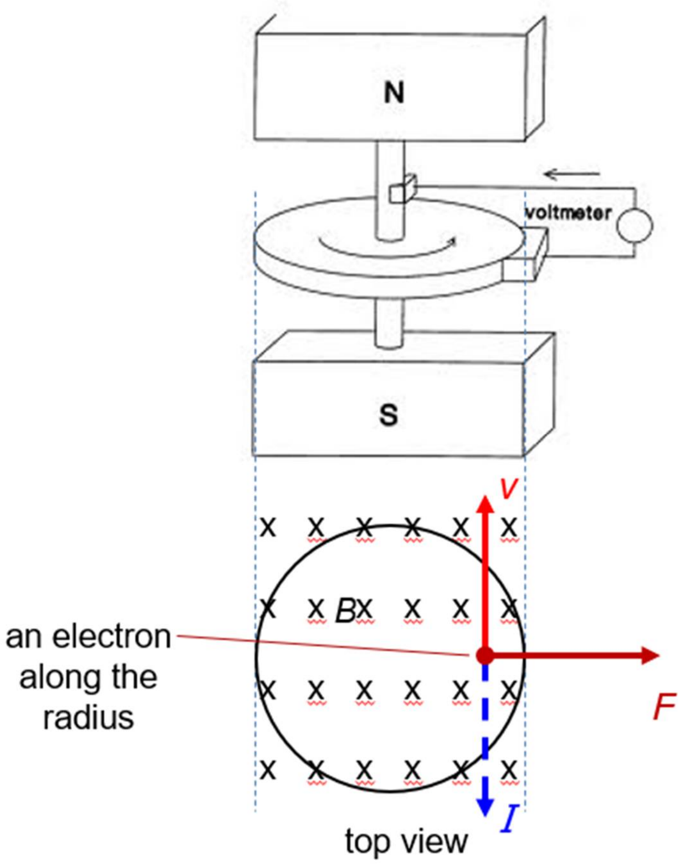


When a disc of area *A* perpendicular to a uniform magnetic field *B* is rotated with constant frequency *f*, determine the e.m.f. induced between the center and the rim of the disc.

Solution:

We can regard the disc as a many-spoked wheel. The radius of the disc is sweeping through the magnetic field.

Thus, $|\varepsilon| = B\pi r^2 f$.



According to Fleming's Left Hand Rule, the electrons in the rotating disc experience a magnetic force directed radially outward.

This force drives the electrons towards the rim of the disc, resulting in a net negative charge at the rim and a net positive charge at the center.

Consequently, the center of the disc has a higher electric potential than the rim.

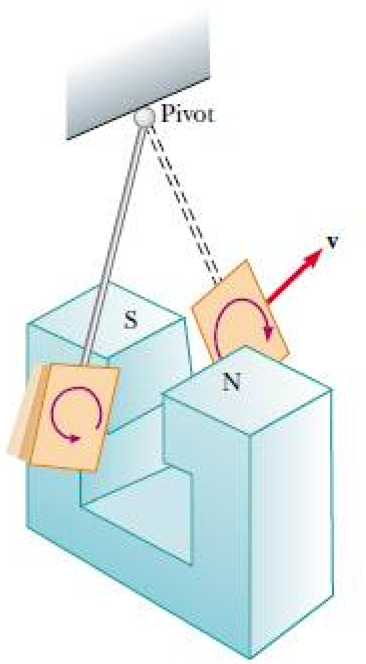
If the rotation of the disc is reversed, the direction of the force changes, causing the rim to have a higher potential than the center.

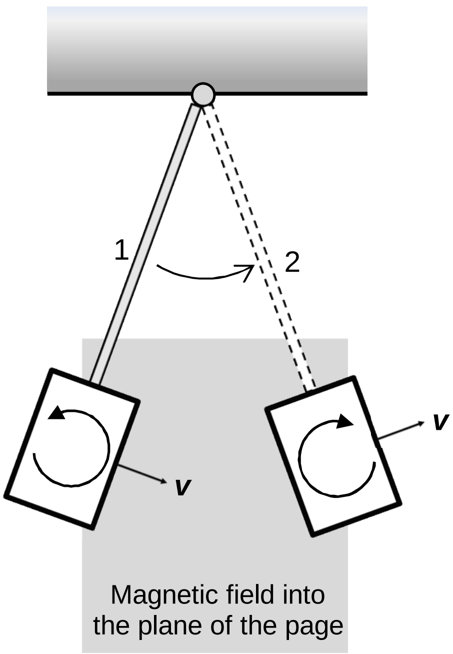
This separation of charges generates an e.m.f. If a circuit is completed, such as by connecting a resistor between the rim and the center of the disc, the induced e.m.f. drives an electric current through the circuit.

17.3 Applications of Electromagnetic Induction

17.3.1 Eddy Currents

When a conductor is subjected to a changing magnetic flux, the induced e.m.f. causes currents to flow. If the conductor is in the shape of a loop, the induced current will flow around the loop. However, if the conductor is a solid plate, induced currents, known as eddy currents, flow simultaneously along many different paths in swirls. This can easily be demonstrated by allowing a flat copper or aluminium plate attached to the end of a rigid bar to swing back and forth through a magnetic field. As the plate enters the field, the plate cuts the magnetic field/flux. The changing magnetic flux through the plate induces an e.m.f. in the plate. The rate of cutting is not the same over the whole plate, so different e.m.f. are induced in different parts of plate. This leads to eddy currents in the plate.





As the swinging plate enters the field at position 1, the flux due to the external magnetic field into the page through the plate is increasing, hence by Lenz's law the induced eddy current must provide its own magnetic field out of the page. Direction of the induced current is thus anticlockwise. The opposite is true as the plate leaves the field at position 2, where the current is clockwise.

