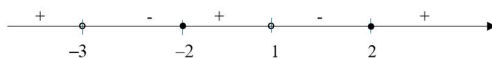


$$\begin{aligned}\text{Q1} \quad & \frac{3x^2 + 4x - 10}{x^2 + 2x - 3} \geq 2 \\ & \frac{3x^2 + 4x - 10 - 2(x^2 + 2x - 3)}{x^2 + 2x - 3} \geq 0 \\ & \frac{x^2 - 4}{x^2 + 2x - 3} \geq 0 \\ & \frac{(x+2)(x-2)}{(x-1)(x+3)} \geq 0\end{aligned}$$



Using a sign test,

$$x < -3 \text{ or } -2 \leq x < 1 \text{ or } x \geq 2$$

$$\begin{aligned}\text{Replace } x \text{ by } |x| : & |x| < -3 \text{ or } -2 \leq |x| < 1 \text{ or } |x| \geq 2 \\ & -1 < x < 1 \text{ or } x \geq 2 \text{ or } x \leq -2\end{aligned}$$

Q2

(a) A vector equation of L is $\mathbf{r} = \begin{pmatrix} 2 \\ b \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}$, where $\lambda \in \mathbb{R}$.

(b) $\begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} = 4$

An equation of p_3 is $\mathbf{r} \cdot \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} = 4$.

(c) Since L lies in p_4 ,

$$L \parallel p_4 \quad \text{and} \quad (2, b, -1) \text{ lies in } p_4.$$

$$\begin{aligned}\begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} &= 0 & \text{and} & \begin{pmatrix} 2 \\ b \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 3 \\ a = -3 & & \text{and} & b = 2\end{aligned}$$

$$\begin{aligned}\text{Q3(a)} \quad \ln(k - 2x) &= \ln \left[k \left(1 - \frac{2x}{k} \right) \right] \\ &= \ln k + \ln \left(1 - \frac{2x}{k} \right) \\ &= \ln k - \frac{2x}{k} - \frac{1}{2} \left(-\frac{2x}{k} \right)^2 + \dots \\ &= \ln k - \frac{2x}{k} - \frac{2x^2}{k^2} + \dots\end{aligned}$$

(b) Expansion in part (a) is valid for $-1 < -\frac{2x}{k} \leq 1$.

$$\Rightarrow \frac{k}{2} > x \geq -\frac{k}{2}$$

$$\Rightarrow -\frac{k}{2} \leq x < \frac{k}{2}$$

(c) $(1+mx)^n = 1 + n(mx) + \frac{n(n-1)}{2}(mx)^2 + \dots$

Comparing the first three terms in both series, we have

$$\ln k = 1,$$

$$nm = -\frac{2}{k}, \quad \text{-----(1)}$$

$$\frac{n(n-1)m^2}{2} = \frac{(nm)^2 - (nm)m}{2} = -\frac{2}{k^2}. \quad \text{-----(2)}$$

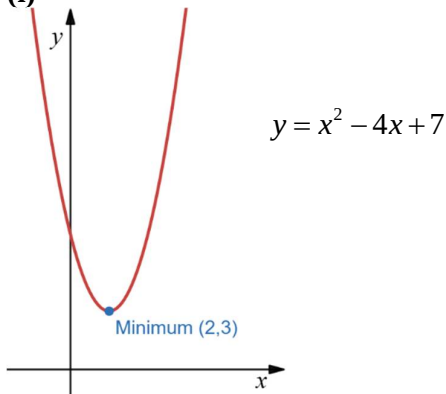
Sub (1) into (2): $\frac{4}{k^2} + \frac{2m}{k} = -\frac{4}{k^2}$

$$\Rightarrow m = -\frac{8}{k^2} \left(\frac{k}{2} \right) = -\frac{4}{k}$$

$$\therefore k = e \text{ and } m = -\frac{4}{e}.$$

Q4

(i)



From graph, greatest value of k is 2

(ii) Method 1

$$y = x^2 - 4x + 7$$

$$y = (x-2)^2 + 3$$

$$(x-2)^2 = y-3$$

$$x = 2 \pm \sqrt{y-3}$$

Method 2

$$y = x^2 - 4x + 7$$

$$x^2 - 4x + (7 - y) = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7-y)}}{2(1)}$$

