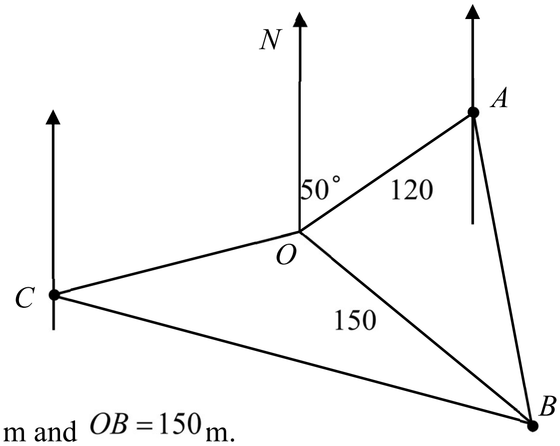
Topical Revision Worksheet 8

Topic: Trigonometry EM

<table>
<tr><td>Question 1 [2008 EOY, Q17]</td><td>marks</td></tr>
<tr><td>The diagram below shows the position of three locations A, B and C with respect to a reference point O. The bearings of A, B and C from O are 050°, 135° and 240° respectively.


It is also given that $OA = 120$ m and $OB = 150$ m.
(i) Find the bearing of B from A.
<u>Solution:</u>
$\measuredangle BOA = 135^\circ - 50^\circ = 85^\circ$
$AB = \sqrt{150^2 + 120^2 - 2(150)(120)\cos 85^\circ} = 183.7$
$\dfrac{\sin OAB}{150} = \dfrac{\sin 85^\circ}{AB} \Rightarrow OAB = 54.4^\circ$
Hence bearing of B from A $= 360^\circ - (180^\circ - 50^\circ) - 54.4^\circ = 175.6^\circ$</td><td>[3]</td></tr>
<tr><td>(ii) If the bearing of B from C is 100°, calculate the length of OC.
<u>Solution:</u>
$\measuredangle COB = 240^\circ - 135^\circ = 105^\circ$ and $\measuredangle OCB = 100^\circ - (180 - 120)^\circ = 40^\circ$
Hence by sine rule $\dfrac{OC}{\sin 35^\circ} = \dfrac{150}{\sin 40^\circ} \Rightarrow OC = 133.8$ m</td><td>[3]</td></tr>
<tr><td>(iii) A mast stands vertically at B such that the angle of elevation of the top of the mast from O is 62°. Find the height of the mast.
<u>Solution:</u>
$\tan 62^\circ = \dfrac{\text{height}}{150} \Rightarrow$ height $= 282.1$ m</td><td>[2]</td></tr>
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<tr><td>Question 2 [2009 EOY, Q14]</td><td>marks</td></tr>
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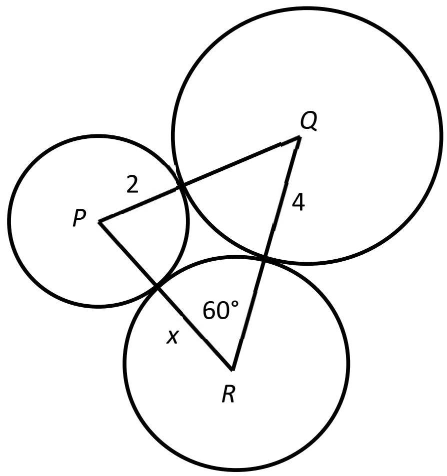
The diagram below shows three circular fields with centres P, Q and R tangential to one another. The radii of the three fields are 2 m, 4 m and x m respectively and $\angle PRQ = 60°$.



(i) Form an equation in x and show that it reduces to $x^2 + 6x - 24 = 0$.

Solution:

Using cosine rule,

$$PQ^2 = PR^2 + QR^2 - 2(PR)(QR)\cos 60^\circ$$

$$6^2 = (x+4)^2 + (x+2)^2 - 2(x+4)(x+2)\left(\frac{1}{2}\right)$$

$$36 = x^2 + 8x + 16 + x^2 + 4x + 4 - (x^2 + 6x + 8)$$

$$36 = x^2 + 6x + 12$$

$$x^2 + 6x - 24 = 0$$

</td><td>[3]</td></tr>
<tr><td>

(ii) Solve the equation, giving your answer in surd form and find the exact area of triangle PQR.

