

Topical Revision Paper 2

Term 1 Unit 2 Surds and Indices

Question 1:

Without using a calculator, simplify $\frac{3}{(2-\sqrt{5})^2} - \frac{2}{(2+\sqrt{5})^2}$. [3]

Solution:

$$\begin{aligned} & \frac{3}{(2-\sqrt{5})^2} - \frac{2}{(2+\sqrt{5})^2} \\ &= \frac{3(2+\sqrt{5})^2 - 2(2-\sqrt{5})^2}{[(2-\sqrt{5})(2+\sqrt{5})]^2} \\ &= \frac{3(9+4\sqrt{5}) - 2(9-4\sqrt{5})}{(1)} \\ &= 9 + 20\sqrt{5} \end{aligned}$$

Question 2:

Solve for x $2(27^x) = 45(3^x) + \frac{243}{3^x}$. [4]

Solution:

$$2(27^x) = 45(3^x) + \frac{243}{3^x}$$

$$2(81^x) = 45(9^x) + 243$$

Let $y = 9^x$, equation becomes

$$2y^2 = 45y + 243$$

$$2y^2 - 45y - 243 = 0$$

$$(2y+9)(y-27) = 0$$

$$y = -4.5(\text{n.a}) \text{ or } y = 27$$

$$9^x = 27$$

$$x = \frac{3}{2}$$

Question 3:

The area of a rectangle $ABCD$ is given by $(1+3\sqrt{2})\text{ m}^2$, and the length of side AB is $(2+\sqrt{8})$ m. Find, without using a calculator, the length of side BC in the form of $(a+b\sqrt{2})\text{ m}$, where a and b are real numbers. [4]

Solution:

Length of the side BC

$$\begin{aligned} &= \frac{1+3\sqrt{2}}{2+\sqrt{8}} \\ &= \frac{(1+3\sqrt{2})(2-2\sqrt{2})}{(2+\sqrt{8})(2-2\sqrt{2})} \\ &= \frac{2-2\sqrt{2}+6\sqrt{2}-12}{-4} \\ &= \frac{-10+4\sqrt{2}}{-4} \\ &= \left(\frac{5}{2}-\sqrt{2}\right) \text{ m} \end{aligned}$$

Question 4:

Without using a calculator, find the fractions, a and b , for which $\frac{6\sqrt{7} + \sqrt{5}}{4\sqrt{5} - \sqrt{7}}$ can be expressed as $a + b\sqrt{35}$. [3]

Solution:

$$\begin{aligned} &\frac{6\sqrt{7} + \sqrt{5}}{4\sqrt{5} - \sqrt{7}} \\ &= \frac{6\sqrt{7} + \sqrt{5}}{4\sqrt{5} - \sqrt{7}} \times \frac{4\sqrt{5} + \sqrt{7}}{4\sqrt{5} + \sqrt{7}} \\ &= \frac{24\sqrt{35} + 6(7) + 4(5) + \sqrt{35}}{(4\sqrt{5})^2 - (\sqrt{7})^2} \\ &= \frac{62 + 25\sqrt{35}}{73} \\ &= \frac{62}{73} + \frac{25}{73}\sqrt{35} \\ \therefore a &= \frac{62}{73} \text{ and } b = \frac{25}{73} \end{aligned}$$

Question 4:

(a) Without using a calculator, evaluate $2^0 - \left| \frac{1}{7} - 49^{\frac{1}{2}} \right| + (16)^{\frac{-1}{4}}$. [2]

(b) Given that $\frac{1}{27^{2x+1}} = 243$, find the value of x . [2]

Solution:

(a)

$$\begin{aligned} 2^0 - \left| \frac{1}{7} - 49^{\frac{1}{2}} \right| + (16)^{\frac{-1}{4}} &= 1 - \left| \frac{1}{7} - 7 \right| + \frac{1}{\sqrt[4]{64}} \\ &= 1 - \frac{48}{7} + \left(\frac{1}{2} \right) \\ &= \frac{-41}{7} + \frac{1}{2} \\ &= \frac{-75}{14} \quad \text{or} \quad -4\frac{9}{14} \end{aligned}$$

(b)

$$\begin{aligned} (3^{-3})^{2x+1} &= 3^5 \\ -6x - 3 &= 5 \\ -6x &= 8 \\ x &= \frac{-4}{3} \end{aligned}$$

Question 5:

Solve for x , $13^{4x+2} - 4(169^{2x}) = 2145$. [4]

Solution:

$$\begin{aligned} 13^{4x+2} - 4(169^{2x}) &= 2145 \\ 13^{4x} \times 13^2 - 4(13^{4x}) &= 2145 \\ 169(13^{4x}) - 4(13^{4x}) &= 2145 \\ 165(13^{4x}) &= 2145 \\ 13^{4x} &= 13 \\ \therefore x &= \frac{1}{4} \end{aligned}$$