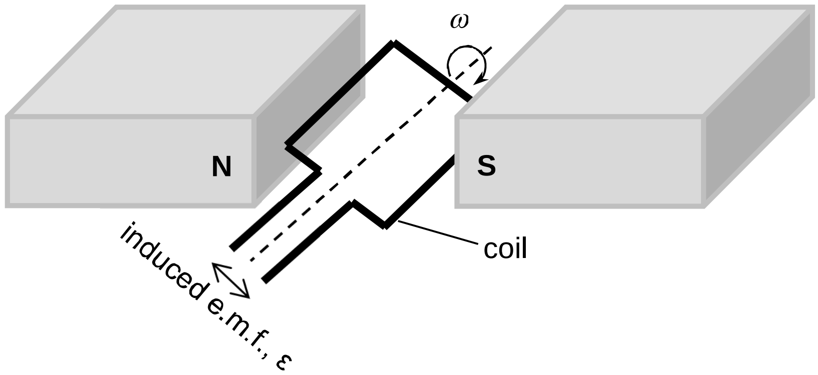
Heat energy is dissipated as eddy currents flows in the conductor (i.e. the conductor gets heated up). Eddy currents can be reduced by eliminating paths for the current flow, for example, by cutting slots in the plate. These slots will prevent large eddy currents from occurring.

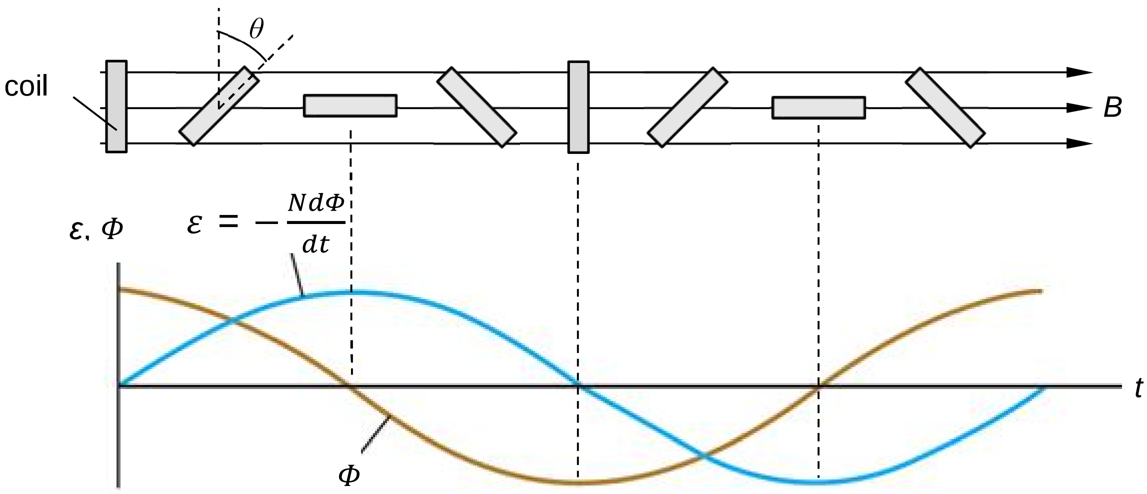
Sometimes eddy currents can be a good thing, for example, it can be used to set up a braking system which can rapidly change kinetic energy to other forms of energy. This can be taken a step further if a circuit can be built to channel the electrical energy from the kinetic energy back into the battery. This is what most hybrid cars do.

17.3.2 Generator

Electric generators convert mechanical energy into electrical energy.

When a coil of N turns rotates with a constant angular velocity ω in a uniform magnetic field B, an e.m.f. ε is induced. The flux Φ and the resulting induced e.m.f. ε between the terminals of the coil during one complete rotation are shown in the figure below.





According to Faraday's and Lenz's laws, the induced e.m.f. is equal to negative the gradient of the magnetic flux linkage against time graph.

- When the flux linking the coil is maximum, the rate of change of flux is zero, and hence the induced e.m.f. is zero.
- When the flux linking the coil is zero, the rate of change of flux is maximum (the Φ-t graph is steepest) and hence the induced e.m.f. is also maximum.

Assume that at time t = 0, the coil is perpendicular to the B field and there is maximum flux linkage through the coil. At any other point in time, the flux linkage through the coil is

$$N\Phi = NBA\cos\theta$$

where θ is the angle between the plane of coil and vertical axis.

For uniform circular motion, $\theta = \omega t$, $N\Phi = NBA\cos(\omega t)$

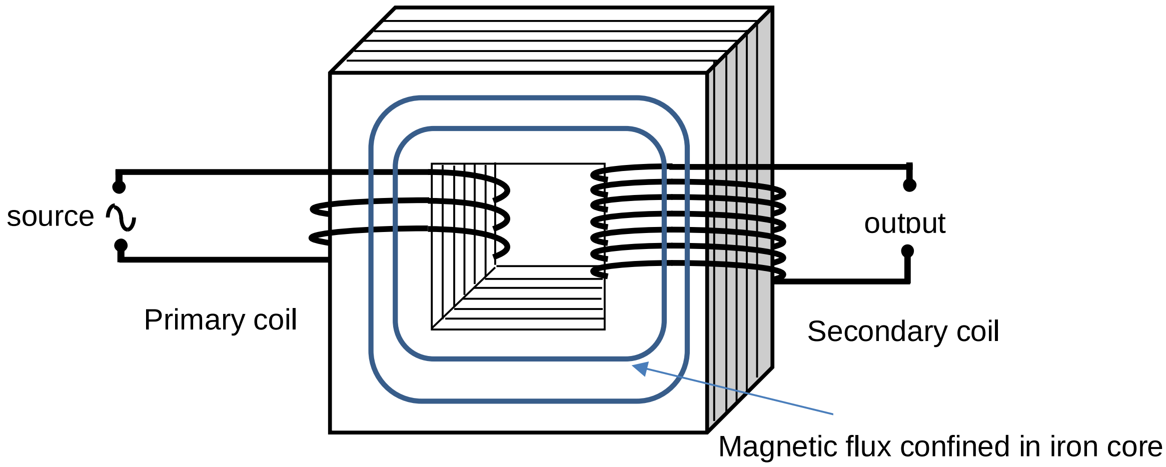
From laws of induction, induced e.m.f. is given by

