

SUMMARY: Permutations and Combinations

EXAMPLE 1

Find the number of ways to seat the 10 students in a class of 6 boys and 4 girls

	<i>Solution</i>
(a) in a single row if (i) there is no restriction (ii) the girls sit together (iii) the girls are seated alternately (iv) all the girls are separated (v) one of the girls sits between John and Andy	(i) $10!$ (ii) $7! 4!$ (iii) $6! 4! 4$ or ${}^6C_3 3! 4! 4!$ (iv) ${}^7C_4 4! 6!$ (v) ${}^4C_1 8! 2!$
(b) at a round table if (i) there is no restriction (ii) the girls sit together (iii) the girls are seated alternately (iv) all the girls are separated (v) one of the girls sits between John and Andy	(i) $(10-1)!$ (ii) $(7-1)! 4!$ (iii) ${}^6C_3 3! (4-1)! 4!$ (iv) $(6-1)! {}^6C_4 4!$ (v) ${}^4C_1 (8-1)! 2!$
(c) at a round table where the seats are numbered if (i) there is no restriction (ii) the girls sit together (iii) the girls are seated alternately (iv) all the girls are separated (v) one of the girls sits between John and Andy	(i) $(10-1)! \times 10$ (ii) $(7-1)! 4! \times 10$ (iii) ${}^6C_3 3! 4! (4-1)! \times 10$ (iv) $(6-1)! {}^6C_4 4! \times 10$ (v) ${}^4C_1 (8-1)! 2! \times 10$
(d) in 2 rows of 5 each, labelled row 1 and row 2, if (i) there is no restriction (ii) the girls sit together (iii) both the girls and boys are seated alternately	(i) $10!$ (ii) ${}^6C_1 2! 4! 5! \times 2$ (iii) $4! 6!$

Refer to **pg 3** for detailed explanation.

EXAMPLE 2

Find the number of different arrangements of the word SLEEPLESSNESS,

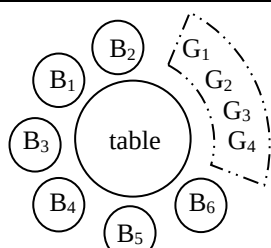
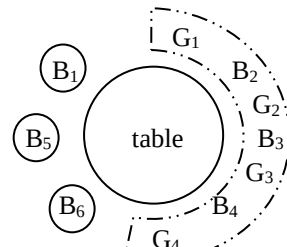
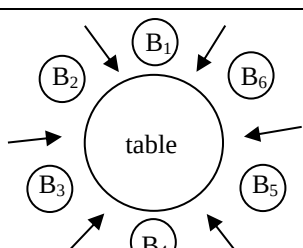
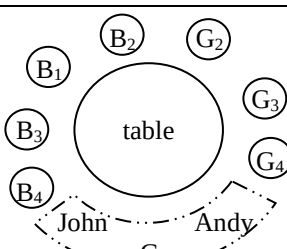
	<i>Solution</i>
(a) using all thirteen letters if (i) there is no restriction (ii) all “S” are adjacent (iii) all “S” are not next to each other (iv) both the first and last letters are “S”	(i) $\frac{13!}{5!4!2!}$ (ii) $\frac{9!}{2!4!}$ (iii) ${}^9C_5 \cdot \frac{8!}{2!4!}$ (iv) $\frac{11!}{4!3!2!}$
(b) forming 4-letter code words if (i) there is no restriction (ii) both the first and last letters are “S”	(i) ${}^5C_4 4!$ (All distinct) $+ {}^3C_1 {}^4C_2 \frac{4!}{2!}$ (1 pair same) $+ {}^3C_2 \frac{4!}{2! 2!}$ (2 pairs same) $+ {}^2C_1 {}^4C_1 \frac{4!}{3!}$ (3 letters same) $+ {}^2C_1$ (All same) (ii) ${}^5C_2 2! + {}^3C_1$

Refer to **pg 6** for detailed explanation.

