Name: ______________________ (   )   Class: __________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS

4049/01

Paper 1

23 August 2021
2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

FOR EXAMINER'S USE

Q1		Q4		Q7		Q10	
Q2		Q5		Q8		Q11	
Q3		Q6		Q9		Q12	

/ 90

This document consists of **15** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $2x = 1 - 3y$ intersects the curve $3y^2 - x^2 = 2$ at points A and B.
The x-coordinate of B is greater than the x-coordinate of A.

(a) Find the coordinates of A. [4]

Point C lies on AB such that $AC : CB = 2 : 1$.

(b) Find the x-coordinate of C. [1]

2 **(a)** The function f is defined by $f(x) = \dfrac{2x+5}{1-3x}$, $x \neq \dfrac{1}{3}$.
Determin whether f is an increasing or decreasing function. [3]

(b) Find $\int \left(\sec^2 x - \dfrac{\pi}{2} \sin(3x-5) \right) dx$. [2]

3 The equation of a curve is $y = 2\cos 2x + c$, where c is a constant.
The graph passes through the point $(30°, 5)$.

(a) Find the value of c. [2]

(b) State the amplitude and period of y. [2]

(c) Sketch the graph of $y = 2\cos 2x + c$ for $0° \leq x \leq 360°$. [3]

4 A curve is such that $\frac{d^2y}{dx^2} = \frac{3}{(2x-1)^2}$ and the point (1, 7) lies on the curve. The gradient of the tangent to the curve at $x = 2$ is $\frac{3}{2}$. Find the equation of the curve. [5]

5 **(a)** Write down and simplify the first 3 terms in the expansion of $(1+3x)^6$, in ascending powers of x.

Hence find the value of a and of b in the expansion of $\left(1+ax+bx^2\right)(1+3x)^6$ if the coefficients of x and x^2 are 23 and 223 respectively. [4]

(b) Find the term independent of x in the expansion of $\left(x-\dfrac{1}{2x^2}\right)^{15}$. [3]

6 The equation of a polynomial is $f(x)=9x^3-6x^2-11x+4$.

(i) Factorise completely the polynomial $f(x)$. [3]

(ii) Sketch the graph of $f(x)=9x^3-6x^2-11x+4$. [2]

(iii) Find the range of values of x for which $f(x)\geq 0$. [2]

7 **(i)** Prove the identity $\dfrac{1}{\sin x+1}-\dfrac{1}{\sin x-1}=2\sec^2 x$. [3]

(ii) Hence, solve the equation $\dfrac{1}{2(\sin x+1)}-\dfrac{1}{2(\sin x-1)}=3+\tan x$ for $0° \le x \le 360°$. [5]

8 Michael bought a brand new Ducati motorcycle. As the motorcycle is a limited edition model, many enthusiasts predict that its value, $\$P$, will appreciate so that after t months, its value can be modelled by $P = 65000e^{kt}$. The motorcycle is expected to have a value of \$72000 after 2 years.

(i) How much did Michael pay for the motorcycle? [1]

(ii) Calculate the value of the motorcycle after 5 years. [4]

(iii) After how many years will the motorcycle have a value of \$90000? [3]

9 **(a)** The equation of a curve is $y = x^2 + (2a-1)x + a^2$, where a is a constant.

(i) Find the range of values of a for which the curve has no real roots. [3]

(ii) In the case where $a = 2$, find the range of values of x for which the curve lies above the line $y = 8$. [3]

(b) Explain why the greatest value of $-2x^2 + 4x - 5$ is -3 for all values of x. [3]

10 **(a)** It is given that $\tan A = c$, where c is a constant and $180° \leq A \leq 270°$.
Express each of the following, in terms of c,

(i) $\cot A$, [1]

(ii) $\sin 2A$. [2]

(b) Given that $\sin x = 2\sin(30° - x)$, find the exact value of $\tan x$. [5]

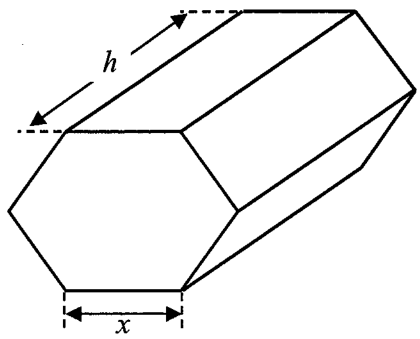
11 A particle P, moves in a straight line so that its velocity, v m/s from a fixed point O is given by $v = 2t^2 - 9t + 7$, where t is the time in seconds after passing O.

(a) Find the deceleration of P when $t = 2$. [3]

(b) Find the values of t when P is instantaneously at rest. [3]

(c) Find the distance P travelled in the fourth second. [4]

12



The diagram shows part of a solid metal allen key that has length h cm and uniform regular hexagonal cross-section of side x cm.

(i) Find an expression, in terms of x, for the cross-sectional area of the allen key. [2]

(ii) The volume of the allen key is 4 cm^3. Show that the surface area of the allen key, A cm^2, is given by $A = 3\sqrt{3}x^2 + \dfrac{16}{\sqrt{3}x}$. [3]

Given that x can vary, find

(iii) the value of x for which A has a stationary value, [3]

(iv) the stationary value of A and determine whether this value is maximum or minimum. [3]