Anglo-Chinese School

(Independent)

FINAL EXAM 2020

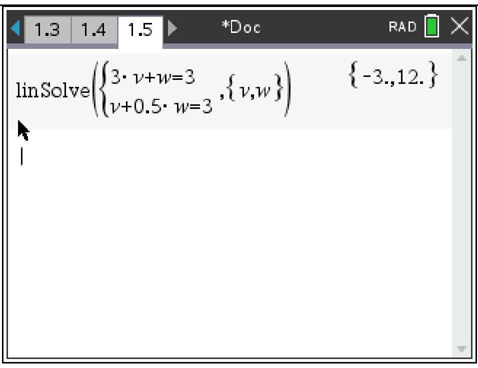
YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

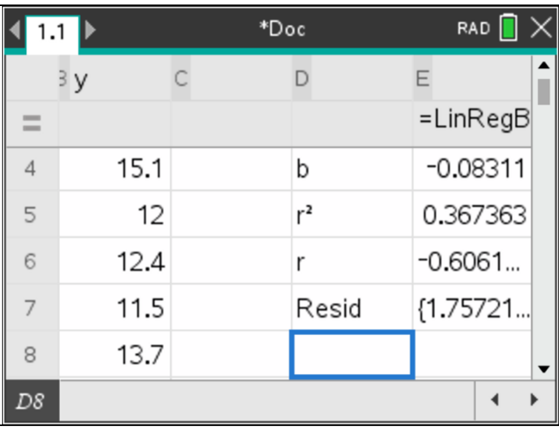
Paper 2 **MARKING SCHEME**

SECTION A

Qn	Solution
1.	
	Eqn 1 : $x^3 y = 8$ $\Rightarrow \quad \log_2 x^3 + \log_2 y = \log_2 8$
	$\Rightarrow \quad 3\log_2 x + \log_2 y = 3$
	Eqn 2 : $\log_2 x + \log_4 y = 3$
	$\Rightarrow \quad \log_2 x + \frac{1}{2}\log_2 y = 3$
	Using GDC, letting $v = \log_2 x$ and $w = \log_2 y$
	 $v = -3$ and $w = 12$
	So $\log_2 x = -3 \quad \Rightarrow \quad x = 2^{-3} = \frac{1}{8}$
	$\log_2 y = 12 \quad \Rightarrow \quad y = 2^{12} = 4096$
	Comments : Generally well attempted.

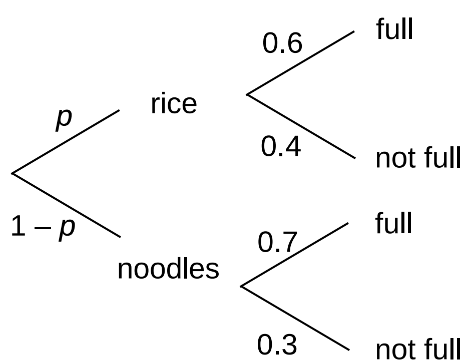
Qn	Solution
2.	
	Let the two numbers be $2k_1+1, 2k_2+1$
	$(2k_1+1)(2k_2+1)=4k_1k_2+2k_1+2k_2+1$ $=2(2k_1k_2+k_1+k_2)+1$
	Comments : generally poorly attempted. Common mistakes included : 1. Letting the two odd numbers be $(2k+1)(2k+1)$ 2. Letting the two odd numbers be consecutive $(2k-1)(2k+1)$ 3. Letting n be an even number and then $(n-1)$ is an odd number As they do not fully prove the result on its own. A few students attempted this question by substituting values. This reveals the students' incorrect concepts as this technique does not prove the result for a high generality. A few students attempted to prove this question using contrapositive. While this attempt is commendable, they proved the result that "the product of two even numbers is even" instead of the correct contrapositive statement "suppose the product of two numbers is even, prove that the two numbers are both even".

