

Name: _____ ()

Class: _____



華僑中學

HWA CHONG INSTITUTION

Paper E

INTEGRATED MATHEMATICS

Level : Secondary Three

Duration : 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 7 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

- 1** **(a)** Simplify $\frac{8x^3+1}{(2x-3)(2x-1)} \div \left(2x - \frac{1}{1-2x}\right)$. [3]
- (b)** Find the values of x satisfying both inequalities $-1 < 4x+5 \leq 3$ and $(x+4)(5-x) \geq 18$. [4]
- 2** **(a)** Find the value of x , given that $8^{2x} = \frac{2^{x-2}}{2^3}$. [3]
- (b)** Using a suitable substitution, or otherwise, solve for x if $4^{x+\frac{1}{2}} = 7(2^x) - 5$. [4]
- 3** Solve each of the following equations.
- (a)** $\log_{\sqrt{5}} x + 2 \log_x \sqrt{5} = 3$ [3]
- (b)** $\sqrt{9-4x} = 2 + \sqrt{5-x}$ [4]
- 4** **(a)** Express $2x^2 - 6x + 13$ in the form $a(x-b)^2 + c$, where a , b and c are constants. [2]
- (b)** Given that $f(x) = x^2 + 2x + k^2 - kx$, find the range of values of k for which $f(x) > 0$ for all real values of x . [4]
- 5** The equation of a circle C_1 is $5y^2 - 20y + 125 = 50x - 5x^2$. Find
- (i)** the coordinates of the centre of the circle C_1 and its radius, [3]
- (ii)** the equation of another circle C_2 which has the same centre as the circle C_1 and passes through the point $(5, -7)$. [2]
- 6** The cubic polynomial $f(x)$ has roots 3 , -3 and $-k$.
- (i)** Given that the coefficient of x^3 is 2 , and that $f(x)$ has a remainder of 8 when divided by $x + 1$, find the value of k . [3]
- (ii)** Find the remainder when $f(x)$ is divided by $x - 5$. [2]

[Turn Over]

7 (a) Given $\mathbf{B} = \begin{pmatrix} 5 & -4 \\ 1 & 1 \end{pmatrix}$, find $\mathbf{P}$ if $\mathbf{P} = 3 \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \mathbf{B}^2$. [2]

(b) The table below shows the price of the items for sale in two mini-markets in Ponggol.

Product	Price in mini-market A	Price in mini-market B
Eggs	\$3.30 per tray of 6	\$3.20 per tray of 6
Meat	\$1.90 per 100 g	\$2.00 per 100 g
Milk	\$3.40 per litre	\$3.45 per litre

Mr Tan's shopping list consists of 2 trays of eggs, 500 g of meat and 3 litres of milk.

If $\mathbf{C} = \begin{bmatrix} 3.30 & 3.20 \\ 1.90 & 2.00 \\ 3.40 & 3.45 \end{bmatrix}$, determine matrix $\mathbf{Q}$ such that the amount spent

in each mini-market can be calculated using matrices $\mathbf{C}$ and $\mathbf{Q}$.

Find the amount spent in each mini-market and hence conclude which mini-market to shop for the items. [3]

8 Functions f and g are defined by

$$f : x \mapsto 2x - 7 \text{ and}$$

$$g : x \mapsto \frac{3x}{x-2}, x \neq 2.$$

Solve the equations

(i) $g^{-1}(x) = f(3)$, [3]

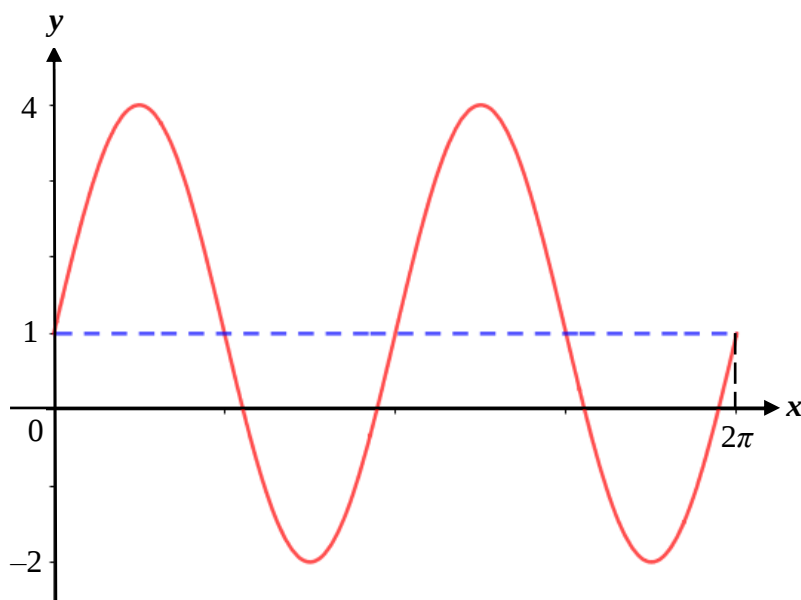
(ii) $|f(x)| = |x - 3|$. [3]

