1. (i) Variables x and y are connected such that when $\lg y$ is plotted against x, a straight line is obtained. The straight line passes through the points $(2, 2)$ and $(5, 11)$. Express y in terms of x. [2]

(ii) Find the value of y when $x = 1$. [2]

2. The variables x and y are related by the equation $y = ax^3 + bx^2$, where a and b are constants. When the graph of $\frac{y}{x^2}$ against x is drawn, a straight line is obtained. The straight line passes through the points (3, 0) and (6,2).

(i) Find the value of a and of b. [4]

(ii) Find the value of y when $x = 6$. [2]

3. In a chemical reaction, a solid substance slowly dissolves in a liquid. At the start of the reaction, there are M grams of the substance. After t minutes, there are x grams of the substance still to be dissolved. It is known that x and t are related by the equation $x = 8e^{-\frac{t}{5}}$.

(i) Find the value of M. [1]

(ii) Estimate the time taken for half of the substance to be dissolved. [2]

4. (a) Given that $\left(5^{1-x}\right)\left(3^{x+3}\right) = 20$, find the value of $\left(\frac{3}{5}\right)^x$. [2]

(b) Hence, solve the equation $\left(5^{1-x}\right)\left(3^{x+3}\right) = 20$. [1]

5. Solve the equation $2\log_3 x = 1 + 5\log_x 9$. [4]

6. Using long division, find the remainder when $-x^4 - x^3 + x^2 + 2x + 8$ is divided by $x^2 - 3$ [2]

7. It is given that $f(x) = ax^3 - (a+3b)x^2 + 2bx + c$ is divisible by $x^2 - 2x$.

(i) Show that $c = 0$ and $a = 2b$. [2]

When $f(x)$ is divided by $x-1$, the remainder is 8 **more than** when it is divided by $x+1$.

(ii) Find the values of a and b. Hence, or otherwise, factorise $f(x)$ completely. [4]

(iii) Sketch the graph of $y = f(x)$, indicate clearly the x and y intercepts. [2]

End of Paper