Solution:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-24)}}{2}$$

$$x = -3 + \sqrt{33} \text{ or } -3 - \sqrt{33} \text{ (rej)}$$

$$(2.74456...) \text{ or } (-8.74456...)$$

$$\text{Area} = \frac{1}{2}(PR)(QR)\sin 60^\circ$$

$$= \frac{1}{2}(-1+\sqrt{33})(1+\sqrt{33})\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4}(32) = 8\sqrt{3} \text{ m}^2$$

</td><td>[4]</td></tr>
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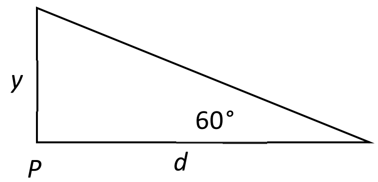
(i) A pole of y m stands at P. A boy walks from R to Q. Given that the maximum angle of elevation of the top of the pole during the walk is 60°, find y.

[3]

Solution:

Maximum angle of elevation occurs when distance, d from P to QR is the smallest.



$$8\sqrt{3}=\frac{1}{2}(1+\sqrt{33})d$$

$$d=\frac{16\sqrt{3}}{1+\sqrt{33}}$$

$$\tan 60^{\circ}=\frac{y}{d}$$

$$y=d\times\tan 60^{\circ}$$

$$=\frac{16\sqrt{3}}{1+\sqrt{33}}\times\sqrt{3}$$

$$=\frac{48}{1+\sqrt{33}} \text{ or } 7.12 \text{ m}$$

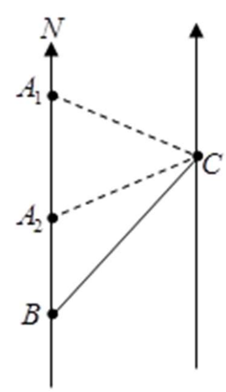
Question 3 [2010 EOY, Q9]

marks

Three points A, B and C on level ground are such that A is due north of B, $BC=20$ km, the bearing of C from B is $042°$ and $AC=16$ km.

(i) Find the possible distances of A from B.

[5]



Solution:

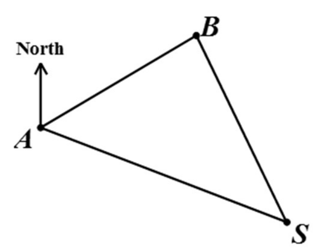
By sine rule, $\dfrac{\sin\measuredangle BAC}{20}=\dfrac{\sin 42°}{16}$ [M1]

Hence $\measuredangle BA_1C=56.76°$ (2 d.p.) and $\measuredangle BA_2C=123.24°$ (2 d.p.)

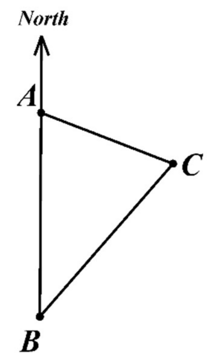
So $\measuredangle BCA_1=81.24°$ (2 d.p.) and $\measuredangle BCA_2=14.76°$ (2 d.p.)

$$\frac{BA_1}{\sin 81.24°}=\frac{16}{\sin 42°}\Rightarrow BA_1=23.6 \text{ km (3 s.f.)}$$

$$\frac{BA_2}{\sin 14.76°}=\frac{16}{\sin 42°}\Rightarrow BA_2=6.09 \text{ km (3 s.f.)}$$