9 The roots of the quadratic equation $2x^2 + 7x + 14 = 0$ are α and β .

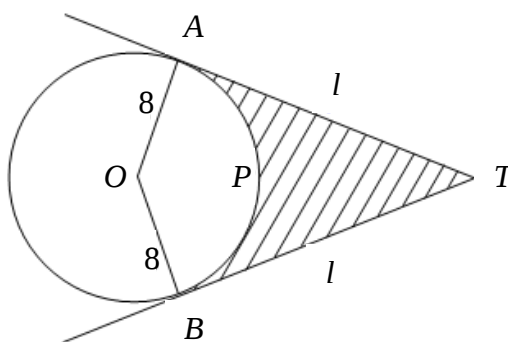
(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Find the quadratic equation whose roots are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$. [3]

- 10 The figure below shows the graph of $y = a \sin bx + c$ for $0 \leq x \leq 2\pi$, where a , b and c are integers. State the values of a , b and c . [3]



- 11 The figure below, not drawn to scale, shows a circle with centre O and radius 8 m. Two tangents, TA and TB , have length l m each. P is a point on the arc AB .



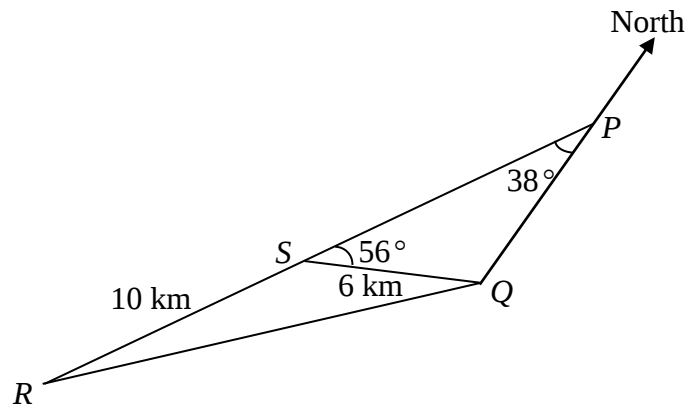
- (i) Write $\angle AOT$ in terms of l . [1]
- (ii) Show that the area, $S \text{ m}^2$, of the shaded region is given by

$$S = 8l - 64 \tan^{-1}\left(\frac{l}{8}\right).$$
 [2]
- (iii) Find the perimeter of the shaded region $ATBP$ in terms of l . [2]
- (iv) The figure above lies on horizontal ground such that a pole of length 12 m stands vertically at O . Find the angle of elevation, in degrees, of the top of the pole from T when $l = 8$. [2]

[Turn Over]

- 12 (a) Given that $\frac{1+\tan x}{1-\tan x} = \sqrt{3}$, express $\tan x$ in exact form. [2]
- (b) Find all angles which satisfy the equation
- (i) $4\sin 2x = -1$, $0^\circ \leq x \leq 180^\circ$, [2]
- (ii) $3\cos^2 x = 4\sin x \cos x$, $0 \leq x \leq 2\pi$. [3]

- 13 In the diagram P , Q , R and S are points on level ground. P , S and R lie on a straight line and P is due north of Q .



Given that $SQ = 6$ km and $RS = 10$ km, $\angle QPS = 38^\circ$ and $\angle PSQ = 56^\circ$, calculate

- (i) QR , [2]
- (ii) $\angle SQR$, [2]
- (iii) the bearing of S from R , [1]
- (iv) the shortest distance from S to QR . [2]
- 14 (i) Given that $f(x) = \ln x$ and $g(x) = 2\ln(3+x)$, describe the transformation of the graph of $f(x)$ to the graph of $g(x)$ in two successive steps. [2]
- (ii) Sketch the graph of $y = 2\ln(3+x)$, showing clearly the asymptote and the x -intercept. [2]
- (iii) Find the equation of the straight line to be drawn and explain how the graphical solution of the equation $e^2 = (x+3)e^{\frac{x}{2}+1}$ could be obtained. [2]

15 Solution to this question by accurate drawing will not be accepted.

ABC is an isosceles triangle with $A(6,6)$, B lies on the x -axis and $AB = AC$.

Given that the equation of BC is $2y = x - 4$ and M is the midpoint of BC , find

- (i) the coordinates of B , C and M , [5]
- (ii) the equation of the perpendicular bisector of BC . [2]
- (iii) Given that coordinates of D are $(10, -2)$, determine the name of this quadrilateral $ABDC$. [2]
- (iv) Find the area of quadrilateral $ABDC$. [2]

16 Answer the whole of this question on a sheet of graph paper.

Two positive variables x and y are related by the equation $y^2 = ae^{bx}$, where a and b are unknown constants.

The table shows some experimental values of the two variables.

x	0.5	1	1.5	2	2.5	3	3.5	4
y	2.87	3.69	4.73	6.08	7.80	10.6	12.9	16.5

- (i) Using a scale of 4 cm to 1 unit on both axes, plot $\ln y$ against x . [3]
- (ii) Use your graph to estimate the values of a and b . [3]
- (iii) Use your graph to estimate the value of y when $x = 1.8$. [1]

End of Paper