Term 3 A.Math Class Test (Revision Paper 2) Solution

Answer all the questions.

Question 1:

It is given that $h(x) = (3x + 1)(x + 5)(2x - 1)$.

(a) Express $h(x)$ in the form $Ax^3 + Bx^2 + Cx + D$, giving the values of the constants A, B, C and D. [2]

(b) Find the value of the constant a, given that $(x + 3)$ is a factor of $h(x) + ax$. [2]

Solution:

(a) Expanding $h(x)$,

$$(3x^2 + 16x + 5)(2x - 1) = 6x^3 + 29x^2 - 6x - 5$$

$$\Rightarrow A = 6,\ B = 29,\ C = -6 \text{ and } D = -5$$

(b) $h(x) + ax = 6x^3 + 29x^2 - 6x - 5 + ax$

Let $g(x) = 6x^3 + 29x^2 - 6x - 5 + ax$.

Since $x + 3$ is a factor of $g(x)$, by Factor Theorem,

$$g(-3) = 0$$

$$6(-3)^3 + 29(-3)^2 - 6(-3) - 5 + (-3)a = 0$$

$$a = 37\frac{1}{3}$$

Question 2:

Given that $P(x) = 3x^3 - 7x^2 - x - 55$ and $Q(x) = 6x^3 - 5x^2 - 3x + 2$, find the value(s) of q if $P(q) = Q(q)$. [4]

Solution:

$P(q) = 3q^3 - 7q^2 - q - 55$ and $Q(q) = 6q^3 - 5q^2 - 3q + 2$.

$$P(q) = Q(q)$$

$$3q^3 - 7q^2 - q - 55 = 6q^3 - 5q^2 - 3q + 2$$

$$\boxed{3q^3 + 2q^2 - 2q + 57 = 0} \longrightarrow \text{to be solved}$$

Let $f(q) = 3q^3 + 2q^2 - 2q + 57$

$f(-3) = 3(-3)^3 + 2(-3)^2 - 2(-3) + 57$

$= 0$

By Factor Theorem, $q + 3$ is a factor of $3q^3 + 2q^2 - 2q + 57$.

So, $3q^3 + 2q^2 - 2q + 57 = (q + 3)(aq^2 + bq + c)$.

Equating coefficient of q^3: $a = 3$

Equating coefficient of q^2: $3a + b = 2$

$$3(3) + b = 2$$
$$b = -7$$

Equating constant term: $3c = 57$

$$c = 19$$

$$3q^3 + 2q^2 - 2q + 57 = 0$$
$$(q + 3)(3q^2 - 7q + 19) = 0$$

$q = -3$ and $3q^2 - 7q + 19 = 0$

Discriminant, $b^2 - 4ac = (-7)^2 - 4(3)(19) < 0$

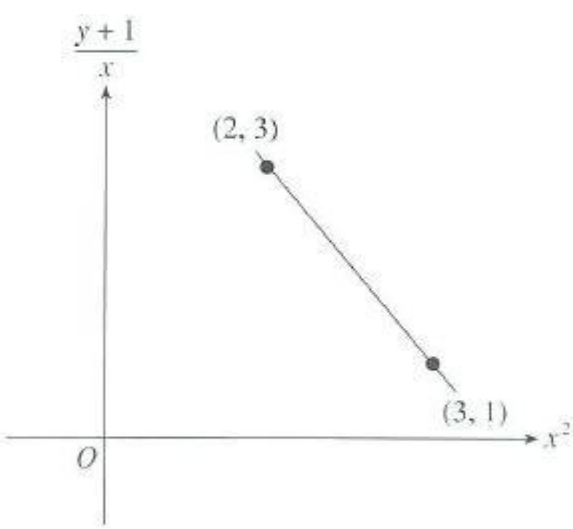
$\Rightarrow 3q^2 - 7q + 19$ has no solutions.

