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CLASS: _____



FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS

4049/02

Paper 2

Date: 28 August 2025
Duration: 2 hours 15 minutes

 Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

 Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

 For π , use either your calculator value or 3.142.

For Examiner's Use

Table of Penalties		Question Number	Parent's/Guardian's Signature	90
Presentation	☐ 1 ☐ 2			
Rounding off	☐ 1			

Setters: Mr Joel Li

This question paper consists of 23 printed pages

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

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Answer **all** the questions.

- 1** In an experiment, a liquid is heated to reach its boiling point and then allowed to cool down. The temperature of the liquid T is given by the formula $T = 25 + 200e^{-0.2t}$ when the liquid is cooling for time t minutes.

(a) Find the

(i) boiling point of the liquid, [1]

(ii) value of t for which $T = 150$, [2]

(iii) rate at which the temperature of the liquid is decreasing at the instance when $t = 10$. [2]

(b) Explain why the temperature of the liquid can never fall to 25°C . [2]

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- 2 It is given that $f(x) = 2hx^3 + kx^2 + 4x + 4$, where h and k are constants, has a factor of $(x-1)$. The remainder when the expression is divided by $(x-2)$ is four times the remainder when it is divided by $(x+1)$.

(a) Find the value of h and of k . [4]

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- (b) Using these values of h and k , solve $f(x) = 0$, leaving your answers in the form $a \pm b\sqrt{3}$ where applicable. [4]

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3 (a) Show that $4\sec\theta + \tan\theta = 3\cot\theta$ can be expressed as $4\sin^2\theta + 4\sin\theta - 3 = 0$. [3]

(b) Hence, solve $4\sec 2x + \tan 2x = 3\cot 2x$ for $-\pi < x < \pi$. [4]

4 (a) State the values between which each of the following must lie:

(i) the principal value of $\tan^{-1} x$, [1]

(ii) the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. [1]

(b) If $\sin \theta = -\frac{3}{4}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the exact value of

(i) $\tan \theta$, [1]

(ii) $\cos\left(\frac{\pi}{2} + \theta\right)$. [1]

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- (c) The acute angles P and Q are such that $\tan(P+Q)=5$ and $\tan Q=2$. Without using a calculator, find the exact value of $\sin P$. [5]

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5 (a) (i) Given that $y = x\sqrt{x+1}$, show $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$. [3]

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- (a) (ii) Hence or otherwise, find $\int_0^8 \frac{x}{\sqrt{x+1}} dx$. [5]

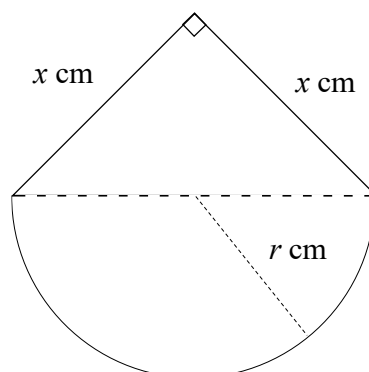
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- 5 (b) Water leaks from a container at a rate of $6 \text{ cm}^3/\text{s}$. The volume of the water left in the container is found to be $V = 10 \sin\left(\frac{h}{4}\right)$ where h cm is the depth of water in it.

When $V = 5 \text{ cm}^3$, find the rate of change of h . [4]

- 6 A keychain of negligible thickness is made up of a semicircle and an isosceles right-angled triangle as shown in the diagram. Two sides of the triangle are x cm each and the radius of the semicircle is r cm. The perimeter of the keychain is 100 cm and the surface area of the keychain is A cm².



- (a) Show that $A = \frac{x^2}{2} + \frac{2}{\pi}(50-x)^2$. [4]

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(b) Given that x can vary, find the value of x for which A is a stationary point. [4]

(c) Determine whether this value of x gives a maximum or minimum value for the surface area of the keychain. [2]

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7 (a) Find the value of x for which $\log_4(2x) + \log_2 8 - 1 = \frac{1}{\log_3 2}$. [4]

(b) Solve the equation $e^{2y}(e^{2y} - 4) = 12$. [3]

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- (c) **Without using a calculator**, find the value of 6^x , given that $12^{x-2} = 18^{2-x}$. [3]

- 8 At a particular harbour, the height of the water varies with the tides. The height of the water, H , in metres, on a particular day is modelled by the equation

$$H = 5 - 2.4 \cos\left(\frac{\pi t}{3}\right) + \sin\left(\frac{\pi t}{3}\right)$$

where t is the time in hours after midnight.