$$\varepsilon = -\frac{d(N\Phi)}{dt} = -\frac{d\left[NBA\cos(\omega t)\right]}{dt} = NBA\omega\sin(\omega t)$$

Factors affecting the magnitude of the induced e.m.f. are (i) magnetic flux density B, (ii) area of the coil A, number of turns in the coil N and angular speed ω or frequency of rotation f.

17.3.3 Transformer

Our system for electricity transmission is based on transformers, devices that use electromagnetic induction to change a.c. voltages. Transformers raise voltages for transmission over long distances across power lines. Transformers then reduce voltages for safe use in businesses and homes. The figure below shows a simple transformer which consists of two coils wound on a common soft iron core, so that nearly all the magnetic field lines produced by the primary coil pass through each turn of the secondary coil.



Recall that a magnetic field is produced by a solenoid, given by

$$B = \mu_0 n I.$$

Also, note the flux (and rate of change of flux) through both coils are the same, but the flux linkage is different, since N is different for primary and secondary coils.

Thus, by Faraday's Law,

$$\varepsilon \propto N \frac{d\Phi}{dt}$$

but since $d\Phi/dt$ is the same for both coils, therefore

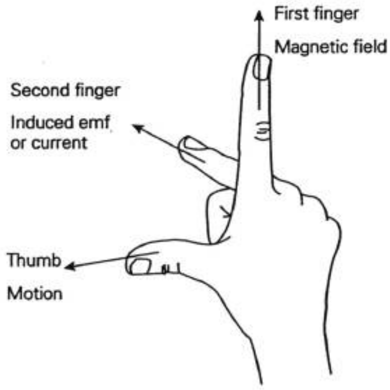
$$\varepsilon \propto N$$

That means we can change the induced voltage based on number of turns in the coil. If the secondary coil has more turns, the output voltage is higher than source, and we call this a <u>step-up transformer</u>.

In contrast, if the secondary coil has less turns, then the output voltage is lower, and it is called a <u>step-down transformer</u>. The transformer only works with AC, since only AC produces a changing flux which then induces an emf. Also, the consequence of stepping up the voltage causes the current to be reduced (since $P = IV$ and P is constant for an ideal transformer), which then reduces power loss in transmission (recall $P = I^2R$). This will be covered in detail in the next topic Alternating Current.

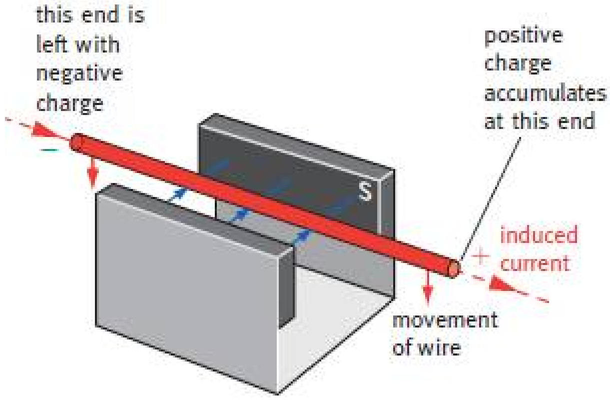
Annex

Fleming's right-hand rule: a technique used to determine the direction of the induced e.m.f. or current in a conductor moving through a magnetic field.



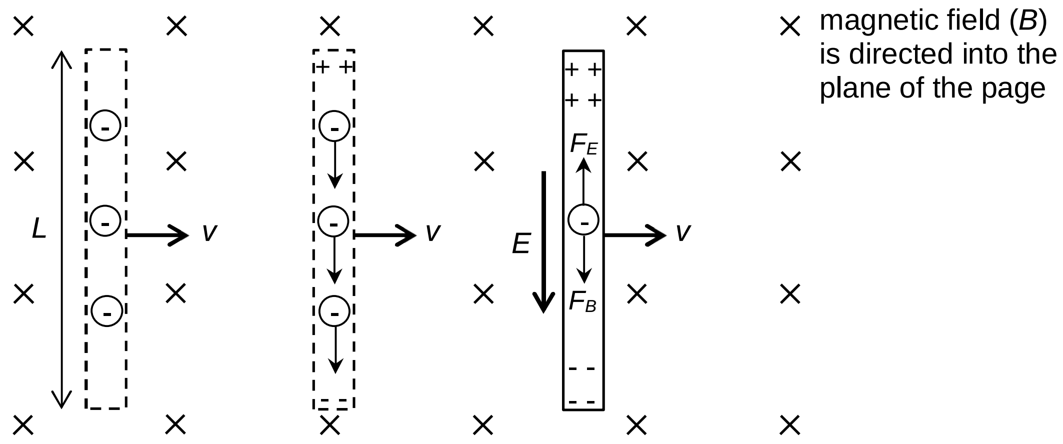
The first finger points in the direction of the magnetic field and the thumb points in the direction of motion, then the second finger gives the direction of the induced e.m.f. or current.

