

Topical Revision Worksheet 3

Topic: Logarithm

Question 1 [2008 EOY, Q7]	marks
Solve the following equations for x: $\log_2(x+6) + \log_2(x-1) = 1 + \log_2(3x)$ <u>Solution:</u> $\log_2(x+6)(x-1) = \log_2 2 + \log_2(3x)$ $\log_2(x+6)(x-1) = \log_2 6x$ $(x+6)(x-1) = 6x \Rightarrow x^2 - x - 6 = 0$ so $x = 3$ $x = -2$ is rejected because $\log_2(3x)$ is undefined	[4]
Question 2 [2009 EOY, Q10]	marks
Solve the following equations: (a) $x^2 - 6 = x$, <u>Solution:</u> $(x^2 - 6)^2 = (x)^2$ $x^4 - 12x^2 + 36 = x^2$ $x^4 - 13x^2 + 36 = 0$ $(x^2 - 9)(x^2 - 4) = 0$ $x^2 = 9 \text{ or } x^2 = 4$ $x = \pm 3 \text{ or } \pm 2 \text{ (rej)}$	[4]
(b) $3^{2x-1} + 3^{x-2} = 3^x - \frac{4}{9}$, <u>Solution:</u> $3^{2x} \cdot 3^{-1} + 3^x \cdot 3^{-2} = 3^x - \frac{4}{9}$ Let $u = 3^x$ $\frac{1}{3}u^2 + \frac{1}{9}u = u - \frac{4}{9}$ $3u^2 + u = 9u - 4$ $3u^2 - 8u + 4 = 0$ $(3u - 2)(u - 2) = 0$ $u = \frac{2}{3} \text{ or } 2$ $3^x = 2$ $x = \frac{\lg 2}{\lg 3} \text{ or } 0.631$ $3^x = \frac{2}{3}$ $x = \frac{\lg \frac{2}{3}}{\lg 3} \text{ or } -0.369$	[5]

(c) $1 + \lg(x + 1.6) = \lg 2 + \lg(2x^2 + 1)$. <u>Solution:</u> $\lg 10 + \lg(x + 1.6) = \lg 2 + \lg(2x^2 + 1)$ $\lg[10(x + 1.6)] = \lg[2(2x^2 + 1)]$ $10x + 16 = 4x^2 + 2$ $4x^2 - 10x - 14 = 0$ $2x^2 - 5x - 7 = 0$ $(2x - 7)(x + 1) = 0$ $x = -1 \text{ or } \frac{7}{2}$ Optional Check: $x = -1 \qquad x = 3.5$ $\lg(0.6) \rightarrow \text{defined} \qquad \lg(5.1) \rightarrow \text{defined}$ $\lg(3) \rightarrow \text{defined} \qquad \lg(25.5) \rightarrow \text{defined}$	[4]
Question 3 [2010 EOY, Q13]	marks
Solve the following equations: (a) $\log_8[2 + \log_2(x - 4)] = \log_{125} 5$ <u>Solution:</u> $\log_8[2 + \log_2(x - 4)] = \log_{125} 5 \Rightarrow \log_8[2 + \log_2(x - 4)] = \frac{1}{3}$ $\Rightarrow \sqrt[3]{8} = 2 + \log_2(x - 4)$ $\Rightarrow \log_2(x - 4) = 0$ $\Rightarrow x - 4 = 1$ $\Rightarrow x = 5$	[3]
(b) $e^{2 \ln y} = \lg 1.6$ <u>Solution:</u> $e^{2 \ln y} = \lg 1.6 \Rightarrow 2 \ln y = \ln(\lg 1.6)$ $\Rightarrow \ln y = \frac{\ln(\lg 1.6)}{2}$ $\Rightarrow y = e^{\frac{\ln(\lg 1.6)}{2}}$ $\Rightarrow y = 0.452 \text{ (3 s.f.)}$	[3]

Question 4 [2010 EOY, Q14]	marks
Solve the pair of simultaneous equations $e^x(e^2) = (e^2)^y$ $\lg(x-1) + \lg(x+1) = \lg y + \lg(2x-y)$ <u>Solution:</u> $e^x(e^2) = (e^2)^y \Rightarrow x+2 = 2y, \text{ and}$ $\lg(x-1) + \lg(x+1) = \lg y + \lg(2x-y) \Rightarrow (x-1)(x+1) = y(2x-y)$ Hence $(2y-2-1)(2y-2+1) = y(4y-4-y) \Rightarrow (2y-3)(2y-1) = y(3y-4)$ $\Rightarrow y^2 - 4y + 3 = 0$ $\Rightarrow y = 1 \text{ or } 3$ But if $y = 1$, $x = 0$ and $\lg(x-1)$ will be undefined, hence solution rejected When $y = 3$, $x = 4$	[6]
Question 5 [2011 EOY, Q13]	marks
Solve the following equations $\log_2 \left(\frac{x^2}{4} + 12 \right) = 2 \log_2 (x+5) - 2$ (a) , <u>Solution:</u> $\log_2 \left(\frac{x^2}{4} + 12 \right) = \log_2 (x+5)^2 - \log_2 4$ $\log_2 \left(\frac{x^2}{4} + 12 \right) = \log_2 \frac{(x+5)^2}{4}$ $\left(\frac{x^2}{4} + 12 \right) = \frac{(x+5)^2}{4}$ $(x^2 + 48) = (x+5)^2 \quad x = 2\frac{3}{10}$	[4]

