A Math Practise solution :)

1. $f(x) = 4x^3 + mx^2 + nx - 3$

$(x-1)$ is a factor, so

$f(1) = 0$

$4(1)^3 + m(1)^2 + n(1) - 3 = 0$

$m + n = -1$ — ① M_1

$(x+1)$ leaves a remainder of 2, so

$f(-1) = 2$

$4(-1)^3 + m(-1)^2 + n(-1) - 3 = 2$

$m - n = 9$ — ② — M_2

From ①,

$m = -1 - n$ — ③ — M_3

Sub ③ into ②

$(-1-n) - n = 9$

$-2n = 10$

$n = -5$ — A_1

Sub $n = -5$ into ③

$m = -1 - (-5)$

$m = 4$ — A_2

$m = 4$

$n = -5$

2. $\int_1^2 x^2 + 1 - \frac{2}{x^2}\, dx$

$= \int_1^2 x^2 + 1 - 2x^{-2}\, dx$

$= \left[\frac{x^3}{3} + x - \frac{2x^{-1}}{-1}\right]_1^2$ — M_1

$= \left(\frac{2^3}{3} + 2 - \frac{2(2)^{-1}}{-1}\right) - \left(\frac{1^3}{3} + 1 - \frac{2(1)^{-1}}{-1}\right)$ — M_2

$= \frac{7}{3}$ // — A_1

3) a. $\frac{dy}{dx} = 9x^2 - 4$ — A_1

b. $\frac{dy}{dx} = 0$

$9x^2 - 4 = 0$

$9x^2 = 4$

$x^2 = \frac{4}{9}$

$x = \pm\frac{2}{3}$

$x = \frac{2}{3}$ OR $x = -\frac{2}{3}$ — M_1

$y = 3\left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right) - 4$ $\qquad$ $y = 3\left(-\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right) - 4$

$y = -\frac{52}{9}$ $\qquad$ $y = -\frac{20}{9}$ — M_2

$\left(\frac{2}{3}, -\frac{52}{9}\right)$ // $\qquad$ $\left(-\frac{2}{3}, -\frac{20}{9}\right)$ // — A_1

c. $\frac{d^2y}{dx^2} = 18x$

$18\left(\frac{2}{3}\right)$ $\qquad$ $18\left(-\frac{2}{3}\right)$

$= 12$ $\qquad$ $= -12$ — M_1

$\left(\frac{2}{3}, \frac{-52}{9}\right)$ is a minimum point — A_1

$\left(-\frac{2}{3}, -\frac{20}{9}\right)$ is a maximum point — A_2

4) a. $(\sin x + \cos x)^2 = 1 + \sin 2x$

LHS $= (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x$ — M_1

$= 1 + \sin 2x$

$=$ RHS (proven) — A_1

b) $1 + \sin 2x = \frac{3}{2}$, $\quad 0 \le x \le 360$

$0 \le 2x \le 720$

$\sin 2x = \frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$ — M_1

$= 30°$

$2x = 30°$ OR $2x = 180 - 30$ OR $2x = 360 + 30$ OR $2x = 540 - 30$

$x = 15°$ // $\quad x = 75°$ // $\quad x = 195°$ // $\quad x = 255°$ //

A_1 $\qquad$ A_2

5. $x - y = 3$ — ①

$x^2 + y^2 - 3xy + 19 = 0$ — ②

From ①,

$x = 3 + y$ — ③

Sub ③ into ②.