EXAMPLE 1

Find the number of ways to seat the 10 students in a class of 6 boys and 4 girls

	Thinking Process	Solution
(a) in a SINGLE ROW if		
(i) no restriction	The 10 of them can re-arrange among themselves, thus gives $10!$	$10!$
(ii) the girls sit together	1) Group the girls together in a bubble. 2) The 7 bubbles can re-arrange in different positions, thus $7!$. 3) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, thus gives $4!$.	$7! 4!$
(iii) the girls are seated alternately	Method 1 (List out the possible ways) 1) $B_1 B_2 B_3 B_4 B_5 B_6$ re-arrange among themselves, gives $6!$. 2) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives $4!$. 3) There are 4 ways for the above.	$6! 4! 4$
	Method 2 1) Of the 6 boys, select any 3 boys to be alternating with the girls, which gives 6C_3. The 3 boys selected can re-arrange among themselves, which gives $3!$. 2) Ensure that the girls are arranged alternately. 3) Group them as a bubble. 4) The 4 bubbles can re-arrange in different positions, thus $4!$. 5) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives $4!$.	${}^6C_3 3! 4! 4!$
(iv) all the girls are separated.	1) List the remaining group (i.e the boys) with spacing in between. The arrows indicate the space to slot the girls. Note the space for first and last position. 2) Of the 7 arrows, select any 4 arrows to make space for the girls, thus 7C_4. This ensures that the girls are separated. 3) $G_1 G_2 G_3 G_4$ re-arrange among themselves, gives $4!$. 4) $B_1 B_2 B_3 B_4 B_5 B_6$ re-arrange among themselves, gives $6!$	${}^7C_4 4! 6!$
(v) one of the girls sits between John and Andy	1) Of the 4 girls, select a girl to sit between John and Andy, gives 4C_1. Group them together. 2) The 8 bubbles can re-arrange in different positions, thus $8!$. 3) John and Andy can swap positions.	${}^4C_1 8! 2!$

(b) at a ROUND TABLE if		
(i) no restriction	The 10 of them can re-arrange among themselves, thus gives $(10-1)!$	$9!$
(ii) the girls sit together	 1) Group the girls together in a bubble. 2) The 7 bubbles can re-arrange in different positions, thus $(7-1)!$. 3) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, thus gives $4!$.	$(7-1)! 4!$
(iii) the girls are seated alternately	 1) Of the 6 boys, select any 3 boys to be alternating with the girls, which gives 6C_3. The 3 boys selected can re-arrange among themselves, which gives $3!$. 2) Ensure that the girls are arranged alternately. 3) Group them as a bubble. 4) The 4 bubbles can re-arrange in different positions, thus $(4-1)!$. 5) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives $4!$.	${}^6C_3 3!(4-1)!4!$
(iv) all the girls are separated.	 1) List the remaining group (i.e the boys) with spacing in between. The arrows indicate the space to slot the girls. Note the space for first and last position. 2) $B_1 B_2 B_3 B_4 B_5 B_6$ can re-arrange among themselves, gives $(6-1)!$. 3) Of the 6 arrows, select any 4 arrows to make space for the girls, thus 6C_4. This ensures that the girls are separated. 4) $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives $4!$.	$(6-1)! {}^6C_4 4!$
(v) one of the girls sits between John and Andy	 1) Select a girl to sit between John and Andy, gives 4C_1 2) The 8 bubbles can re-arrange in different positions, thus $(8-1)!$. 3) John and Andy can swop positions.	${}^4C_1 (8-1)! 2!$

(c) at a ROUND TABLE where the seats are NUMBERED if		
(i) no restriction	The considerations for all questions are same as in (b), but because the seats are numbered, multiply the answers in (b) by the number of seats. This is because for every way of arranging, there are in fact 10 different ways of having the same orientation.	$(10-1)! \times 10$
(ii) the girls sit together		$(7-1)! \cdot 4! \times 10$
(iii) the girls are seated alternately		${}^6C_3 \cdot 3! \cdot 4! (4-1)! \times 10$
(iv) all the girls are separated .		$(6-1)! \cdot {}^6C_4 \cdot 4! \times 10$
(v) one of the girls sits between John and Andy		${}^4C_1 \cdot (8-1)! \cdot 2! \times 10$

(d) in 2 rows of 5 each, labelled row 1 and row 2, if		
(i) no restriction	The 10 of them can re-arrange among themselves, thus gives 10!	10!
(ii) the girls sit together	Row 1: $\textcircled{B_4} \textcircled{G_1 G_2 G_3 G_4}$ Row 2: $\textcircled{B_5} \textcircled{B_6} \textcircled{B_1} \textcircled{B_2} \textcircled{B_3}$ - Group the girls together in a bubble. Of the 6 boys, select 1 boy to be in the same row as the girls, thus 6C_1 - The 2 bubbles in Row 1 can re-arrange in different positions, thus 2!. $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives 4!. - The 5 bubbles in Row 2 can re-arrange in different positions, thus 5!. - Row 1 and 2 can swop, hence 2!.	${}^6C_1 \cdot 2! \cdot 4! \cdot 5! \times 2$
(iii) both the girls and boys are seated alternately	Row 1: $B_1 G_1 B_2 G_2 B_3$ Row 2: $B_4 G_3 B_5 G_4 B_6$ There's only one case to consider, which is 3 boys and 2 girls in each row. $G_1 G_2 G_3 G_4$ can re-arrange among themselves, gives 4!. $B_1 B_2 B_3 B_4 B_5 B_6$ can re-arrange among themselves, gives 6!.	4! 6!