- (a) Express H in the form $k + R \sin\left(\frac{\pi t}{3} - \alpha\right)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

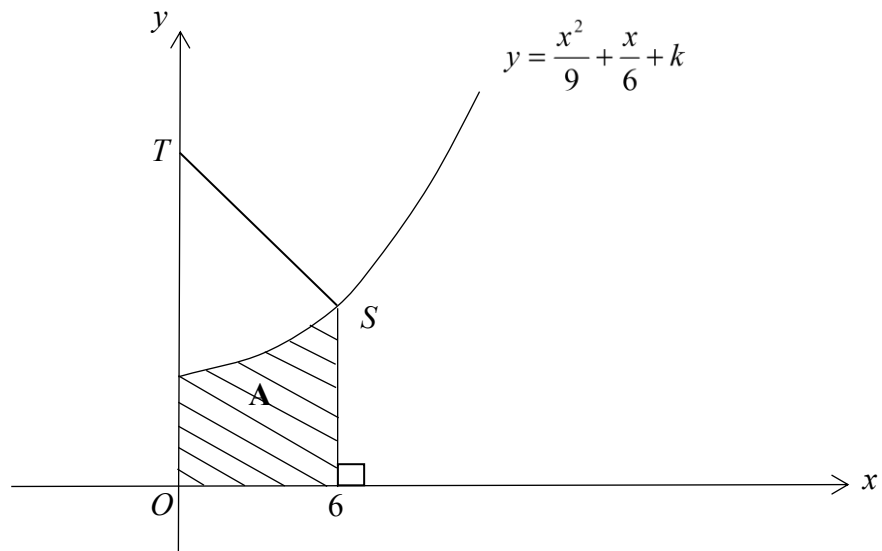
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(b) Find the maximum and minimum heights of the water. [2]

(c) Find the time interval between the first high tide and the first low tide. [3]

- 9 The diagram below shows part of the curve $y = \frac{x^2}{9} + \frac{x}{6} + k$, where k is a constant. S is the point on the curve where $x = 6$ and T is a point on the y -axis.



- (a) Given that the area of region **A** is 23 units², show that $k = 2$.

[3]

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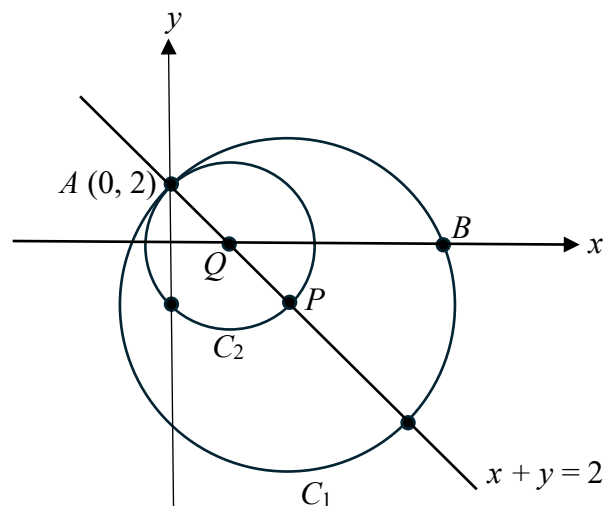
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ST is a normal to the curve at S .

(b) Find the coordinates of T .

[4]

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The diagram shows two circles C_1 and C_2 . Circle C_1 has its centre at P and circle C_2 has its centre at Q . Point P lies on the circumference of circle C_2 and point Q lies on the x -axis. The line $x + y = 2$ passes through the centres of both circles and intersects the circumference of both circles at point $A(0, 2)$.

- (a) Find the equations of C_1 and C_2 in the form $x^2 + y^2 + 2gx + 2fy + c = 0$. [5]

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Continuation of working space for question **10(a)**

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$B(k, 0)$ is a point on the circle C_1 , where $k > 0$.