$(3+y)^2 + y^2 - 3y(3+y) + 19 = 0$ — M_1

$y^2 + 6y + 9 + y^2 - 9y - 3y^2 + 19 = 0$

$-y^2 - 3y + 28 = 0$

$y^2 + 3y - 28 = 0$

$(y-4)(y+7) = 0$ — M_2

$y = 4$ OR $y = -7$

when $y = 4$, $\qquad$ when $y = -7$,

$x = 3 + 4$ $\qquad$ $x = 3 - 7$

$x = 7$ $\qquad$ $x = -4$

$P(7, 4)$ M_3 and $Q(-4, -7)$ M_4

Midpoint $PQ = \left(\frac{7 + (-4)}{2}, \frac{4 + (-7)}{2}\right)$

$= \left(\frac{3}{2}, -\frac{3}{2}\right)$ //

A_1

6. a) not thrown, then $t = 0$

$h = -9(0)^2 + 12(0) + 1$

$h = 1\text{m}$ — A_1

b) $h = -9t^2 + 12t + 1$

$h = -9\left(t^2 - \frac{4}{3}t - \frac{1}{9}\right)$ — M_1

$h = -9\left(t^2 - \frac{4}{3}t + \left(\frac{-\frac{4}{3}}{2}\right)^2 - \left(\frac{-\frac{4}{3}}{2}\right)^2 - \ldots\right.$

$h = -9\left[\left(t - \frac{2}{3}\right)^2 - \frac{5}{9}\right]$

$h = -9\left(t - \frac{2}{3}\right)^2 + 5$ // — A_1

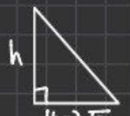
c) Max height

5m //

Time $= \frac{2}{3}$ s

A

7.



Area $= 16+6\sqrt{2}$

$\frac{1}{2}(4-2\sqrt{2})h = 16+6\sqrt{2}$ — M1

$h = \frac{32+12\sqrt{2}}{4-2\sqrt{2}}\left(\frac{4+2\sqrt{2}}{4+2\sqrt{2}}\right)$ — M2 + M3

$h = \frac{128+112\sqrt{2}+24(2)}{8}$

$h = \frac{176+112\sqrt{2}}{8}$ — A1

$h = 22+14\sqrt{2}$ cm

8. a) $\frac{d}{dx}\left[(3x+2)\sqrt{x+3}\right]$

Let $u = 3x+2$ and $v = (x+3)^{\frac{1}{2}}$

$\frac{du}{dx} = 3$ M1 $\frac{dv}{dx} = \frac{1}{2}(x+3)^{-\frac{1}{2}}$

$\frac{d}{dx}\left[(3x+2)\sqrt{x+3}\right] = 3(x+3)^{\frac{1}{2}} + \frac{1}{2}(3x+2)(x+3)^{-\frac{1}{2}}$ — M1

$= \frac{1}{2}(x+3)^{-\frac{1}{2}}\left[6(x+3)+3x+2\right]$

$= \frac{6x+18+3x+2}{2\sqrt{x+3}}$

$= \frac{9x+20}{2\sqrt{x+3}}$ — A1

b) $\int_1^3 \frac{9x+22}{2\sqrt{x+3}}\,dx = \int_1^3 \frac{9x+20}{2\sqrt{x+3}}\,dx + \int_1^3 \frac{2}{2\sqrt{x+3}}\,dx$ — M1

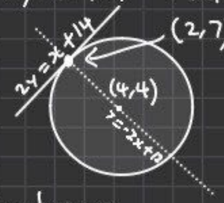
$= \left[(3x+2)\sqrt{x+3}\right]_1^3 + \int_1^3 (x+3)^{-\frac{1}{2}}\,dx$

$= \left((3(3)+2)\sqrt{3+3}\right) - \left((3(1)+2)\sqrt{1+3}\right) + \left[\frac{(x+3)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^3$ M2 + M3

