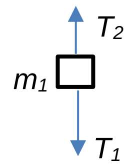
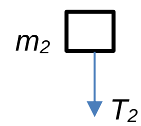
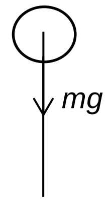
Circular Motion Discussion Questions Suggested Solutions

D1		Given mass, $m = 2.0 \times 10^{-3}$ kg (S.I unit) and time for 3 revolutions, $3T = 3.14$ s $\Rightarrow T = 3.14/3$ s (a) $\theta = \omega t = \frac{2\pi}{3.14/3} \times (2) = 12.0 \text{ rad} = 12.0 - 2\pi = 5.7 \text{ rad}$ (b) $r = 0.05$ m $v = r\omega = \frac{2\pi}{3.14/3} \times 0.05 = 0.30\ ms^{-1}$ (c) $a = r\omega^2$ or $a = \frac{v^2}{r} = \frac{(0.3001)^2}{5.0 \times 10^{-2}}$ = 1.800 = <u>1.8 m s^{-2}</u> (d) Note**: Frictional force, *f*, provides the centripetal force, F_c.** Writing this statement is necessary to justify why friction is equated to the centripetal force in the next step of working. $f = F_c = ma_c$ $= (2.0 \times 10^{-3})(1.800)$ $= 3.600 \times 10^{-3}$ = <u>3.6×10^{-3} N</u>
D2		Tension, $T = kx = 40 \times 0.20 = 8.0$ N **Tension provides the centripetal force**, $T = \frac{mv^2}{r}$ $v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{8 \times 0.70}{0.050}} = 10.58 = 11\ \text{m s}^{-1}$ Answer: **A**
D3		The external forces acting on the pendulum are tension (strictly speaking, force of string on pendulum) and weight. The **horizontal component of the tension provides the centripetal force** for the pendulum to turn right. (Note that the **centripetal force should not be drawn as a third force**.) Answer: **C**
D4		**Frictional force <u>of road on car</u> *F* provides for centripetal force** for car to round the corner. $F = m\frac{v^2}{r}$ Since m and r remain the same, $F \propto v^2$ $\frac{F_{wet}}{F_{dry}} = (\frac{v_{wet}}{v_{dry}})^2$ $\frac{1}{2} = (\frac{v_{wet}}{20})^2$ $v_{wet} = \frac{20}{\sqrt{2}}$ Answer: **D**

D5	(a)	Diagram 1. **Frictional force of wall on person** keeps him/her from sliding down.
	(b)	$a_c = r\,\omega^2 = 3.00 \times 5.00^2 = 75.0\ \text{m s}^{-2}$
	(c)	$F = m a_c = 60 \times 75 = 4500\ \text{N}$ **Normal contact force of wall on person provides for the centripetal acceleration**.
D6	(a)	For m_1, $T_1 - T_2 = m_1 a_1$ (Newton's second law) $a_1 = \frac{T_1 - T_2}{m_1} = \frac{4.5 - 2.9}{2.5} = 0.64\ \text{ms}^{-2}$  For m_2, $T_2 = m_2 a_2$ (Newton's second law) $a_2 = \frac{T_2}{m_2} = \frac{2.9}{3.5} = 0.83\ \text{ms}^{-2}$ 
	(b)	$a = \frac{v^2}{r}$ $v_1 = \sqrt{a_1 r_1} = \sqrt{0.64 \times 1.0} = 0.80\ \text{ms}^{-1}$ $v_2 = \sqrt{a_2 r_2} = \sqrt{0.83 \times 1.3} = 1.04 = 1.0\ \text{ms}^{-1}$
D7	(a)	The **tension in the string** created by the weight, *W* of the washers **provides the centripetal force** for the bung to perform circular motion. If we let *M* = mass of washers and *m* = mass of rubber bung, then $Tension = W = Mg = mr\left(\frac{2\pi}{T}\right)^2$ $0.035 \times 9.81 = m(0.57)\left(\frac{2\pi}{\frac{18.2}{20}}\right)^2 = 12.6g$
	(b)	If glass rod is twirled at just the right pace the paper clip can be maintained in a position just below the bottom of the glass tube. This ensures that the radius is kept constant at a known radius.
	(c)(i)	$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{95 \times 10^{-3} \times 9.81 \times 0.57}{12.6 \times 10^{-3}}} = 6.49\ \text{m s}^{-1}$
	(ii)	There must exist an upward vertical component of the tension to balance the weight of the rubber bung. If the string was purely horizontal no such component could exist.
	(iii)	In the vertical direction, forces acting on rubber bung $T \sin\theta = \text{mg}$ Hence, $\sin\theta = \frac{mg}{Mg} = \frac{12.6 \times 10^{-3}}{95 \times 10^{-3}}$ $\theta = 7.62^{\circ}$

<table>
<tr><td>D8</td><td>(a)</td><td>String is just taut when particle reaches C → Tension $T = 0$
Weight of particle provides for centripetal force
$mg = \frac{m{v_C}^2}{L}$
$v_C = \sqrt{gL}$ (shown)

</td></tr>
<tr><td></td><td>(b)</td><td>By conservation of energy,
Initial total energy at the bottom = Final total energy at the top
$\frac{1}{2}mv^2 = \frac{1}{2}m{v_C}^2 + mg(2L)$
$\frac{1}{2}v^2 = \frac{1}{2}gL + g(2L)$
$v^2 = gL + 4gL$
$v = \sqrt{5gL}$</td></tr>
<tr><td>D9</td><td>(a)</td><td>Let v be the tangential velocity at A and m be the mass of the roller coaster.