In the example shown on the right, the conductor is being moved downwards across the magnetic field. There is an induced e.m.f. in the conductor as shown. When the conductor is not part of a complete circuit, there cannot be an induced current. Instead, negative charge will accumulate at one end of the conductor, leaving the other end positively charged. We have induced e.m.f. across the ends of the conductor. If the conductor is part of a complete circuit, then there is an induced current in the conductor as shown.



Motional e.m.f.: Cutting of Magnetic Field Lines

In earlier sections, we considered cases in which an e.m.f. is induced when there is a change in the magnetic flux linkage. In this section, we will study what is called motional e.m.f., which is the e.m.f. induced in a conductor moving through a uniform magnetic field.



When a conductor (wire) moves at a constant velocity in a uniform magnetic field (see Example 6), the moving conductor cuts the magnetic field (or flux). An induced e.m.f. is generated when there is a cutting of flux. In the above figure, the conductor is being pushed to move to the right cutting across a uniform magnetic field (B) which is directed into the page. Now, think of the free electrons in the conductor. They are moving to the right, so the conventional current is flowing to the left. Based on *Fleming's left-hand rule*, each electron experiences a downward magnetic force, $F_B = Bqv$. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field E (directed downward) is produced inside the conductor. The charges accumulate at both ends until the downward magnetic force on charges remaining in the conductor is balanced by the upward electric force, $F_E = qE$.

electric force on remaining charge = magnetic force on the charge

$$F_E = F_B$$

$$qE = qvB$$

$$E = vB$$

Since the electric field in the conductor is uniform, the potential difference across the ends of the conductor of length L (from the topic on Electric Fields) is given by

$$\varepsilon = EL$$

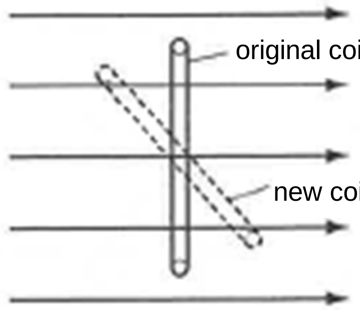
$$= BLv$$

Therefore an e.m.f. (ε) is induced as long as the conductor continues to move through the uniform magnetic field. If the direction of the motion is reversed, the polarity of the induced e.m.f. is also reversed.

From the formula, magnitude of the induced e.m.f. is proportional to the rate of flux cutting.

Tutorial Questions

1 A coil is situated with its plane perpendicular to a uniform magnetic field and within the field.

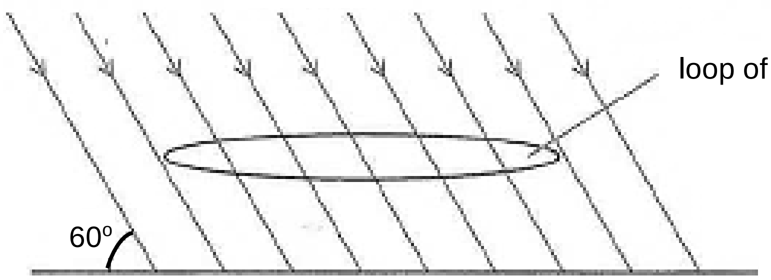


The coil is rotated to the new position shown.

How do the magnetic flux density and the total magnetic flux passing through the coil change during the rotation?

	magnetic flux density	total magnetic flux
A	decreases	decreases
B	Increases	increases
C	no change	decreases
D	no change	increases

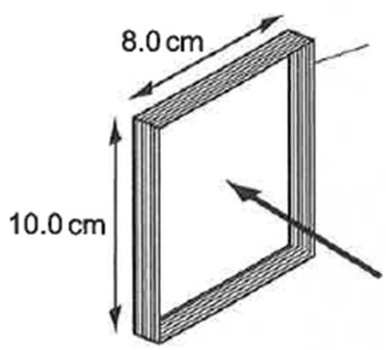
2 A flat, circular loop of wire, enclosing an area of 12 cm^2, is held horizontally above the Earth's surface. The Earth's magnetic flux density B_E is 65 μT in a direction making an angle of 60° with the horizontal.



What is the magnetic flux through the loop of wire?

A 3.9×10^{-8} Wb **B** 6.8×10^{-8} Wb **C** 3.9×10^{-4} Wb **D** 6.8×10^{-4} Wb

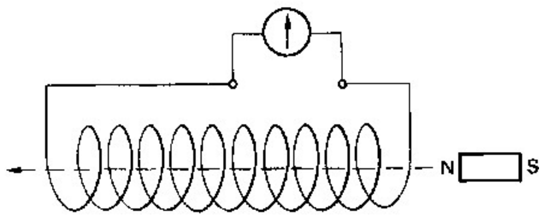
3 A rectangular coil made of 800 turns of wire has dimensions 8.0 cm × 10.0 cm. It is placed at right angles to a magnetic field of uniform magnetic flux density.