(ii) Taking the shorter distance between *A* and *B*, determine the bearing of *A* from *C*. **Solution:** Bearing of A_2 from $C = 180° + 42° + 14.8°$ $= 236.8°$ (1 d.p.)	[2]
Question 4 [2011 EOY, Q7]	marks
The variables *x* and *y* are related by the equation $\sqrt{2^y} = ay + \frac{b}{\sqrt{x}}$. A straight line passes through (5, 7) is obtained when $\sqrt{2^y x}$ is plotted against $y\sqrt{x}$. (i) Given that the gradient of this line is 2, calculate the value of *a* and of *b*. **Solution:** $\sqrt{2^y} = ay + \frac{b}{\sqrt{x}}$ $\Rightarrow \sqrt{2^y x} = ay\sqrt{x} + b$ $a = gradient = 2$ A1 $\Rightarrow (7) = 2(5) + b \Rightarrow b = -3$	[3]
(ii) Given that this line also passes through (*k*, 13), find the value of *k*. **Solution:** $\Rightarrow (13) = 2(k) - 3$ $\Rightarrow k = \frac{16}{2} = 8$	[2]
Question 5 [2011 EOY, Q15]	marks
In the diagram *A*, *B* and *S* represent three points in the sea. *B* is 3 km from *A* on a bearing of $060°$. The bearings of *S* from *A* and *B* are $110°$ and $155°$ respectively. Calculate the distance *AS*.  **Solution:** $\angle BAS = 110^0 - 60^0 = 50^0$ $\angle ABS = 60^0 + (180^0 - 155^0) = 85^0$ $\angle ASB = 180^0 - 85^0 - 50^0 = 45^0$ $\frac{AS}{\sin 85^0} = \frac{3}{\sin 45^0}$ $AS = \frac{3siin85^0}{\sin 45^0} = 4.226 \approx 4.23km$	[3]

(ii) A ship at *S* sails directly to *A* at a steady speed of 2 kmh^{-1}. After 40 minutes, the ship is at *T*, a point on *AS*. (a) Calculate the distance *BT*. **<u>Solution:</u>** Distance = Speed x Time $= 2 \times \frac{40}{60} = \frac{4}{3}$ km AT = 4.226 – 1.333 km = 2.893 km….M1 Using cosine rule, $BT^2 = 3^2 + 2.893^2 - 2(3)(2.893)\cos 50^0$ $BT = 2.492$ $BT = 2.49km$	[3]
(b) Find the bearing of *B* from *T*. **<u>Solution:</u>** $\frac{\sin \angle ABT}{2.892} = \frac{\sin 50^0}{2.492}$ $\angle ABT = \frac{2.892 \sin 50^0}{2.492} = 62.74^0$ $\angle y = 62.74^0 - 60^0 = 2.74^0$ *The bearing of B from T is* $360^0 - 2.74^0 = 357.26^0 \approx 357.3^0$	[3]
(iii) A hot air balloon, *H*, is lowering at a point 600 m vertically above ***B***. Calculate the greatest angle of elevation of ***H*** from an observer on the ship as the ship sails from ***S*** to ***A***. **<u>Solution:</u>** Shortest distance from observer to *B* = $3\sin 50^0$ $\theta = \tan^{-1}\left(\frac{0.6}{3\sin 50^0}\right)$ $\approx 14.6^0$	[2]

Question 6 [2012 EOY Q13]	marks
ABC represents a horizontal triangular field such that *A* is due north of *B*, *AB* = 240 m, *AC* = 180m and the bearing of *C* from *A* is 110°.  (i) Find the distance *BC*. **<u>Solution:</u>** **Using Cosine Rule** $BC^2 = 240^2 + 180^2 - 2(240)(180)\cos 70^\circ$ $BC \approx 245.864$ $BC \approx 246\ m (3 s.f.)$	[3]
(ii) Find the bearing of *C* from *B*. **<u>Solution:</u>** **Using Sine Rule** $\frac{\sin ABC}{180} = \frac{\sin 70^\circ}{BC}$ $\qquad A\hat{B}C = 43.468...$ Bearing of C from B is 043.5°.	[3]
(iii) A vertical tree, of height 25 m, is planted at *C*. A farmer walks along a footpath from *A* to *B*. Find the maximum angle of elevation of the top of the tree from the farmer during this walk. You may ignore the height of the farmer. **<u>Solution:</u>** Maximum angle of elevation occurs when distance between tree and farmer is the ***shortest.*** Let this shortest distance be *d*. $d = 180 \sin 70^\circ$ Let the maximum angle of elevation be θ $\tan\theta = \frac{25}{180\sin 70^\circ}$ $\theta = 8.4075...$ $\approx 8.4^\circ$	[4]