Question 3:

The figure shows part of a straight line graph obtained by plotting $\dfrac{y+1}{x}$ against x^2.

Express y in terms of x.

[3]



Solution:

Gradient of line $m = \dfrac{3-1}{2-3}$

$= -2$

Equation of the line $\Rightarrow$ $Y = -2X + c$

(2, 3) lies on the line $\Rightarrow$ $3 = -2(2) + c$

$c = 7$

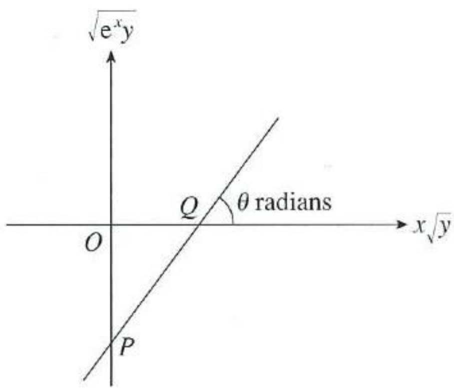
Hence, the equation of the line is

$\dfrac{y+1}{x} = -2x^2 + 7$

$y = -2x^3 + 7x - 1.$

Question 4:

In the figure, x and y are related by $e^{\frac{1}{2}}x = 2x - \dfrac{3}{\sqrt{y}}$.



Find

(a) the value of θ, [3]

(b) the coordinates of P and Q. [2]

Solution:

a) $e^{\frac{1}{2}x} = 2x - \dfrac{3}{\sqrt{y}}$

$\sqrt{e^x y} = 2x\sqrt{y} - 3$

$Y \quad = \quad mX + \qquad c$

Gradient = tan θ

$= 2$

$\Rightarrow \;\; = \tan^{-1} 2$

$= 1.11$

(b) Equation of line $\Rightarrow$ $Y = 2X - 3$

P lies on y-axis $\Rightarrow$ coordinates of P are $(0, Y)$

$Y = -3$

Thus, the coordinates of P are $(0, -3)$.

Q lies on x-axis $\Rightarrow$ coordinates of Q are $(X, 0)$

$X = 1.5$

Thus, the coordinates of Q are $(1.5, 0)$.

<u>Question 5:</u>

Given that x and y are known to be related by the equation $\sqrt[3]{y} = st^x$ where s and t are constants. Express the equation in a form suitable for drawing a straight line graph. [3]

<u>Question 6:</u>

Solve $\log_3 (x + 6) = 2 - \log_3 (x - 4) + \log_9 x^2$, giving your answers in exact values. [5]

<u>Solution:</u>

$$\log_3 (x + 6) = \log_3 3^2 - \log_3 (x - 4) + \frac{\log_3 x^2}{\log_3 9}$$

$$\log_3 (x + 6) = \log_3 3^2 - \log_3 (x - 4) + \frac{2\log_3 x}{2\log_3 3}$$

$$\log_3 (x + 6) = \log_3 3^2 - \log_3 (x - 4) + \log_3 x$$

$$\log_3 (x + 6) = \log_3 \frac{3^2(x)}{x-4}$$

$$x + 6 = \frac{9x}{x-4}$$

$$(x + 6)(x - 4) = 9x$$

$$x^2 + 2x - 24 = 9x$$

$$x^2 - 7x - 24 = 0$$

$$x = \frac{7 \pm \sqrt{49+4(24)}}{2}$$

$$x = \frac{7+\sqrt{145}}{2} \quad \text{or} \quad x = \frac{7-\sqrt{145}}{2}$$

Question 7 :

Given that $(\log_3 7)(\log_7 k) = 2$, find the value of k. [2]

Solution :

$$\left(\log_3 7\right)\left(\frac{\log_3 k}{\log_3 7}\right) = 2$$

$$\log_3 k = 2$$

$$k = 3^2$$

$$= 9$$

Thus, $k = 9$.