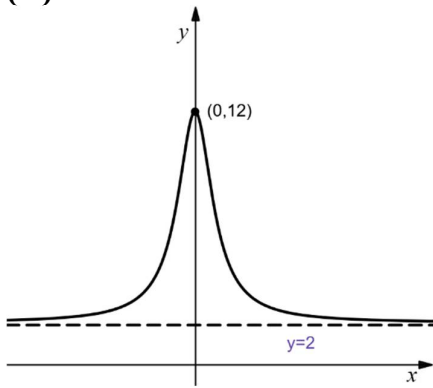
$$= \frac{4 \pm \sqrt{4y-12}}{2}$$

$$= 2 \pm \sqrt{y-3}$$

(Reject positive since $x \leq 2$)

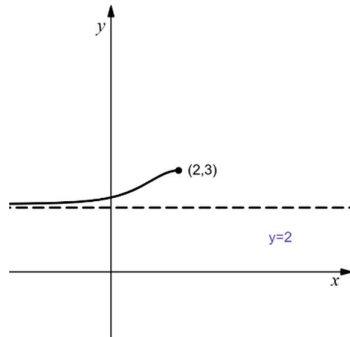
$$f^{-1} : x \mapsto 2 - \sqrt{x-3}, \quad x \in \mathbb{R}, x \geq 3$$

(iii)



(iv) Since $R_f = [3, \infty)$ and $D_g = (-\infty, \infty)$, $R_f \subseteq D_g$. Hence gf exists.

(v) Method 1: To find range of gf using graph of gf: $gf(x) = 2 + \frac{10}{(x^2 - 4x + 7)^2 + 1}, x \leq 2$

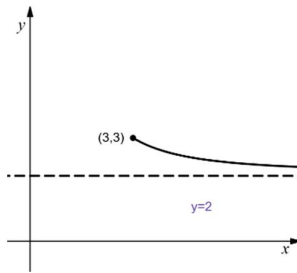


From graph, $R_{gf} = (2, 3]$

Method 2: Mapping:

$$(-\infty, 2] \xrightarrow{f} [3, \infty) \xrightarrow{g} (2, 3]$$

(refer to graph below)



Q5(a)(i)

Month	Amount owed at the start of month	Amount owed at the end of month
1	$(5000)1.025$	$(5000)1.025 - 400$
2	$[(5000)1.025 - 400]1.025 =$ $(5000)1.025^2 - 400(1.025)$	$(5000)1.025^2 - 400(1.025) - 400$
3	$[(5000)1.025^2 - 400(1.025) - 400]1.025$ $=$ $(5000)1.025^3 - 400(1.025)^2 - 400(1.025)$	$(5000)1.025^3 - 400(1.025)^2 - 400(1.025) - 400$
n		$5000(1.025)^n - 400(1.025)^{n-1} - \dots - 400(1.025) - 400$

Amount owed at the end of n months

$$= (5000)1.025^n - 400(1.025)^{n-1} - 400(1.025)^{n-2} - \dots - 400(1.025) - 400$$

$$= (5000)1.025^n - 400[1 + 1.025 + 1.025^2 + \dots + 1.025^{n-1}]$$

$$= (5000)1.025^n - 400 \left[\frac{(1.025^n - 1)}{1.025 - 1} \right]$$

$$= (5000)1.025^n - 16000(1.025^n - 1)$$

$$= 16000 - (11000)1.025^n$$

(a)(ii) To repay his loan $16000 - (11000)1.025^n \leq 0$

Method 1

$$(11000)1.025^n \geq 16000$$

$$n \geq \frac{\ln\left(\frac{16000}{11000}\right)}{\ln 1.025}$$

$$n \geq 15.174$$

Method 2

GC table

n	$16000 - (11000)1.025^n$
15	$68.72 > 0$
16	$-329.6 < 0$
17	$-737.8 < 0$

He would take 16 complete months to repay his loan.