Qn	Solution
3.	
(a)	From GDC,



	The product moment correlation coefficient is -0.606 (3 sf)
	There is moderate negative correlation
(b)	$y=-0.0831x+27.5404$
(c)	(i) $y=-0.0831(168)+27.5404=13.6$
	(ii) We find the regression line of x on y
	From GDC,
	The regression line is $x=229.219-4.4202y$
	So the estimated height when $y=100$ is $-4.4202(100)+229.219=177 \quad (3\text{ sf})$
	Comment : a surprising number of students used the regression line of y on x to answer part (c), when the regression line of x on y ought to be used. For (a), the key word markers were looking out for was "moderate".

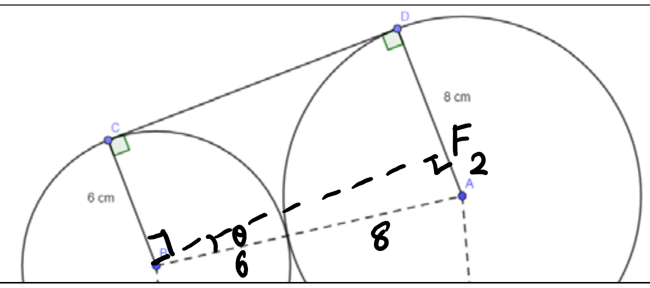
Qn	Solution
4.	
(a)	There are 2! ways of arranging the man and woman.
	So there are $2!\times 13!=12454041600$ ways
(b)	Fix the position of the women. There are 8! ways of doing so
	___ W ___ W ___ W ___ W ___ W ___ W ___ W ___ W ___
	Now fix the position of the men in the gaps, for doing so, no two men will be standing next to each other. There are ${}^{9}P_{6}$ ways of doing so
	The number of ways is ${}^{9}P_{6}\times 8!=2438553600$
	Comment : generally well attempted. Some students rounded the answer off to 3 significant figures when there is no need to as the answer is exact. Students must be reminded that they only need to round their answer off to 3 significant figures when their answers are not exact.

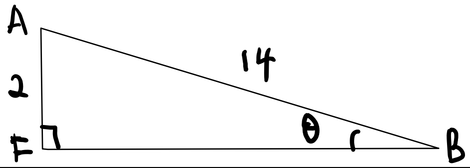
Qn	Solution
5.	
(a)	Draw a probability tree diagram:
	
	$P(\text{Ryan is full})=0.64$
	So $p(0.6)+(1-p)(0.7)=0.64$
	$0.6p+0.7-0.7p=0.64$ $0.1p=0.06$ $p=0.6$
(b)	$P(\text{Ryan eats chicken rice} \mid \text{he is not full})$ $=\dfrac{P(\text{Ryan eats chicken rice and he is not full})}{P(\text{Ryan is not full})}$ $=\dfrac{0.6(0.4)}{0.6(0.4)+(0.4)(0.3)}$ $=0.667=\dfrac{2}{3}$
	Comment : generally well done. Some students were unable to identify that part (b) is a conditional probability, instead finding the probability that Ryan eats chicken rice and he is not full.

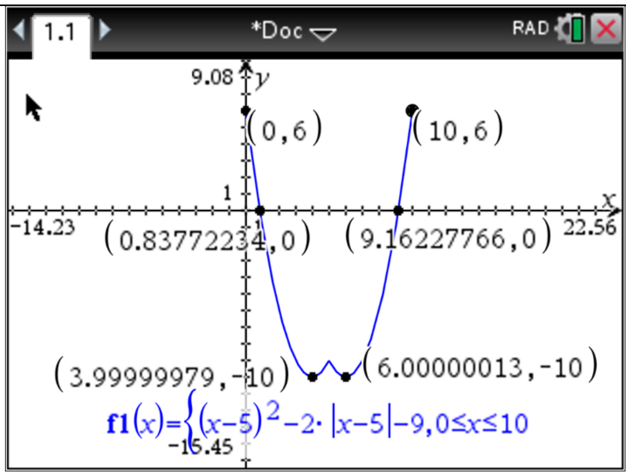
Qn	Solution
6.	
(a)	Let X be the number of times the target is hit in a training set.
	$X \sim B(6,0.68)$
	$P(X>3)=P(X\geq 4)=binomcdf(6,0.68,4,6)=0.706441=0.706$
	Comment : Some students incorrectly found $P(X\geq 3)$ costing them precious marks.
(b)	Let N be the number of training sets in a day where the archer will hit the target more than thrice.
	Then $N \sim B(8,0.706)$
	$E(N)=8(0.706)=5.65$
	Comment : generally well done.
(c)	Let Y be the number of times the target is hit out of n times
	From GDC,
	When $n=7$, $P(Y\geq 4)=0.8465$
	When $n=8$, $P(Y\geq 4)=0.925$
	Therefore number of trials required is 8.
	Comment : generally well attempted if students knew how to use their GDC. Students may have lost marks by getting engaged in repetitive calculations when the answer could have been obtained by a few steps on GDC. Students must realize that since GDCs are allowed for P2, they must know how the GDC can be used to make their lives easier.