(b) $\log_4(x^2) + 3\log_{x^2} 4 = 4$ Solution: $\log_4 y + 3\log_y 4 = 4$ $\log_4 y + 3\left[\frac{\log_4 4}{\log_4 y}\right] = 4$ $(\log_4 y)^2 + 3 = 4(\log_4 y)$ Let $a = \log_4 y$ $a^2 - 4a + 3 = 0$ $(a-3)(a-1) = 0$ $a = 3, a = 1$ $\log_4 y = 3, \log_4 y = 1$ $y = 4^3 = 64, y = 4$ $x^2 = 64 \Rightarrow x = \pm 8, x^2 = 4 \Rightarrow x = \pm 2$	[4]
Question 6 [2012 EOY, Q3]	marks
Solve the equation $\log_x(2x+1) - 2\log_{x^2}(4x-1) + 1 = 0$. Solution: $\log_x(2x+1) - \log_x(4x-1) = -1$ $\log_x \frac{2x+1}{4x-1} = -1$ $\frac{2x+1}{4x-1} = \frac{1}{x}$ $2x^2 + x = 4x - 1$ $2x^2 - 3x + 1 = 0$ $x = \frac{1}{2} \text{ or } x = 1 \text{ (rej. } \because x \neq 1)$	[4]
Question 7 [2013 EOY, Q1c]	marks
Solve the equation $\log_5 x + 1 = 6\log_x 5$. Solution: $\log_5 x + 1 = \frac{6}{\log_5 x}$ $(\log_5 x)^2 + \log_5 x - 6 = 0$ $(\log_5 x + 3)(\log_5 x - 2) = 0$ $\log_5 x = -3 \text{ or } \log_5 x = 2$ $x = \frac{1}{125} \text{ or } x = 25$	[3]

Question 8 [2014 EOY, Q3]	marks
(a) Solve the equations $\log_{(x-1)}(3x^2 - 7x - 2) = 2$ <u>Solution:</u> $\log_{(x-1)}(3x^2 - 7x - 2) = 2$ $\log_{(x-1)}(3x^2 - 7x - 2) = \log_{(x-1)}(x-1)^2$ $3x^2 - 7x - 2 = x^2 - 2x + 1$ $2x^2 - 5x - 3 = 0$ $(2x+1)(x-3) = 0$ $x = -\frac{1}{2}$ (rejected) or 3	[4]
(b) $\ln \sqrt{x} - 2 + \frac{1}{2}\ln x = 0$, where $x > 0$ <u>Solution:</u> $\ln \sqrt{x} - 2 + \frac{1}{2}\ln x = 0$, where $x > 0$ $\ln \sqrt{x} - 2 + \ln\sqrt{x} = 0$ $\ln x - 2\sqrt{x} = 0$ $x - 2\sqrt{x} = 1$ $x - 2\sqrt{x} = 1$ or -1 (Two cases) Case 1: $x - 2\sqrt{x} = 1$ $2\sqrt{x} = x - 1$ $x^2 - 2x + 1 = 4x$ $x^2 - 6x + 1 = 0$ $x = 3 \pm 2\sqrt{2}$ (reject $3 - 2\sqrt{2}$) OR $x = 5.83$ or 0.172 (rejected, as it does not satisfy the Case 1 equation) [A1] Case 2: $x - 2\sqrt{x} = -1$ or $x - 2\sqrt{x} + 1 = 0$ $2\sqrt{x} = x + 1$ $(\sqrt{x} - 1)^2 = 0$ $x^2 + 2x + 1 = 4x$ $x = 1$ $(x - 1)^2 = 0$ $x = 1$ Hence $x = 3 + 2\sqrt{2}$ or 1	[5]

Question 9 [2015 EOY, Q6]	marks
Solve the following equations. (a) $\log_3 x = \ln 2$, <u>Solution:</u> $\log_3 x = \ln 2$ $x = 3^{\ln 2} \approx 2.14$ (3 s.f.)	[2]
(b) $1 + \log_{\sqrt{2}}(y - 1) = \log_2(22 - 7y)$. <u>Solution:</u> $1 + \log_{\sqrt{2}}(y - 1) = \log_2(22 - 7y)$ $\Rightarrow \log_2 2 + \log_2(y - 1)^2 = \log_2(22 - 7y)$ Hence $2(y - 1)^2 = 22 - 7y \Rightarrow 2y^2 + 3y - 20 = 0$ Solving, $y = \frac{5}{2}$ or -4 (rej.)	[4]