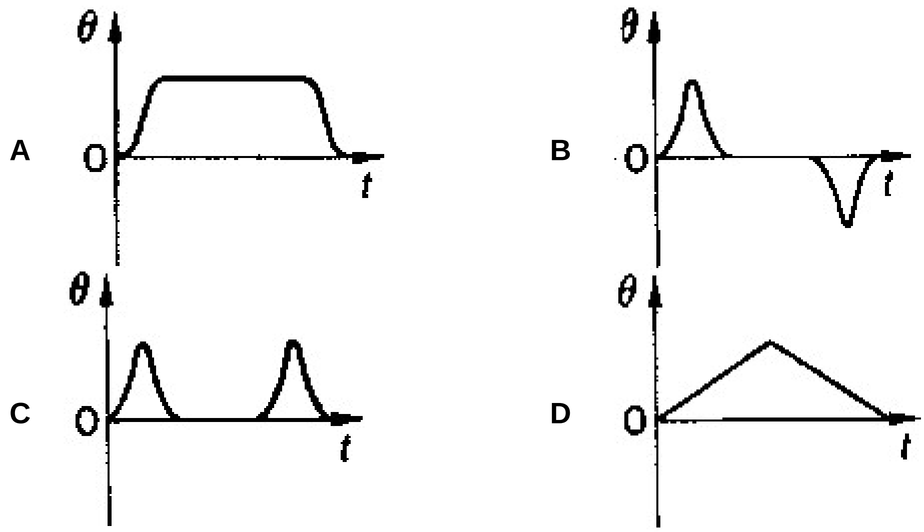
What are the possible values for the magnetic flux density, the magnetic flux and the magnetic flux linkage for the coil?

	magnetic flux density / T	magnetic flux / Wb	magnetic flux linkage / Wb turns
A	3.4	2.7×10^{-2}	220
B	2.7×10^{-2}	3.4	2700
C	3.4×10^{-4}	4.2×10^{-2}	34
D	4.2×10^{-2}	3.4×10^{-4}	0.27

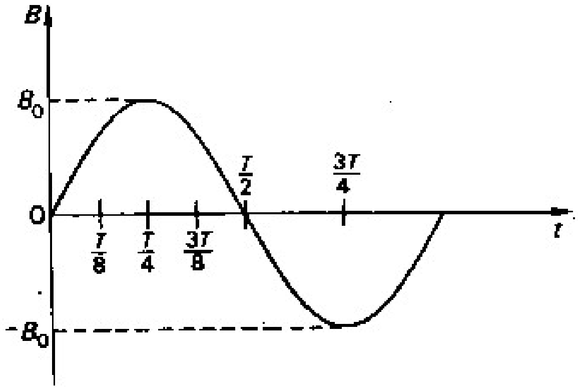
4 A short bar magnet passes at a steady speed right through a long solenoid. A galvanometer is connected across the solenoid.



Which graph best represents the variation of the galvanometer deflection θ with time t?



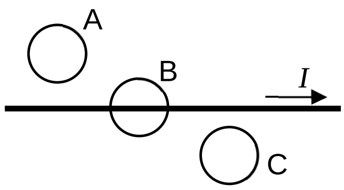
5 A magnetic field is applied perpendicular to the plane of a flat coil of copper wire. The time variation of the magnetic flux density is as shown graphically below.



At which of the following values of t is the magnitude of the emf induced in the coil a maximum?

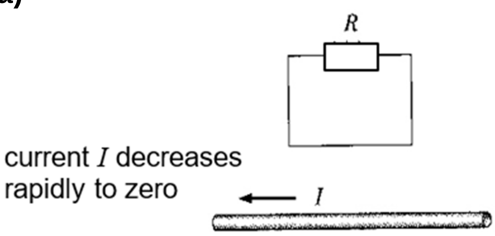
A $\frac{T}{8}$ **B** $\frac{T}{4}$ **C** $\frac{3T}{8}$ **D** $\frac{T}{2}$

6 If the current is the wire is increased, indicate the direction of the induced current in each loop A, B and C.



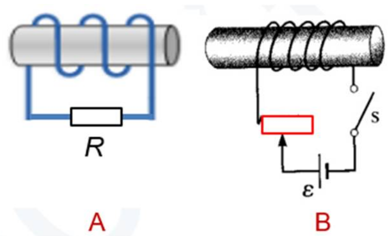
7 Determine the direction of the induced current in *R* in each of the following cases:

(a)



current *I* decreases rapidly to zero

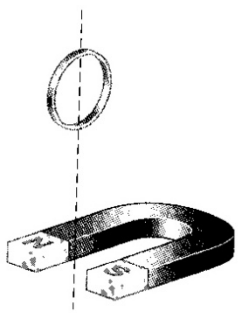
(b)



i. first, the switch is closed
ii. then the variable resistor in series with the cell is decreased
iii. then the circuit containing resistor *R* is moved to the left
iv. then the switch is opened.

8 **(a)** A loop of wire is allowed to fall between the poles of a magnet along the dotted line as shown. Determine the direction of the induced current in the loop when it is

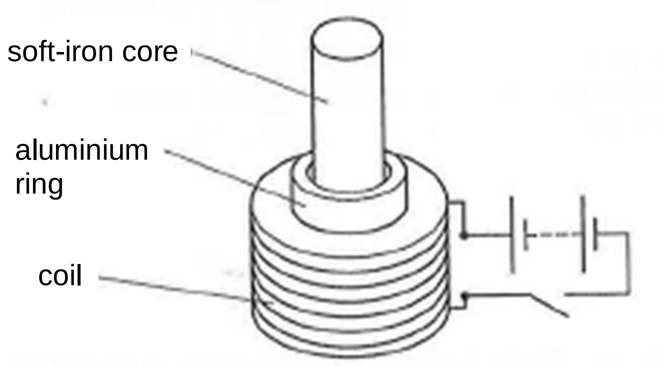
i above the magnet, [2]
ii. below the magnet. [2]



(b) Rather than let the loop fall downwards, suppose we attach a string to it and raise it upwards with constant speed along the path indicated by the dotted line. How does the tension in the string compare with the weight of the loop when the loop is