(b) (i) For the first n weeks:

First term = 100, common difference = 10

$$S_n = \frac{n}{2}[2(100) + (n-1)(10)]$$

$$= \frac{n}{2}(190 + 10n)$$

(b)(ii) For the remaining $(20-n)$ weeks:First term = $100 + (n-1)(10) - 10 = 80 + 10n$

Common difference = -10

$$S_{20-n} = \frac{(20-n)}{2}[2(80+10n) + (20-n-1)(-10)]$$

$$= \frac{(20-n)}{2}(30n-30)$$

Since total number of steel rods delivered is 3180,

$$\frac{n}{2}(190+10n) + \frac{(20-n)}{2}(30n-30) = 3180$$

$$95n + 5n^2 + 300n - 300 - 15n^2 + 15n = 3180$$

$$-10n^2 + 410n - 300 = 3180$$

$$10n^2 - 410n + 3480 = 0$$

$$n^2 - 41n + 348 = 0$$

$$(n-12)(n-29) = 0$$

$$n = 12 \text{ or } n = 29 \text{ (reject since } n < 20)$$

Q6**(i)** $P(X = -0.5) = P(\text{BG or RG})$

$$= \frac{4}{10} \times \frac{2}{9} \times 2 + \frac{4}{10} \times \frac{2}{9} \times 2$$

$$= \frac{16}{45} \left(\text{or } \frac{{}^4C_1 \times {}^2C_1 + {}^4C_1 \times {}^2C_1}{{}^{10}C_2} \right)$$

(ii) $P(X = -1) = P(\text{BR or RB})$

$$= \frac{4}{10} \times \frac{4}{9} \times 2 \left(\text{or } \frac{{}^4C_1 \times {}^4C_1}{{}^{10}C_2} \right) = \frac{16}{45}$$

 $P(X = m) = P(\text{BB or RR})$

$$= \frac{4}{10} \times \frac{3}{9} + \frac{4}{10} \times \frac{3}{9} \left(\text{or } \frac{{}^4C_2 + {}^4C_2}{{}^{10}C_2} \right) = \frac{4}{15}$$

$$\begin{aligned}
 P(X = m + 0.5) &= P(GG) \\
 &= \frac{2}{10} \times \frac{1}{9} \left(\text{or } \frac{{}^2C_2}{{}^{10}C_2} \right) = \frac{1}{45}
 \end{aligned}$$

The probability distribution of X is

x	-1	-0.5	m	$m + 0.5$
$P(X = x)$	$\frac{16}{45}$	$\frac{16}{45}$	$\frac{4}{15}$	$\frac{1}{45}$

$$\begin{aligned}
 \text{(iii)} \quad E(X) &= -\left(\frac{16}{45}\right) - \frac{1}{2}\left(\frac{16}{45}\right) + m\left(\frac{4}{15}\right) + \left(m + \frac{1}{2}\right)\left(\frac{1}{45}\right) \\
 &= \frac{13}{45}m - \frac{47}{90}
 \end{aligned}$$

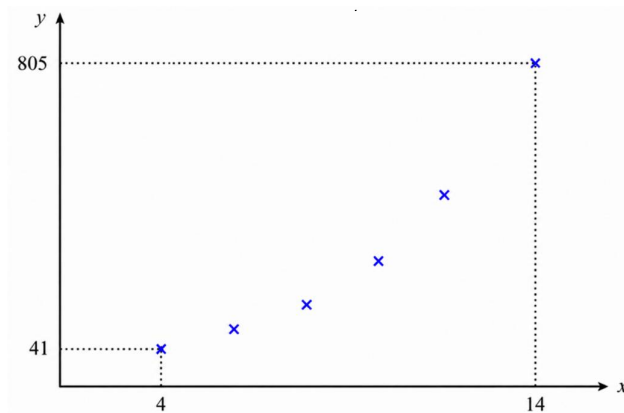
For game to be favourable

$$\frac{13}{45}m - \frac{47}{90} > 0$$

$$m > 1.8077$$

i.e. least $m = 1.81$

Q7(a)



$y = ax^3 + b$ is most appropriate because as x increases, y increases at an increasing rate.

$$\text{(b)} \quad y = 0.28668x^3 + 20.702 \approx 0.287x^3 + 20.7$$

$$r = 0.999905 \text{ (6dp)}$$

(c) $x = 11$ lies **within the data range**, hence it is an **interpolation**

r is close to +1, which indicates **strong positive linear correlation** between y and x^3

(d) When $x = 11$,

$$y = 0.28668(11)^3 + 20.702 \approx 402$$

The original turbine would have generated 402 kW

Since $450 > 402$, the 2nd turbine appears to perform better

Q8

(a)(i) Number of teams = ${}^5C_2 \times {}^4C_2 \times {}^3C_1 = 180$

(a)(ii)