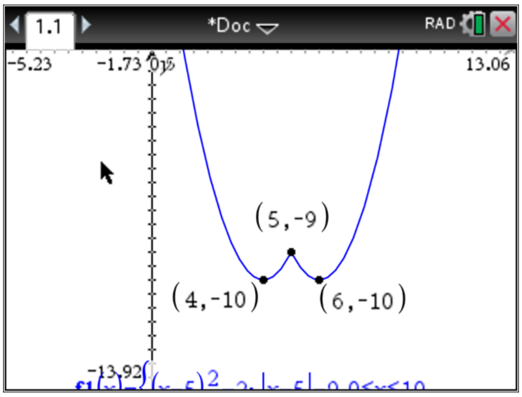
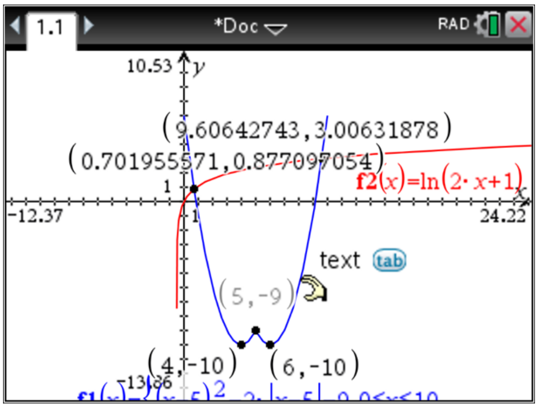
Qn	Solution
7.	
(a)	$P(X>61)=0.25$ $\Rightarrow \dfrac{61-\mu}{\sigma}=invNorm(0.75)$ $\Rightarrow 61-\mu=0.67449\sigma \qquad (1)$
	$P(X<41)=0.21$ $\Rightarrow \dfrac{41-\mu}{\sigma}=invNorm(0.21)$ $\Rightarrow 41-\mu=-0.80642\sigma \qquad (2)$
	Solve equation (1) and (2)
	$\mu=51.8909, \sigma=13.5052$
(b)	$invNorm(0.99,51.8909,13.5052)=83.3$
	Comment : generally well done

Qn	Solution
8.	
(a)	$\frac{z^*}{z^2}=\frac{\alpha-i\beta}{(\alpha+i\beta)^2}$ $=\frac{\alpha-i\beta}{\alpha^2-\beta^2+2\alpha\beta i}$ $=\frac{(\alpha-i\beta)(\alpha^2-\beta^2-2\alpha\beta i)}{(\alpha^2-\beta^2)^2+4\alpha^2\beta^2}$
	Since it is purely imaginary, $\mathrm{Re}\left(\frac{z^*}{z^2}\right)=0$
	So $\alpha^3-\alpha\beta^2-2\alpha\beta^2=0$
	$\alpha(\alpha^2-3\beta^2)=0$
	So either $\alpha=0$ (NA) or $\beta^2=\frac{1}{3}\alpha^2$
	Therefore $z=\alpha+i\frac{\alpha}{\sqrt{3}}$ or $\alpha-i\frac{\alpha}{\sqrt{3}}$
(b)	If $\arg(z)>0$, z is in the first or second quadrant
	If z is in the first quadrant, $\arg(z)=\arctan\left(\frac{\alpha/\sqrt{3}}{\alpha}\right)=\frac{\pi}{6}$
	If z is in the second quadrant, $\arg(z)=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$
	Comment : generally poorly done by students who are weak in algebraic manipulation.