Using conservation of energy,
Initial KE + initial GPE = final KE + final GPE

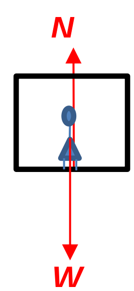
$\frac{1}{2}mv_o^2 + mgh = \frac{mv^2}{2} + \frac{2}{3}mgh$
$\frac{1}{2}mv_o^2 + \frac{1}{3}mgh = \frac{mv^2}{2} \quad ----(1)$

The resultant force on the roller coaster at A provides the centripetal force,
Using N2L,
$W - N = \frac{mv^2}{R}$
When $N = 0$, v is maximum
$mg = \frac{mv^2}{R}$
$\frac{1}{2}Rmg = \frac{mv^2}{2} \quad ----(2)$

Sub (2) into (1), when v is maximum, v_o will also be maximum:
$\frac{1}{2}mv_o^2 + \frac{1}{3}mgh = \frac{1}{2}Rmg$

$v_o^2 = Rg - \frac{2}{3}gh$

$v_o = \sqrt{g(R - \frac{2}{3}h)}$</td></tr>
</table>

	(b)	**By conservation of energy,** **Initial total energy = Final total energy at B** $\frac{1}{2}mv_o^2 + mgh = mgh'$ (just makes it to B $\Rightarrow$ final KE = 0) $gh' = \frac{1}{2}v_o^2 + gh$ $h' = \frac{1}{2}\frac{v_o^2}{g} + h$ $h' = \frac{1}{2}\frac{Rg - \frac{2}{3}gh}{g} + h$ $h' = \frac{2}{3}h + \frac{1}{2}R$
D9	(c)	The acceleration is not uniform.
D10		At point C, particle's velocity is entirely horizontal. i.e. $v = -3.0$ m s^{-1} By conservation of energy, Total energy at A = total energy at C $\text{KE}_A = \text{GPE}_C + \text{KE}_C$ $\frac{1}{2}m\left(v_A^2 - v_C^2\right) = mg(2R)$ $R = \frac{5.0^2 - (-3.0)^2}{4g} = 0.408$ m $a_c = \frac{v^2}{R} = \frac{3^2}{0.408} = 22.1 \text{ m s}^{-2}$ Note that $a_c = r\omega^2$ is not applicable in this context. This is because the angular speed of the particle is not constant throughout its vertical circular motion. The centripetal acceleration varies as the speed of the particle changes. Answer: **D**
D11	(a)	$v = \frac{2\pi r}{T} = \frac{2\pi \times 7.0}{20.0} = 2.20 \text{ m s}^{-1}$
	(b)	$F_c = \frac{mv^2}{r} = \frac{50 \times 2.2^2}{7.0} = 34.6 \text{ N}$
	(c)	Considering the forces acting on Sasha at the top of the ride and using N2L: $W - N = \frac{mv^2}{r}$  For Sasha to feel weightless at the top of the ride, $N = 0$ $\frac{mv^2}{r} = mg$ $v = \sqrt{rg} = \sqrt{7.0 \times 9.81} = 8.29 \text{ m s}^{-1}$
	(d)	At the bottom of the Ferris wheel, $\frac{mv^2}{r} = N - mg$ $N = mg + \frac{mv^2}{r} = 2mg = 2 \times 50 \times 9.81 = 981 \text{ N}$ O

D12

(a)

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{37 \times 60} = 2.8303 \times 10^{-3} = 2.83 \times 10^{-3} \text{ rad s}^{-1}$$

(b)

$$v = r\,\omega = \left(\frac{150}{2}\right) \times (2.8303 \times 10^{-3}) = 0.21227 = 0.212 \text{ m s}^{-1}$$

(c)

$$t = \frac{\Delta\theta}{\omega} = \frac{2 \times \left(\frac{2\pi}{32}\right)}{2.8303 \times 10^{-3}} = 138.75 = 139 \text{ s}$$

or

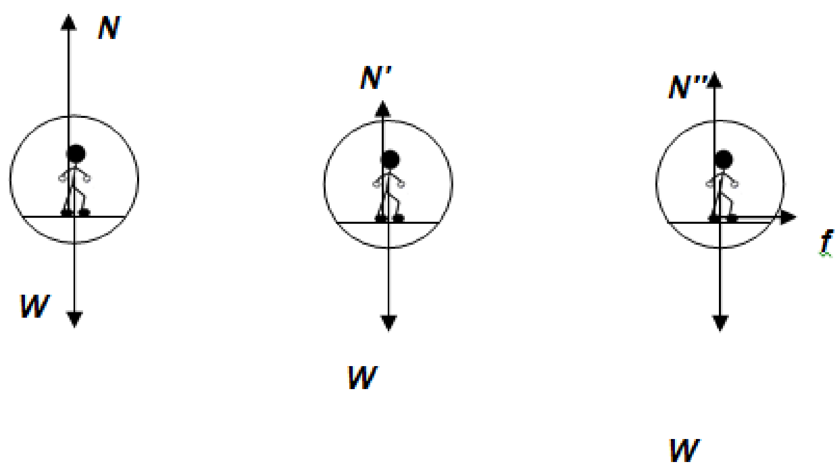
$$t = \frac{37 \times 60}{32} \times 2 = 139 \text{ s}$$

(d)

At the bottom

At the top

At the side (horizontal)



N significantly longer than *W* ($N \approx 1.5W$)

N' significantly shorter than *W* ($N \approx 0.5\ W$)

N'' = *W*
$f \approx 0.5\ W$, so about half its length

N: Normal contact force with the capsule floor
W: weight of passenger
f: friction between passenger's feet and capsule floor