Case 1: 1 goalkeeper, 2 attackers, 2 defenders

$${}^3C_1 \times {}^5C_2 \times {}^4C_2 = 180$$

Case 2: 1 goalkeeper, 1 attacker, 3 defenders

$${}^3C_1 \times {}^5C_1 \times {}^4C_3 = 60$$

Case 3: 1 goalkeeper, 0 attacker, 4 defenders

$${}^3C_1 \times {}^5C_0 \times {}^4C_4 = 3$$

Total number of ways = ${}^{12}C_5 = 792$

Hence probability that team has at most 2 attackers = $\frac{180 + 60 + 3}{792}$
 $= \frac{243}{792}$ or 0.307

(b) Req probability

$$= P(AAAD) + P(AADA) + P(ADAA) + P(DAAA)$$

$$= \left(\frac{5}{9}\right)^3 \left(\frac{4}{9}\right) \times 4 = \frac{2000}{6561} \text{ or } 0.305$$

(c) Method 1:

Req probability

$$= P(AAA_) + P(_AAA) - P(AAAA)$$

$$= \left(\frac{5}{9}\right)^3 + \left(\frac{5}{9}\right)^3 - \left(\frac{5}{9}\right)^4$$

$$\frac{1625}{6561} \text{ or } 0.248$$

Method 2:

Req prob

$$= P(AAAD) + P(DAAA) + P(AAAA)$$

$$= \left(\frac{5}{9}\right)^3 \left(\frac{4}{9}\right) + \left(\frac{4}{9}\right) \left(\frac{5}{9}\right)^3 + \left(\frac{5}{9}\right)^4$$

$$\frac{1625}{6561} \text{ or } 0.248$$

Q9(a)(i) Let X and Y be the masses of oranges from Farm A and Farm B respectively.

$$X \sim N(245, 8^2) \quad Y \sim N(212, 7^2).$$

Then $X - Y \sim N(33, 113)$

$$P(|X - Y| > 30) = P(X - Y > 30) + P(X - Y < -30)$$

$$= 0.61111 \approx 0.611$$

- (ii) $W = X_1 + X_2 + X_3 + Y_1 + Y_2$
 $W \sim N(3(245) + 2(212), 3(64) + 2(49))$
 $W \sim N(1159, 290)$
 $P(W > 1180) = 0.10876 \approx 0.109$

(iii) The mass of **every** orange is independent of one another.

- (b) (i) $X \sim N(\mu, \sigma^2)$

$$P(X < m) = 0.25 \Rightarrow P\left(Z < \frac{m - \mu}{\sigma}\right) = 0.25$$

$$\begin{aligned} P(X > 2\mu - m) &= P\left(Z > \frac{2\mu - m - \mu}{\sigma}\right) \\ &= P\left(Z > \frac{\mu - m}{\sigma}\right) \\ &= 0.25 \end{aligned}$$

- (ii) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$$P(\bar{X} \geq \mu + 0.01\sigma) > 0.4$$

$$P\left(Z \geq \frac{\mu + 0.01\sigma - \mu}{\frac{\sigma}{\sqrt{n}}}\right) > 0.4$$

$$P(Z \geq 0.01\sqrt{n}) > 0.4$$

$$P(Z < 0.01\sqrt{n}) < 0.6$$

$$0.01\sqrt{n} < 0.25335$$

$$n < 641.85$$

$$\text{greatest } n = 641$$

Q10(a) $\bar{x} = \frac{\sum (x - 150)}{30} + 150 = 153.49$ (exact)

$$s^2 = \frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]$$

$$= \frac{1}{29} \left(3367.5 - \frac{104.7^2}{30} \right) = 103.52 \approx 104$$

- (b) Let X = mass of meat in a serving prepared by Josh.
 μ is the **population mean mass** of meat in a serving prepared by Josh.

$$H_0 : \mu = 150$$

$$H_1 : \mu > 150$$

Under H_0 , since $n = 30$ is **large**, by **Central Limit Theorem**, $\bar{X} \sim N(150, \frac{103.52}{30})$ approximately.