Question 8:

(a) **(i)** Solve $-3x^3 + 7x^2 + 7x - 3 = 0$, showing your working clearly. [4]

(ii) Sketch the graph of $y = -3x^3 + 7x^2 + 7x - 3$. Indicate clearly, the coordinates of the x-intercept(s) and y-intercept. [3]

(b) When a polynomial $P(x)$ is divided by $(x-1)$ and $(x-2)$, the remainders are 2 and 3 respectively. Find the remainder when $P(x)$ is divided by $(x-1)(x-2)$. [4]

Solution:

(a) (i)

$-3x^3+7x^2+7x-3=0$

Let $f(x)=-3x^3+7x^2+7x-3$

$f(-1)=3+7-7-3$

$f(-1)=0$

$(x+1)$ is a factor of $f(x)$.

$f(x)=(x+1)(-3x^2+kx-3)$

By comparing the coefficient of x^2 :

$k-3=7$

$k=10$

$f(x)=(x+1)(-3x^2+10x-3)$

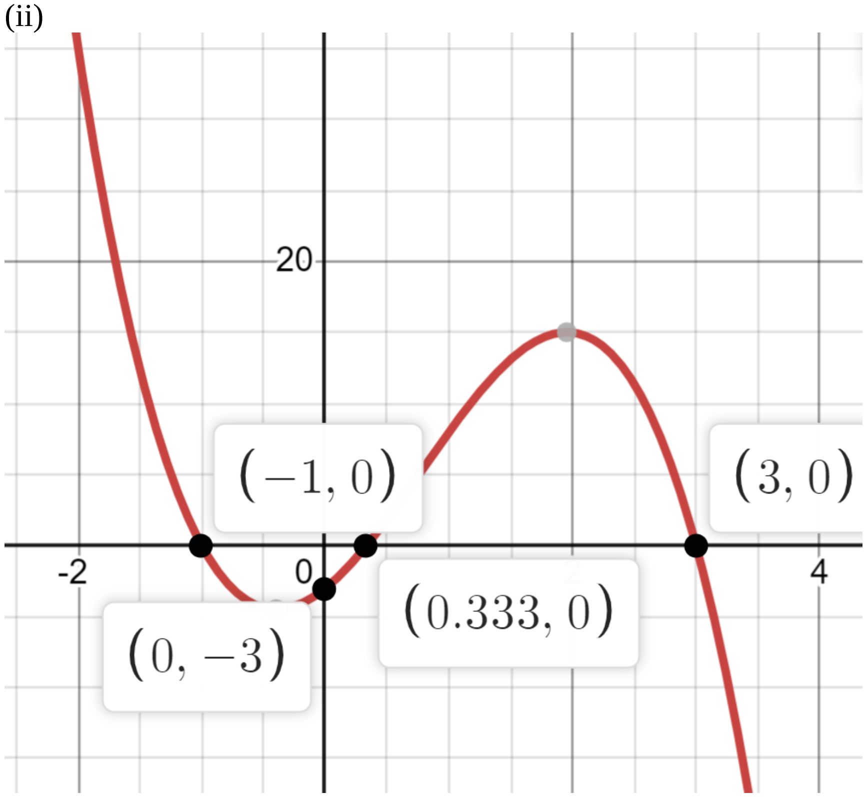
$f(x)=(x+1)(-3x+1)(x-3)$

$f(x)=0$

$(x+1)(-3x+1)(x-3)=0$

$x=-1$ or $x=-\frac{1}{3}$ or $x=3$

(b) (ii)



(b)

Remainder $= ax+b$

$P(x) = Q(x) \times D(x) + R(x)$

$P(x) = Q(x) \times (x-1)(x-2) + ax + b$

$P(1) = 2$

$P(2) = 3$

$a + b = 2 --- (1)$

$2a + b = 3 --- (2)$

$(2) - (1):$

$a = 1$

$b = 1$

$\mathrm{Re}\,mainder = x + 1$