Qn	Solution
9.	
(a)	Project a line that is perpendicular to both BC and AD from B to AD.
	
	By Pythagoras' theorem, $BF^2+AF^2=AB^2$ $\Rightarrow \quad BF^2=14^2-2^2=192$
	Therefore $CD=BF=\sqrt{192}$
(b)	In ΔAFB,

Qn	Solution
	
	$\sin\theta = \frac{2}{14} = \frac{1}{7}$
	$\angle CBA = \angle CBF + \angle ABF$ $= \frac{\pi}{2} + \arcsin\left(\frac{1}{7}\right)$ $= 1.714 = 1.71$
	By symmetry, $\angle CBA = \angle ABE = 1.71$
	Therefore, obtuse $\angle CBE = 2\pi - 2(1.714) = 2.855$
(c)	By symmetry, EF = CD = $\sqrt{192}$
	Length of minor arc $CE = 6(2.855) = 17.13$
	Also, $\angle DAB = \arccos\left(\frac{1}{7}\right)$, so reflex $\angle DAF = 2\pi - 2\arccos\left(\frac{1}{7}\right)$
	Length of major arc DF = $8\left(2\pi - 2\arccos\frac{1}{7}\right) = 27.43$
	Therefore, the length of string is $2\sqrt{192} + 27.43 + 17.13 = 72.27 = 72.3$
	Comment : A surprising number of students gave a reflex angle for the answer in (b). Students who skipped (a) in order to proceed with the question tended to get (b) and (c) correct. Some students computed the area of the sector instead of the arc length. Some students wrongly assumed that $\angle CBA = \angle DAB = 90°$.

	Solution
10.	 (a)
	(b) $0.838 \le x \le 9.16$

	(c) range of $f: -10 \le y \le 6$
	(d) 
	$-10 < k < -9$
	(e) 
	$x = 0.702, 9.61$
	Comments : generally well done. Students lost marks for not putting down the maximum point at (10,6) on the graph. A surprising number of students were careless in the direction of the inequality for (d) – giving their answer as $-9 < k < -10$.

	Solution
11.	(a) $P(X \ge 5) = 1 - F(X < 5) = 1 - \int_0^5 \frac{x}{50} dx$
	$= \frac{3}{4}$

	Comment : Some students have mistakenly find the probability using discrete method, i.e. summation.
	(b) $E(X) = \int xf(x)dx = \int_0^{10} x\left(\frac{x}{50}\right)dx$
	$= \frac{20}{3}$
	Comment : Well attempted, take note that GDC can be used to evaluate definite integral to save time.
	$Var(x) = \int_0^{10} x^2 f(x)dx - \left(E(X)\right)^2$
	$= \int_0^{10} x^2\left(\frac{x}{50}\right)dx - \left(\frac{20}{3}\right)^2$
	$= \frac{50}{9}$
	Comment : Some students mistook E(X^2) as the formula for Var(X); also some forgot to square the E(X)
	(c) $\sum P(Y = y) = 1$
	$a + 2a + 3a + 4a + 5a = 1$
	$15a = 1$
	$a = \frac{1}{15}$
	Comment : Well attempted, though some students mistakenly use methods for continuous variable (integration) to find sum of probability
	(d)(i) $P(Y < 3) = P(Y = 1) + P(Y = 2)$
	$= a + 2a = 3a = \frac{3}{15} = \frac{1}{5}$
	Comment: For discrete variable,take note that X=3 is not included in X<3
	(ii) $E(Y) = \sum yP(Y = y)$
	$= 1\left(\frac{1}{15}\right) + 2\left(\frac{2}{15}\right) + 3\left(\frac{3}{15}\right) + 4\left(\frac{4}{15}\right) + 5\left(\frac{5}{15}\right)$

