Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4051/01**
Paper 1

6 August 2025 Wednesday 1 hour 45 minutes

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2025 SECONDARY FOUR NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
70

Setter: Mrs Yvonne Ng
Vetter: Miss Li Feiqing

This question paper consists of **15** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer all questions.

1 Solve the simultaneous equations.

$$x - 2y = 6$$

$$2x^2 + 3xy - 3y = 9 \qquad [5]$$

2 Express, in terms of π, the principal value of $\cos^{-1}\left(-\sin\frac{\pi}{4}\right)$. [2]

3 (a) Find the derivative of $\left(3x^2-4\right)^5$. [2]

(b) (i) Factorise x^3+1. [1]

(ii) Hence find $\int \frac{x^3+1}{x+1}\,dx$. [2]

4 Given that $x-1=\sqrt{2}(2-x)$, show that $x=a+b\sqrt{2}$, where a and b are integers. [5]

5 The diagonals of a rhombus are of lengths $\left(4\sqrt{3}+3\sqrt{2}\right)$ cm and $\left(4\sqrt{3}-3\sqrt{2}\right)$ cm respectively.

Find, the perimeter of the rhombus, giving your answers in surd form where necessary. [4]

6 Express $\dfrac{8x^2-22x+17}{(3x-1)(x^2+2)}$ in partial fractions. [5]

7 Given that A is an acute angle and that $\sec A = \frac{13}{12}$, find the exact value of $\tan 2A$. [4]

8 The function f is defined by $f(x)=2x^2-6x+9$.

(a) Express $2x^2-6x+9$ in the form $p(x-q)^2+r$, where p, q and r are constants. [2]

(b) Explain why the lowest point on the curve $y=f(x)$ has coordinates $(1.5,\ 4.5)$. [2]

(c) State the range of values of k for which $f(x)=k$ has no solutions. [1]

9 (a) Find the range of values of k for which the curve $y = x^2 - 3x + 2$ meets the line $y - kx = 1$ at two distinct points. [4]

(b) Explain why $ax^2 - 7x - 3$ cannot be always positive. [3]

Do not use a calculator in this question.

10 **(a)** Find the exact value of $\cos 85° \cos 40° + \sin 85° \sin 40°$. [2]

(b) Show that $\sin \frac{5\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$. [3]

11 A curve is such that $\frac{dy}{dx} = 12(a + bx)^2$, where a and b are integers.

(a) In the case where the curve has a stationary value at (3, 2),

(i) find an equation connecting a and b, [2]

(ii) find a possible equation of the curve. [5]

(b) In the case where $a = 1$ and $b = -1$, determine the nature of the stationary point of the curve. [2]

12 **(a)** Given that $y = 3x\sqrt{7+x^3}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\frac{a+bx^3}{2\sqrt{7+x^3}}$. [4]

(b) Hence or otherwise, evaluate $\int_0^1 \frac{14+5x^3}{2\sqrt{7+x^3}}\,\mathrm{d}x$, leaving your answer in surd form. [4]

13 The equation of a circle is $x^2 - 4x + y^2 - 6y + c = 0$, where c is a constant to be determined.

(a) Find the coordinates of the centre of the circle. [2]

(b) Given that $y = 7$ is a tangent to the circle, show that the value of c is -3. [2]

(c) The equation of a normal to the circle is $2y + x = k$. Find the value of k. [2]

END OF PAPER

Answer Key

1	$x=3,\ y=-\dfrac{3}{2}$ or $x=0,\ y=-3$	10a	$\dfrac{\sqrt{2}}{2}$
2	$\dfrac{3\pi}{4}$	10b	$\dfrac{\sqrt{6}+\sqrt{2}}{4}$
3a	$30x(3x^2-4)^4$	11a(i)	$a+3b=0$
3bi	$x^3+1=(x+1)(x^2-x+1)$	11a(ii)	$a=-3,\ b=1,\ c=2$
3bii	$\dfrac{x^3}{3}-\dfrac{x^2}{2}+x+c$	11b	Point of inflexion
4	$3-\sqrt{2}$	12a	$\dfrac{42+15x^3}{2\sqrt{7+x^3}}$
5	$4\sqrt{33}$ cm	12b	$2\sqrt{2}$
6	$\dfrac{5}{3x-1}+\dfrac{x-7}{x^2+2}$	13a	(2, 3)
7	$\dfrac{120}{119}$	13b	Since $y=7$ is the tangent to the circle, radius = 4. $\sqrt{13-c}=4$ $13-c=16$ $c=-3$ (shown)
8a	$2\left(x-\dfrac{3}{2}\right)^2+\dfrac{9}{2}$		
8b	Since $\left(x-\dfrac{3}{2}\right)^2\geq 0$, $2\left(x-\dfrac{3}{2}\right)^2\geq 0$ $2\left(x-\dfrac{3}{2}\right)^2+\dfrac{9}{2}\geq\dfrac{9}{2}$ $y\geq\dfrac{9}{2}$ The least value of y is 4.5, and it occurs when $x=1.5$. $\therefore$ The lowest point on the curve y = f(x) is (1.5, 4.5).	13c	$k=8$
8c	$k<\dfrac{9}{2}$		
9a	$k<-5$ or $k>-1$		
9b	For ax^2-7x-3 to be always positive, $D<0$ and a > 0. $49+12a<0$ $12a<-49$ $a<\dfrac{-49}{12}$ Since there is no positive value of a that satisfies $D<0$, ax^2-7x-3 cannot be always positive.		