$$z = \frac{\bar{x} - \mu}{\sqrt{\frac{103.52}{30}}} \approx 1.88$$

$$z = 1.88$$

$$p\text{-value} = 0.0301$$

Level of significance = 5%

Since $p\text{-value} < \alpha$, we **reject H_0** . There is **sufficient evidence**, at the 5% level, to indicate that the population mean mass of meat is more than 150 g. **Hence manager's suspicion is justified.**

- (c) Let Y = mass of meat in a serving dispensed by machine
 μ is the **population mean mass** of meat in a serving dispensed by machine

$$H_0 : \mu = 150$$

$$H_1 : \mu \neq 150$$

Under H_0 , $Y \sim N(150, 100)$

$$\bar{Y} \sim N(150, \frac{100}{30})$$

$$\text{Test statistics } \frac{\bar{y} - 150}{\sqrt{\frac{100}{30}}}$$

Level of Significance: 0.05

Critical Region: $z < -1.95996$ or $z > 1.95996$

$$\frac{\bar{y} - 150}{\sqrt{\frac{100}{30}}} < -1.95996 \quad \text{or} \quad \frac{\bar{y} - 150}{\sqrt{\frac{100}{30}}} > 1.95996$$

$$\bar{y} < 146.42 \quad \text{or} \quad \bar{y} > 153.58$$

- (d) Since the mass of meat dispensed by the machine is **normally distributed**, a sample of fewer than 30 servings could still be used.
- (e) If the sample is not chosen randomly, it may be **biased /not representative** of all servings dispensed by the machine.

Q11(i) The event that a senior relationship manager has a successful interaction is independent of other interactions.

The probability that a senior relationship manager has a successful interaction is constant (at 0.24).

- (ii) Let X be the no. of successful interactions (out of 6) for Sky

$$X \sim B(6, 0.24)$$

$$P(X < 2) = P(X \leq 1)$$

$$= 0.55782 \approx 0.558$$

- (iii) Let Y be the no. of days (out of 20) that Sky secures at least 2 successful interactions.
 $Y \sim B(20, 0.44218)$
 $P(Y > 5) = 1 - P(Y \leq 5)$
 $= 0.93644 \approx 0.936$
- (iv) Let W be the no. of successful interactions (out of 9) for Jules. Then $W \sim B(9, p)$
 Given mode of W is 2,

$$\begin{aligned}
 P(W=2) > P(W=1) & \quad \text{and} \quad P(W=2) > P(W=3) \\
 \binom{9}{2} p^2 (1-p)^7 > \binom{9}{1} p (1-p)^8 & \quad \binom{9}{2} p^2 (1-p)^7 > \binom{9}{3} p^3 (1-p)^6 \\
 36p > 9(1-p) & \quad \text{and} \quad 36(1-p) > 84p \\
 p > \frac{1}{5} & \quad p < \frac{3}{10} \\
 \therefore \frac{1}{5} < p < \frac{3}{10}
 \end{aligned}$$

- (v) $W \sim B(9, 0.2)$
 Required probability
 $= (0.2)^2 \times (0.8)^3 \times \frac{5!}{2!3!} \times (0.2)$
 $= 0.04096 \approx 0.0410$

Alternative Method

Let V be the the number of successful interactions (out of 5) for Jules.

$$V \sim B(5, 0.2)$$

$$\begin{aligned}
 \text{Required probability} &= P(V=2) \times 0.2 \\
 &= 0.04096 \approx 0.0410
 \end{aligned}$$

- (vi) $X \sim B(6, 0.24)$, $W \sim B(9, 0.2)$
 $E(X) = 6(0.24) = 1.44$ $E(W) = 9(0.2) = 1.8$
 $\text{Var}(X) = 6(0.24)(0.76) = 1.0944$ $\text{Var}(W) = 9(0.2)(0.8) = 1.44$
 Since sample size is large, by Central Limit Theorem,

$$\bar{X} \sim N\left(1.44, \frac{1.0944}{50}\right) \text{ approximately and } \bar{W} \sim N\left(1.8, \frac{1.44}{50}\right) \text{ approximately.}$$

$$\bar{X} - \bar{W} \sim N\left(-0.36, \frac{2.5344}{50}\right) \text{ approximately}$$

$$P(\bar{X} > \bar{W}) = P(\bar{X} - \bar{W} > 0) = 0.054910 \approx 0.0549$$