RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 7B: Arithmetic Progression and Geometric Progression

Section A (Basic Questions)

1 In the sequence 1.0, 1.1, 1.2, … , 99.9, 100.0, each number after the first is 0.1 greater than the preceding number. Find

(a) how many numbers there are in the sequence,

(b) the sum of all the numbers in the sequence.

[**(a)** 991 **(b)** 50045.5]

2 The first term of a geometric progression is 10 and its sum to infinity is 15. Find

(a) the third term of the progression.

(b) the sum of the first 5 terms of the progression.

[**(a)** $\frac{10}{9}$ **(b)** $\frac{1210}{81}$]

3 The ninth term of an arithmetic progression is 43 and the sum of the first 15 terms is 570. It is given that the sum of the first n terms is greater than 2265. Find the least possible value of n.

[31]

Section B (Discussion Questions)

1 **(a)** The first four terms u_1, u_2, u_3, u_4 of an arithmetic progression are such that

$$u_4 - u_2 = 15 \quad \text{and} \quad 4u_3 = 9u_1.$$

Find the value of u_1.

(b) The sum to infinity of a geometric series is 5. By taking every third term after the first term, another series is formed. Given that the sum to infinity of this series is 4, find the common ratio of the first series in exact form.

(c) The first term of a geometric progression is 10 and its sum to infinity is 15. Find the third term of the progression.

[(a) $u_1 = 12$ **(b)** $r = \frac{-1+\sqrt{2}}{2}$ **(c)** $u_3 = \frac{10}{9}$**]**

2 The sum of the first 16 terms of an increasing arithmetic progression is 322. The first, fifth and thirteenth terms of this progression are three consecutive terms of a geometric progression. Find the first term and the common difference of the arithmetic progression.

[7 ; $\frac{7}{4}$]

3 You drop a ball from 1 metre above a flat surface. Each time the ball hits the surface after falling a distance h, it rebounds a distance rh, where r is positive but less than 1. Find the total vertical distance the ball travels up and down.

[$\dfrac{1+r}{1-r}$]

4 The sum of the first n terms of a series is given by $S_n = 3 - \dfrac{1}{3^{n-1}}$.

(a) Show that the series is geometric.

(b) Explain why the sum to infinity exists and write down this value.

[(b) 3**]**

5 The even positive integers, starting at 2, are grouped into sets containing 1, 3, 5, 7, ... integers, as indicated below, so that the number of integers in each set is two more than the number of integers in the previous set.

$$\{2\},\ \{4,6,8\},\ \{10,12,14,16,18\},\ \{20,22,24,26,28,30,32\},\ \ldots$$

Find, in terms of r, an expression for

(i) the number of integers in the r^{th} set, [1]

(ii) the last integer in the r^{th} set. [2]

Given that the n^{th} set contains the integer 2016, find n. [2]

[(i) $2r-1$ **(ii)** $2r^2$ **;** 32 **]**

6 A gardener is cutting off pieces of string from a long roll of string. The first piece he cuts off is 128cm long and each successive piece is $\dfrac{2}{3}$ as long as the preceding piece.

(i) The length of the nth piece of string cut off is p cm. Show that

$$\ln p = (An + B)\ln 2 + (Cn + D)\ln 3,$$

for constants A, B, C and D to be determined.

(ii) Show that the total length of string cut off can never be greater than 384 cm.

(iii) How many pieces must be cut off before the total length cut off is greater than 380 cm? You must show sufficient working to justify your answer.

[(i) $A = 1, B = 6, C = -1, D = 1$ **(iii)** 12]

7 **(a)** Mr Lam saves \$120 on 1 January 2019. On the first day of each subsequent month, he saves \$10 more than in the previous month, so he saves \$130 on 1 February 2019, \$140 on 1 March 2019, and so on. On what date will he have first saved over \$10 000 in total? [4]

(b) Mr Lui has opened a special investment account that guarantees an interest rate of 8% per annum, starting from the first year. He deposits \$50 000 into the account at the start of the first year and a fixed amount, \$ x, at the beginning of each year from the second year onwards. The interest earned is based on the total amount in the account at the end of that particular year and it is deposited into the account.