(b) Find the value of k in the form of $a + b\sqrt{7}$ where a and b are integers. [3]

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- (c) Show that the gradient of the tangent to the circle C_1 at point B is $-\sqrt{7}$ and that this tangent intersects the y -axis at $4\sqrt{7} + 14$. [3]

~ End of Paper ~

Answer Key

1ai	225°C	6a	$2x + \frac{1}{2} \times 2\pi r = 100$ $r = \frac{100 - 2x}{\pi}$ $A = \frac{x^2}{2} + \frac{1}{2} \times \pi r^2$ $= \frac{x^2}{2} + \frac{1}{2} \pi \left(\frac{100 - 2x}{\pi} \right)^2$ $= \frac{x^2}{2} + \frac{1}{2} \pi \left[\frac{4}{\pi^2} (50 - x)^2 \right]$ $= \frac{x^2}{2} + \frac{2}{\pi} (50 - x)^2$
1aii	$t = 2.35 \text{ mins}$		
1aiii	$\frac{dT}{dt} = -5.413$		
1b	$T = 25 + 200e^{-0.2t}$ but $e^{-0.2t} > 0$ for all values of t . $\therefore T > 25^{\circ}\text{C}$ and hence the liquid can never fall to 25°C		
2a	$h = -\frac{1}{2}, k = -7$		
2b	$x = 1, -4 + 2\sqrt{3}$ or $-4 - 2\sqrt{3}$		
3a	$4\sec\theta + \tan\theta = 3\cot\theta$ $\frac{4}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{3\cos\theta}{\sin\theta}$ $\frac{4 + \sin\theta}{\cos\theta} = \frac{3\cos\theta}{\sin\theta}$ $4\sin\theta + \sin^2\theta = 3\cos^2\theta$ $= 3(1 - \sin^2\theta)$ $= 3 - 3\sin^2\theta$ $4\sin^2\theta + 4\sin\theta - 3 = 0$ (shown)	6b	$\frac{200}{\pi + 4}$ or 28.0 (3s.f)
		6c	A is a minimum at $x = \frac{200}{\pi + 4}$
		7a	$x = \frac{9}{32}$
		7b	$y = \frac{1}{2} \ln 6$ or 0.896(3sf)
		7c	$6^x = 36$
		8a	$H = 5 + 2.6 \sin\left(\frac{\pi t}{3} - 1.18\right)$
3b	$x = -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12}$ or $x = -2.88, -1.83, 0.262, 1.31$ (3sf)	8b	Minimum height is 2.4 m Maximum height is 7.6 m
4ai	$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$	8c	3 hours
4aii	$-\frac{\pi}{6}$ or -30°	9a	Area = 23 $\int_0^6 \left(\frac{x^2}{9} + \frac{x}{6} + k \right) dx = 23$ $\left[\frac{x^3}{27} + \frac{x^2}{12} + kx \right]_0^6 = 23$ $\left[\frac{6^3}{27} + \frac{6^2}{12} + 6k \right] = 23$ $8 + 3 + 6k = 23$ $6k = 12$ $k = 2$ (Shown)
4bi	$-\frac{3}{\sqrt{7}}$ or $-\frac{3\sqrt{7}}{7}$		
4bii	$\frac{3}{4}$		
4c	$\sin P = \frac{3}{\sqrt{130}}$ or $\frac{3\sqrt{130}}{130}$		
5ai	$\frac{dy}{dx} = 1 \cdot \sqrt{x+1} + \frac{1}{2\sqrt{x+1}}x$ $= \frac{2x+2+x}{2\sqrt{x+1}}$ $= \frac{3x+2}{2\sqrt{x+1}}$		
5aii	$13\frac{1}{3}$ or $\frac{40}{3}$	9b	$T = (0, 11)$
5aiii	$\frac{dh}{dt} = -2.77 \text{ cm/s}$ (3s.f)	10a	$x^2 + y^2 - 8x + 4y - 12 = 0$

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10b	$k = 4 + 2\sqrt{7}$ (ans) or $4 - 2\sqrt{7}$ (rej)	10c	$m_{\text{radius}} = \frac{0+2}{4+2\sqrt{7}-4} = \frac{1}{\sqrt{7}}$ $m_{\text{tangent}} = -\sqrt{7}$ (shown) $y = mx + c$ $0 = -\sqrt{7} (4 + 2\sqrt{7}) + c$ $c = 4\sqrt{7} + 2(7)$ $c = 4\sqrt{7} + 14$ (shown)
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