<table>
<tr><td>Question 7 [2013 EOY, Q12]</td><td>marks</td></tr>
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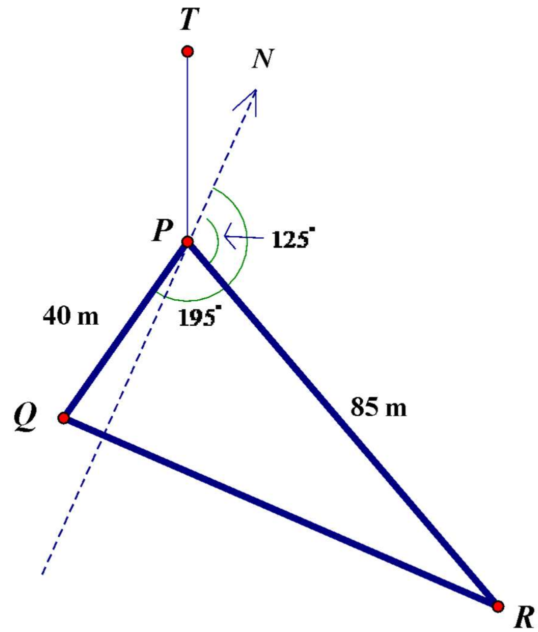
In the diagram, PQR represents a horizontal triangular field such that $PR = 85$ m and $PQ = 40$ m. The bearing of R from P and the bearing of Q from P is $125°$ and $195°$ respectively.

(i) Calculate the distance from Q to R.

Solution:

In $\Delta PQR,\quad \angle QPR = 195^{\circ} - 125^{\circ} = 70^{\circ}$

$$QR = \sqrt{40^2 + 85^2 - 2(40)(85)\cos 70°}$$
$$= \sqrt{1600 + 7225 - 2325.736}$$
$$= \sqrt{6499.24} = 80.61 \simeq 80.6 \text{ m}$$



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(ii) Calculate the area of the triangular field PQR.

Solution:

Area of the triangular field

$$\Delta PQR = \frac{1}{2} \times 40 \times 85 \times \sin 70°$$
$$= 1597.47 \simeq 1600 \text{ m}^2$$

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(iii) Calculate the shortest distance from P to the road QR.

Solution:

In $\Delta PQR,\quad \dfrac{40}{\sin \angle PRQ} = \dfrac{80.61}{\sin 70°}$

$$\Rightarrow \quad \angle PRQ = 27.79°$$

Hence the shortest distance from P to QR is $d = 85 \sin 27.79° = 39.629 \simeq 39.6$ m

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TP represents a building at the corner P. Given that the angle of depression of the top of the building T to the corner R is $20°$,

(iv) calculate the height of the building TP.

Solution:

In $\Delta PTR,\ \tan 20° = \dfrac{h}{85}$, where h is the height of the building

Hence $h = 85 \tan 20° = 30.93 \simeq 30.9$ m

</td><td>[2]</td></tr>
<tr><td>

A man walks along the road from Q to R,

(v) calculate the greatest angle of elevation of the top of the building.