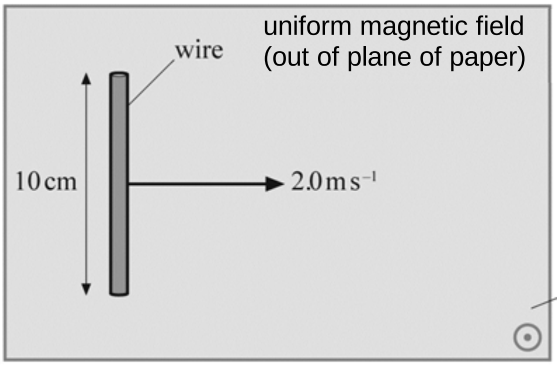
i. below the magnet, [2]
ii. above the magnet. [2]

9 A coil of wire is wound around one end of a soft-iron core as shown.



The axis of the coil is vertical. An aluminium ring is placed over the other end of the soft-iron core. The ring can move freely on the core. The switch is closed and there is a direct current in the coil. Explain why the aluminium ring moves vertically when the switch is closed. [4]

10 The diagram shows a straight wire of length 10 cm moved at a constant speed of 2.0 m s^{-1} in a uniform magnetic field of flux density 0.050 T.



For a period of 1 second, calculate:

(a) the distance travelled by the wire
(b) the area swept by the wire
(c) the change in the magnetic flux for the wire
(d) the e.m.f. induced across the ends of the wire using your answer to (c)
(e) the e.m.f. induced across the ends of the wire using $E = BLv$. [2.0 m, 0.20 m^2, 1.0 × 10^{-2} Wb, 1.0 × 10^{-2} V, 1.0 × 10^{-2} V]

11 A square coil of wire with side 5.00 cm contains 100 loops and is positioned perpendicular to a uniform magnetic field of 0.600 T.

It is quickly pulled from the field at constant speed (moving perpendicular to the magnetic field) to a region of zero magnetic flux density.

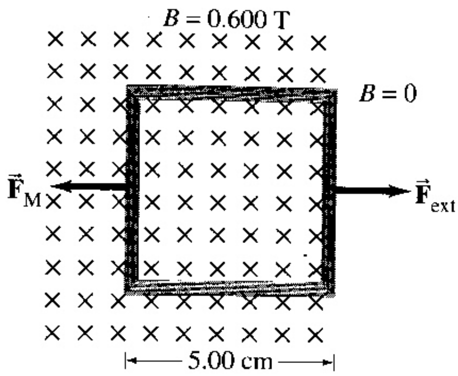
At t = 0, the right edge of the coil is at the edge of the magnetic field. It takes 0.100 s for the whole coil to reach the field-free region.

The coil's total resistance is 100 Ω.

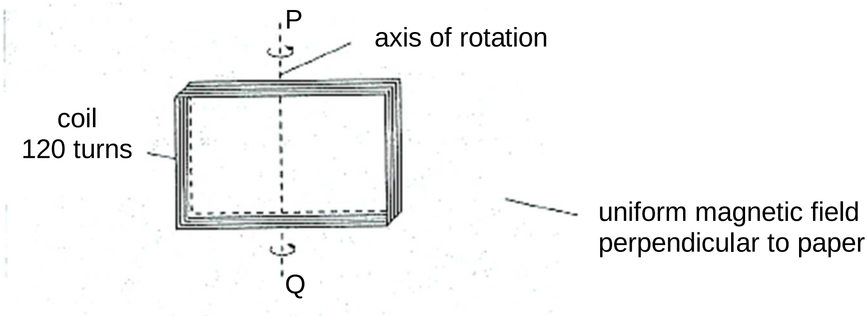
Determine

(a) the change in magnetic flux linkage through the coil
(b) the e.m.f. induced
(c) the current induced
(d) the energy dissipated in the coil
(e) F_{ext} required to pull the coil

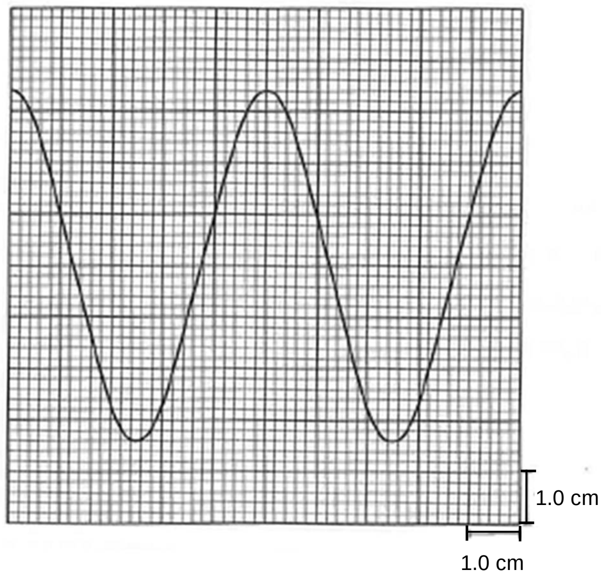
[−0.15 Wb, 1.5 V, 15 mA, 0.00225 J, 0.045N]



12 The coil in a generator is situated in a uniform magnetic field as shown. The coil is rotated about the axis PQ.



An e.m.f. is induced in the coil. The e.m.f. is measured with a cathode-ray oscilloscope (c.r.o.) The waveform displayed on the c.r.o. is shown below.



The Y-plate sensitivity of the c.r.o. is 0.050 V cm^{-1} and the time-base setting is 8.0 ms cm^{-1}.