	$=\frac{11}{3}$
	Comment : Well attempted
	(e) $E(2Y+5)=2E(Y)+5$
	$=2\times\frac{11}{3}+5=\frac{37}{3}$
	Comment : Well attempted
	$Var(2Y+5)=2^2Var(Y)$
	$=4\times\frac{14}{9}=\frac{56}{9}$
	Comment: Some students add a 5 to the variance. Note that adding a constant to the variable only contribute to a vertical shift of mean, no effect on the square distance away from the mean, since both mean and the data shifted the same amount.
	(f) $P(X\geq 5)\times P(Y<3)=\frac{3}{4}\times\frac{1}{5}=\frac{3}{20}$
	$P(X\geq 5)\times P(Y<3)\neq P(X\geq 5\cap Y<3)$
	X and Y are **not independent** variables.
	Comment: Well attempted

	Solution
3.	(a) $\angle AOB=180°-30°\times 2$
	$\angle AOB=120° \;\; =\frac{2\pi}{3}$
	Comment : Well attempted
	(b) $w=2e^{i\frac{2\pi}{3}}$
	Comment: Some students mistook basic angle as the argument of a complex number
	(d) (i) $w^3=\left(2e^{i\frac{2\pi}{3}}\right)^3=2^3e^{i\left(\frac{2\pi}{3}\times 3\right)}$

	$w^3 = 8e^{i(2\pi)} = 8$
	Comment : Well attempted
	(ii) $w^3 = 8$
	$w^3 - 8 = 0$
	The real root of $w = 2$ which is represented by point A
	So $(w-2)$ is a factor of $w^3 - 8$
	Long division $w^3 - 8$ by $(w-2)$
	$w^3 - 8 = (w-2)(w^2 + 2w + 4) = 0$
	$w - 2 = 0$ (rej.) or $w^2 + 2w + 4 = 0$
	Comment: Some students substituted w=2 to obtain the result, take note that w has been stated explicitly as a complex number.
	OR substitute $w = 2e^{i\left(\frac{2\pi}{3}\right)}$
	$w^2 + 2w + 4 = 4e^{i\left(\frac{4\pi}{3}\right)} + 2\left[2e^{i\left(\frac{2\pi}{3}\right)}\right] + 4$
	$= 4\cos\left(\frac{4\pi}{3}\right) + i4\sin\left(\frac{4\pi}{3}\right) + 4\cos\left(\frac{2\pi}{3}\right) + i4\sin\left(\frac{2\pi}{3}\right) + 4$
	$= 4\left(-\frac{1}{2}\right) + i(-4\cdot\frac{\sqrt{3}}{2}) + 4\left(-\frac{1}{2}\right) + i\left(4\cdot\frac{\sqrt{3}}{2}\right) + 4$
	$= 0$
	Comment : Some students did not show enough working to obtain the mark here
(d)	(i) $f(z) = (z^2 - \frac{1}{2}zw - z + \frac{1}{2}w)(z - \frac{w^2}{4})$
	$= z^3 - \frac{1}{4}z^2w^2 - \frac{1}{2}z^2w - \frac{1}{8}zw^2$ $-\frac{1}{8}zw^3 - z^2 - \frac{1}{4}zw^2 + \frac{1}{2}zw - \frac{1}{8}w^3$

	$= z^3 - \frac{1}{4}z^2(w^2+2w+4) - \frac{1}{8}zw(w^2+2w+4) - \frac{1}{8}w^3$
	$f(z) = z^3 - \frac{w^3}{8}$
	Comment: Well attempted. Some students did not group into terms that can be reduced to 0 or did not show sufficient working
	(ii) $z^3 - \frac{w^3}{8} = 26$
	$z^3 = 27$ since $w^3 = 8$
	$r^3 e^{i(3\theta)} = 27e^{i(2k\pi)}$
	$r = \sqrt[3]{27} = 3$
	$3\theta = 2k\pi$
	$\theta = \frac{2k\pi}{3}$
	$k = 0,1,2$ or $k = 0, \pm 1$ or any three consecutive integers
	$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\theta = 0, \frac{2\pi}{3}, \frac{-2\pi}{3}$
	$z = 3, 3e^{i\frac{2\pi}{3}}, 3e^{-i\frac{2\pi}{3}}$
	Comment : Well attempted. Some students did not convert into correct range.