Solution:

The biggest angle of elevation α is

$$\alpha = \tan^{-1}\left(\frac{30.93}{39.62}\right) = 37.97° \simeq 38.0°$$

</td><td>[2]</td></tr>
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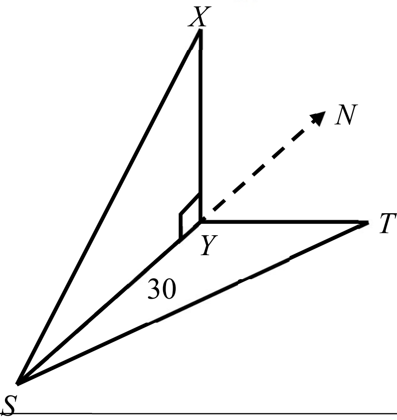
Question 8 [2014 EOY, Q14] — marks

Three points, S, Y and T are on level ground. Y is the foot of a vertical flagpole XY, and S is 30 m due south of Y.



(i) The angle of elevation of X from S is $25°$.
Calculate the height of the flagpole.

Solution:

Height of the flagpole XY

$= 30\tan 25°$ [M1]

$\approx 13.989 = 14.0$ m (3 s.f.)

[2]

(ii) The bearing of T from S and the bearing of T from Y are $042°$ and $075°$ respectively, calculate the distance ST.

[3]

Solution:

The bearing of T from S and the bearing of T from Y are $042°$ and $075°$ respectively.

$\Rightarrow \angle YST = 42°, \angle SYT = 180° - 75° = 105°$

$\therefore \angle YTS = 180° - 42° - 105° = 33°$

Alternatively, by stating that $\angle NYT = 75°$,

$\angle YTS = 75° - 42° = 33°$

$$\frac{ST}{\sin 105°} = \frac{SY}{\sin 33°}$$

$$\therefore ST = \frac{30}{\sin 33°} \times \sin 105°$$

$\approx 53.205 = 53.2$ m (3 s.f.)

(iii) A man is standing at the midpoint of ST. Calculate the angle of elevation of X from the man.

[5]

Solution:

Let the midpoint of S and T be M.

$$SM = \frac{1}{2} \times ST \approx 26.6025$$

Using Cosine Rule,

$MY^2 = SY^2 + SM^2 - 2 \times SY \times SM \times \cos \angle YSM$

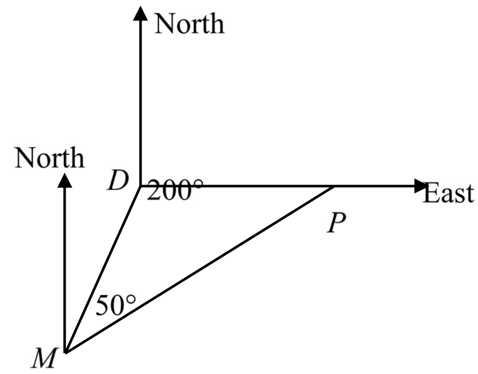
$\therefore MY = \sqrt{30^2 + 26.6025^2 - 2 \times 30 \times 26.6025 \times \cos 42°}$

≈ 20.531

Angle of elevation of X from the man

$$= \tan^{-1}\frac{XY}{MY} = \tan^{-1}\frac{13.989}{20.531}$$

$= 34.3°$

Question 9 [2015 EOY, Q11]	marks
A man, at point M, is 80 metres at a bearing of $200°$ from an observation deck, D, which stands 25 metres tall. He begins to walk to a point P which is due East of D at a bearing of $050°$.  Ignoring the man's height, calculate (i) the angle of elevation of the top of the deck from the man at M, **Solution:** Angle of elevation $= \tan^{-1}\dfrac{25}{80}$ $= 17.4°$ (1 d.p.)	[2]
(ii) the distance he walks when the angle of elevation of the top of the deck from his position is the greatest, and **Solution:** $\measuredangle DMP$ $= 50° - (200° - 180°)$ $= 30°$ Distance he walks $= 80\cos 30°$ m $= 40\sqrt{3}$ or 69.3 (3 s.f.) m	[3]
(iii) the distance DP. **Solution:** By Sine Rule, $\dfrac{DP}{\sin 30°} = \dfrac{80 \text{ m}}{\sin(180° - 30° - 110°)}$ $\Rightarrow DP = \dfrac{80}{2\sin 40°}$ m $\Rightarrow DP = 62.2$ (3 s.f.) m	[2]