(a) **(i)** Use Faraday's law of electromagnetic induction to explain the variation of the e.m.f. induced in the coil. [4]

(ii) Determine, for the induced e.m.f.,

1. the maximum value, [1]

2. the frequency. [2]

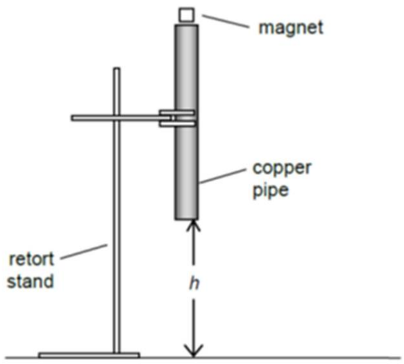
(b) The coil has 120 turns and a cross-sectional area of 1.3×10^{-3} m^2. The maximum induced e.m.f. E_o is given by the expression

E_o = maximum magnetic flux linkage × angular speed of the coil

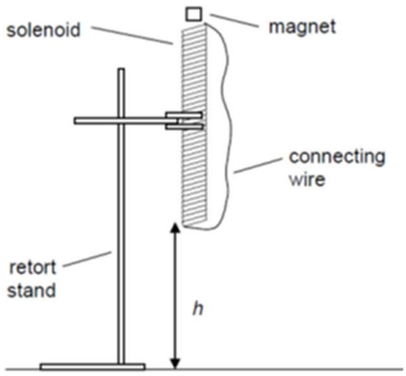
Use this expression and your answers in **(a)(ii)** to calculate the magnetic flux density of the field. [3]

13 **(a)** A magnet is released from rest from the top of a copper pipe as shown below.

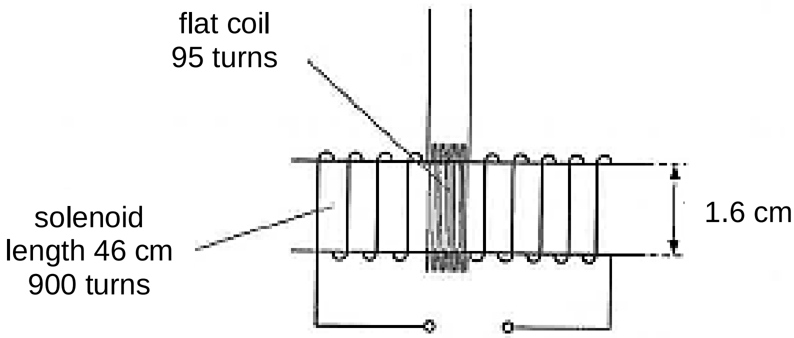
Explain why the time taken for the magnet to fall is considerably longer than when the magnet is released from the same height without the copper pipe. [3]



(b) The copper pipe is then replaced by a solenoid as shown in the Figure. The ends of the solenoids are connected by a wire. Explain why the time taken for the magnet to fall is shorter as compared to the case in **(a)** when the copper pipe was used. [2]



14 A straight solenoid of length 46 cm is wound evenly with 900 turns of insulated wire. The solenoid has a circular cross-section of diameter 1.6 cm. A flat coil having 95 turns of wire is wound tightly around the centre of the solenoid, as shown.

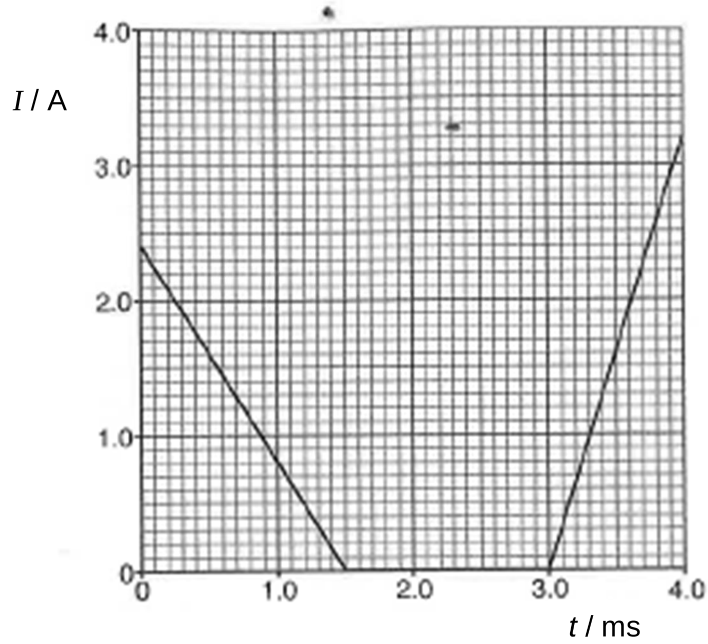


The direct current in the solenoid is 2.4 A.

(a) **(i)** Calculate the magnetic flux density B at the centre of the solenoid. [2]

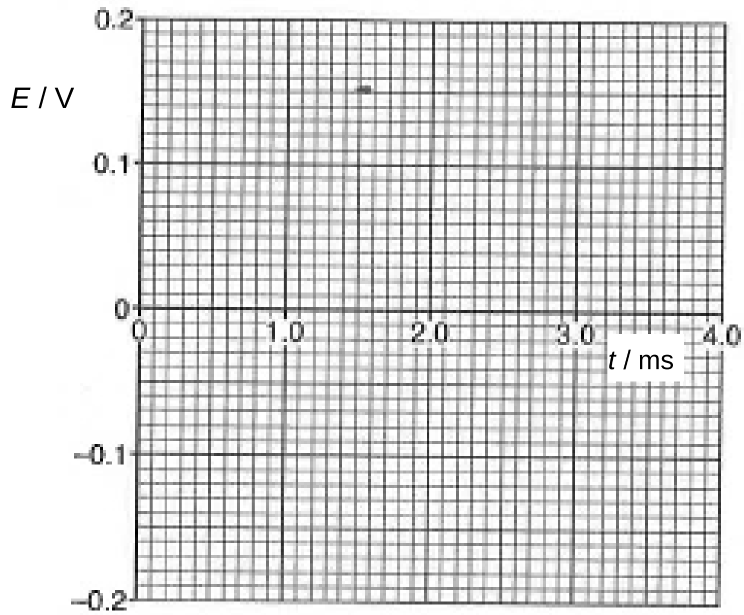
(ii) By reference to the flux pattern of the magnetic field in a solenoid, suggest the difference, if any between the magnitude of the magnetic flux density at the ends of the solenoid compared with that at the centre. [2]

(b) **(i)** The current I in the solenoid is now made to vary with time t as shown.