$= 16.944... + \left(2(3+3)^{\frac{1}{2}} - 2(1+3)^{\frac{1}{2}}\right)$

$= 16.0$ (3 s.f.) — A1

9. $2y = x+14 \rightarrow$ tangent



$y = \frac{1}{2}x+7$

Gradient line $= \frac{1}{2}$

Gradient $\perp = -2$

equation of normal $= y-4 = -2(x-4)$

$y = -2x+12$ — M1

1 point of circle = point of intersection of the 2 lines

$\frac{1}{2}x+7 = -2x+12$ — M2

$\frac{5}{2}x = 5$

$x = 2$

$y = \frac{1}{2}(2)+7$

$y = 8$ — M3

$\therefore (2,8)$

Radius $= \sqrt{(4-2)^2+(4-8)^2}$ — M4

$= \sqrt{20}$

Equation of circle $= (x-4)^2+(y-4)^2 = 20$ — A5

empty space

10. $\frac{d^2y}{dx^2} = 12x^2-6$

gradient when $(1,5) = 7$

$\frac{dy}{dx} = \int 12x^2-6\,dx$

$= \frac{12x^3}{3} - 6x + C$, where C is a constant — M1

when $x=1$, $\frac{dy}{dx} = 7$

$\frac{12(1)^3}{3} - 6(1) + C = 7$

$C = 9$ — M2

$\frac{dy}{dx} = 4x^3-6x+9$

$y = \int 4x^3-6x+9\,dx$

$= \frac{4x^4}{4} - \frac{6x^2}{2} + 9x + C$ — M3

when $y=5$, $x=1$.

$5 = \frac{4(1)^4}{4} - \frac{6(1)^2}{2} + 9(1) + C$

$C = -2$ — M4

$y = x^4 - 3x^2 + 9x - 2$ — A1

11. a) $8x^3-125$

$= (2x)^3 - 5^3$

$= (2x-5)(4x^2+10x+25)$ — A2

b) $(2x-5)(4x^2+10x+25) = 0$

$2x-5=0$ OR $4x^2+10x+25=0$

$x = \frac{5}{2}$ $\quad b^2-4ac$

$= 10^2-4(4)(25)$

M1 — $= -300$

$\therefore$ Since b^2-4ac of $4x^2+10x+25 = -300$ which is negative, it has no real solutions, therefore $x = \frac{5}{2}$ is the only real solution in $8x^3-125=0$ — A1

12. $\frac{8x^3-x^2-12x+12}{x^2-9}$

$$\begin{array}{r} 8x-1 \\ x^2-9\,\big)\overline{\,8x^3-x^2-12x+12} \\ -(8x^3 \quad\quad -72x) \\ \hline -x^2+60x+12 \\ -(-x^2 \quad\quad +9) \\ \hline 60x+3 \end{array}$$

$= 8x-1 + \frac{60x+3}{x^2-9}$ — M1

From $\frac{60x+3}{(x+3)(x-3)}$,

Let $\frac{60x+3}{(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{x-3}$ — M2

$60x+3 = A(x-3)+B(x+3)$ — M3

Let $x=-3$

$-6A = -177$

$A = \frac{59}{2}$ — M4

Let $x=3$,

$183 = 6B$

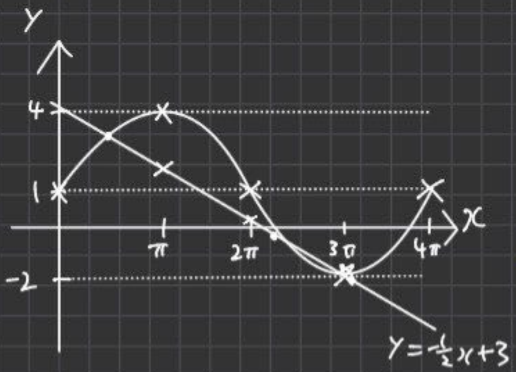
$B = \frac{61}{2}$ — M5

$\therefore \frac{8x^3-x^2-12x+12}{x^2-9} = \frac{59}{2(x+3)} + \frac{61}{2(x-3)} + 8x-1$ — A1

13. $y = 3\sin\frac{x}{2} + 1$

a) amplitude $= 3$ — A1

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$ — A2

b) 

— A3

c) $3\sin\frac{x}{2} = -\frac{1}{2}x + 2$

$3\sin\frac{x}{2} + 1 = -\frac{1}{2}x + 3$

$y = -\frac{1}{2}x + 3$

draw $y = -\frac{1}{2}x + 3$ — M1

x	π	2π	3π
y	1.429..	-0.141..	-1.712

Based on the graph, 3 solutions. — A2

end of answer :)