For example,
In Year One, at the beginning of the year there are 50000 dollars in the account, and at the end of that year there are (1.08)(50000) dollars.
In Year Two, at the beginning of the year there are $\left[(1.08)(50000)+x\right]$ dollars in the account, and at the end of that year there are $(1.08)\left[(1.08)(50000)+x\right]$ dollars, which simplifies to $\left[(1.08)^2(50000)+(1.08)x\right]$ dollars.

(i) Show that the total amount of money, in dollars, in the investment account at the end of the nth year is

$$50000(1.08)^n+13.5x\left(1.08^{n-1}-1\right).$$ [3]

(ii) If the yearly fixed amount that Mr Lui puts into the account from the second year onwards is \$5 000, find the total interest that he would earn at the end of the 10th year, correct to the nearest dollar. [2]

Mr Lui is interested in opening another investment account that is similar to the special investment account but with an interest rate of r % per annum. If his deposit for the first year is still \$50 000, and the yearly fixed deposit thereafter is \$5 000, find the value of r that would enable Mr Lui to have \$300 000 in the account after 10 years. [3]

[(a) 1 Nov 2021 **(b)(ii)** \$80379 **(b)(iii)** 15.1%**]**

8 An arithmetic progression has first term a and common difference d, and a geometric progression has first term b and common ratio r, where a, b and d are non-zero. The second, fourth and sixth term of the arithmetic progression are equal to the first, fourth and eighth term of the geometric progression respectively.

(i) Show that $r^7 - 2r^3 + 1 = 0$. [2]

It is given that the geometric progression has positive terms.

(ii) Find the value of r and justify that this is the only answer. Deduce whether the geometric progression is convergent. [3]

(iii) Another arithmetic progression has first term k and common difference $3k$, where $k > 0$. The difference between the sum of the first $2n$ terms of this arithmetic progression and the nth term of the geometric progression with the common ratio found in part **(ii)** is at most $1000k$. Given that $b = 4k$, write down an inequality satisfied by n, and hence find the largest possible value of n. [5]

(iv) Let u_n denote the nth term of the geometric progression. Show that a new sequence with nth term $\ln\left(\frac{1}{u_n}\right)$ is an arithmetic progression. [2]

[(ii) 0.921 **(iii)** 13**]**

9 The coach of Besto running club designed a training programme such that runners begin with a 400 m run on the first training session. On each subsequent session, the distance covered is 250 m more than the distance covered on the previous session.

(i) Find the minimum number of sessions required for runners from Besto on the training programme to run at least 20 km in a training session. [3]

For another group of runners in Besto, a circuit training exercise was designed to build up their stamina.

In this exercise, this group of runners from Besto run from a starting point O to and from a series of points, $P_1, P_2, P_3, \cdots$, increasingly far away in a straight line. In the exercise, they start at O and run stage 1 from O to P_1, and back to O, then stage 2 from O to P_2, and back to O, and so on.

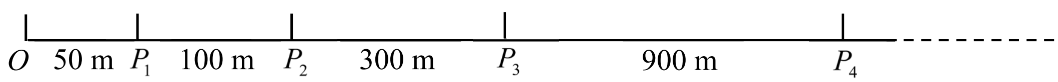

Fig. 1

The distances between the points are such that $OP_1 = 50$ m, $P_1P_2 = 100$ m, $P_2P_3 = 300$ m and $P_nP_{n+1} = 3P_{n-1}P_n$ (see Fig. 1).

(ii) Find an expression for the distance run by a runner from Besto who completes n stages of the circuit training exercise. [3]

(iii) Hence, find the distance from O and the direction of travel, of a runner from Besto undergoing the circuit training exercise after he has run exactly 42 km. [3]

Another running club, Choco, designed a different training programme. The runners in Choco began with running 400 m on the 1st session. On each subsequent session, the distance covered was increased by 10% of the distance covered on the previous session. From the 11th session onwards and for all subsequent sessions, the distance covered was increased by r% of the distance covered on the previous session.

(iv) Given that the runners from Choco club covered at least 20 km on the 70th training session, find the range of values of r. [4]

[(i) 80 **(ii)** $50\left(3^n - 1\right)$ **(iii)** 5600m (iv) $r \geq 5.22$ **]**

THE END