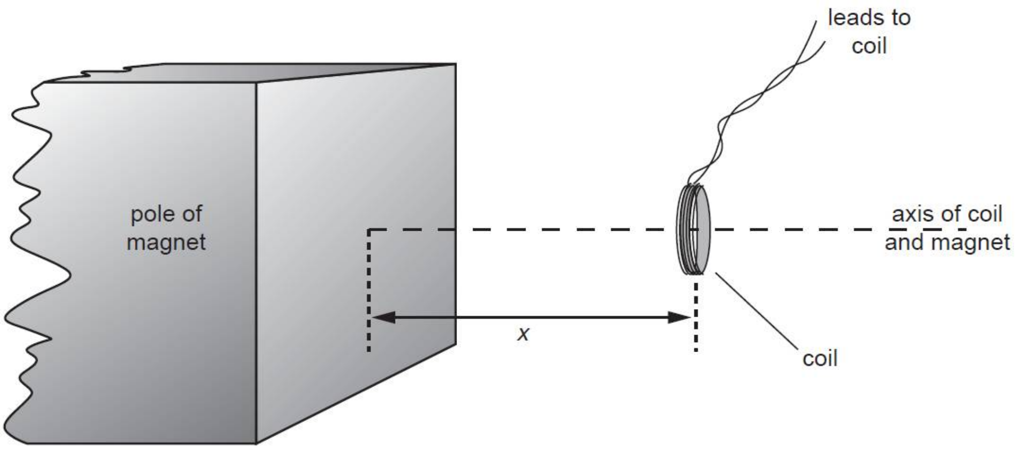


Use your answer in **(a)(i)** to calculate the e.m.f. induced in the flat coil during the time t = 0 to t = 1.5 ms. [2]

(ii) On the figure below, show the variation of the induced e.m.f. E with time t during the time t = 0 to t = 4.0 ms. [3]

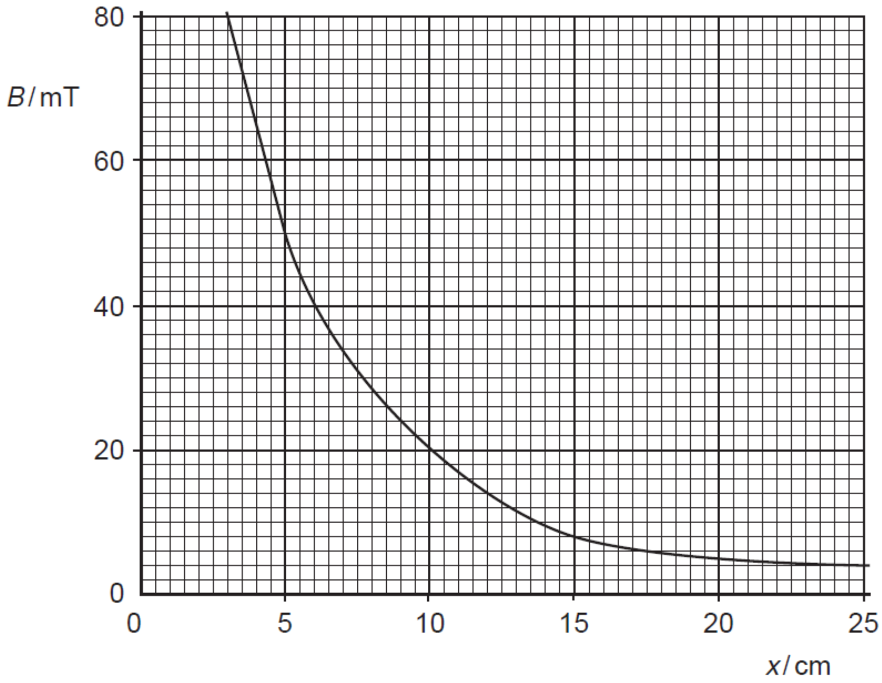


15 A small coil is positioned so that its axis lies along the axis of a large bar magnet, as shown.



The coil has a cross-sectional area of 0.40 cm^2 and contains 150 turns of wire.

The average magnetic flux density *B* through the coil varies with distance *x* between the face of the magnet and the plane of the coil as shown in the figure.



(a) **(i)** The coil is 5.0 cm from the face of the magnet. Use the figure to determine the magnetic flux density in the coil.

(ii) Hence show that the magnetic flux linkage of the coil is 3.0×10^{-4} Wb. [2]

(b) The coil is moved along the axis of the magnet so that the distance *x* changes from *x* = 5.0 cm to *x* = 15.0 cm in a time of 0.30 s. Calculate

(i) the change in flux linkage of the coil, [2.52×10^{-4} Wb] [2]

(ii) the average e.m.f. induced in the coil. [8.4×10^{-4} V] [2]

(c) State and explain the variation, if any, of the speed of the coil so that the induced e.m.f. remains constant during the movement in **(b)**. [3]

- End -