EXAMPLE 2

Find the number of different arrangements of the word SLEEPLESSNESS,

	Thinking process...	Solution
(a) using all thirteen letters if		
(i) there is no restriction	S S S S S E E E E L L P N 1) can re-arrange among themselves, but with repetition of 5 “S”s, 4 “E”s and 2 “L”s, we have $\frac{13!}{5!4!2!}$.	$\frac{13!}{5!4!2!}$
(ii) all “S” are adjacent	(L)(L)(P)(N)(S S S S S)(E)(E)(E)(E) 1) Group the “S”s together in a bubble. 2) The 9 bubbles can re-arrange in different positions, but with repetition of 4 “E”s and 2 “L”s, we have $\frac{9!}{2!4!}$.	$\frac{9!}{2!4!}$
(iii) all “S” are not next to each other	$\uparrow$ L $\uparrow$ L $\uparrow$ E $\uparrow$ E $\uparrow$ E $\uparrow$ E $\uparrow$ P $\uparrow$ N $\uparrow$ 1) List the remaining group (i.e non “S”s) with spacing in between. The arrows indicate the space to slot the “S”s. Note the space for first and last position. 2) Of the 9 arrows, select any 5 arrows to make space for the “S”s, thus 9C_5. This ensures that the “S”s are separated. 3) The “S”s are all the same, thus it’s not necessary for re-arranging. 4) L L E E E P N can re-arrange among themselves, but with repetition of 4 “E”s and 2 “L”s, we have $\frac{8!}{2!4!}$.	${}^9C_5 \times \frac{8!}{2!4!}$
(iv) both the first and last letters are “S”	S (S)(S)(S)(L)(L)(E)(E)(E)(E)(P)(N) S 1) Fix the two “S”s, then the remaining 11 bubbles can re-arrange in different positions, with repetition of 3 “S”, 2 “L”s and 4 “E”s, thus $\frac{11!}{3!2!4!}$.	$\frac{11!}{3!2!4!}$

(b) forming 4-letter code words if		
(i) there is no restriction	CASE 1: (All letters are distinct. Eg SELN) (S S S S S)(E E E E)(L L)(P)(N) 1) Group the repeated letters together in a bubble. Treat the repeated letters in each bubble as 1 letter. Of these 5 bubbles, select any 4 bubbles, which gives 5C_4 (i.e. 4 different letters are selected). 2) These 4 bubbles can re-arrange in different positions, thus 4!.	${}^5C_4 4!$ (All distinct)

	CASE 2: (1 repeated letter and 2 other distinct letters. Eg EESN)  - Only S, E or L can be repeated. Of these 3 letters, select 1 of these letter to repeat, thus 3C_1. - Suppose “E” is selected to repeat, we are left with “S”, “L”, “P” and “N” to select as 2 other distinct letters. Regardless of whether “S”, “E” or “L” is selected to repeat, we are always left with 4 other letters to select as 2 distinct letters, thus 4C_2. - With the 4 letters selected in steps 1 and 2, there are 2 repeated letters, thus $\frac{4!}{2!}$. CASE 3: (2 pairs of repeated letters. Eg SSEE)  - Only “S”, “E” or “L” can be repeated. Of these 3 letters, select 2 of these letters to repeat, thus 3C_2. - With the 4 letters selected in step 1, there are 2 pairs of repeated letters, thus $\frac{4!}{2!2!}$. CASE 4: (1 letter repeated thrice. Eg SSSE)  - Only “S” or “E” can be repeated thrice. Of these 2 letters, select 1 of these letters to repeat, thus 2C_1. - Suppose “E” is selected to repeat thrice, we are left with “S”, “L”, “P” and “N” to select as the other distinct letter. Regardless of whether “S” or “E” is selected to repeat, we are always left with 4 other letters to select as a distinct letter, thus 4C_1. - With the 4 letters selected in steps 1 and 2, there are 3 repeated letters, thus $\frac{4!}{3!}$. CASE 5: (All letters are the same. Eg SSSS)  Only “S” or “E” can be repeated four times. Of these 2 letters, select 1 of these letters to repeat, thus 2C_1.	+ ${}^3C_1 {}^4C_2 \frac{4!}{2!}$ (1 pair same) + ${}^3C_2 \frac{4!}{2!2!}$ (2 pairs same) + ${}^2C_1 {}^4C_1 \frac{4!}{3!}$ (3 letters same) + 2C_1 (All same)
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