Mathematical Formulae

General

$speed = \frac{dist}{time}$

$density = \frac{mass}{volume}$

Quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Compound interest: Total amount = $P\left(1+\frac{r}{100}\right)^n$

Simple Interest:

$I = \frac{PRT}{100}$

total amount, A = P + I

Coordinate Geometry

Length of line segment $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Gradient of line $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of line $y = mx + c$

x-intercept : $y = 0$

y-intercept : $x = 0$

- Parallel lines must always have the same gradient and the lines never meet or intersect one another

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of Trapezium

$= \frac{1}{2} \times$ sum of parallel sides $\times$ height

Area of Parallelogram

= base $\times$ height

Polygons

Sum of interior angles in a polygon: $(n-2)\times 180^\circ$

For a regular polygon, each interior angle: $\frac{(n-2)\times 180^\circ}{n}$

Sum of exterior angles: 360°

For a regular polygon, each exterior angle: $\frac{360^\circ}{n}$

Curved SA of cylinder $= 2\pi rh$

Volume of a pyramid

$= \frac{1}{3} \times$ base area $\times$ height

Area of Circle $= \pi r^2$

Circumference of Circle $= 2\pi r$ or πd

Arc length of a sector $= \frac{\theta}{360^\circ} \times 2\pi r$

Area of a sector $= \frac{\theta}{360^\circ} \times \pi r^2$

θ : deg *rees*

Special Algebraic Identities:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

Additional

$x^4 - y^4 = \left(x^2 - y^2\right)\left(x^2 + y^2\right)$

$= (x+y)(x-y)\left(x^2 + y^2\right)$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radian

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

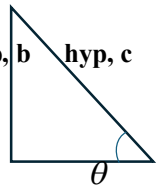
$a^2 = b^2 + c^2 - 2bc\cos A$

Trigonometry

TOA: $\tan\theta = \frac{opp}{adj}$

CAH: $\cos\theta = \frac{adj}{hyp}$

SOH: $\sin\theta = \frac{opp}{hyp}$



Pythagoras' Theorem: $c^2 = a^2 + b^2$

Prefixes:

micro: 10^{-6} nano: 10^{-9}

pico: 10^{-12}

million: 10^6 billion: 10^9 trillion: 10^{12}

Statistics

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

Mode: highest frequency

Where grouped data is given, find the mid-value, x

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

Σ : sum of

f : frequency

$\frac{\Sigma fx}{\Sigma f}$: mean, $\bar{x}$

Probability

P(heads) = 1 – P(tails)

P(A or B)

=P(A) + P(B)

P(A and B)

=P(A) X P(B)

PROBABILITY

Addition of Probability

$P(A \text{ or } B) = P(A) + P(B)$

Multiplication of Probability

$P(A \text{ and } B) = P(A) \times P(B)$

Probability Diagram

	Head	Tail
Head	H, H	H, T
Tail	H, T	T, T

Product of the above matrix **I** × 3 and (3 × 2) will produce a matrix of (1 × 2).
*Knowing this concept allows you to rearrange or produce a new matrices for solving.

Geometric reasons for lines/angles

1)(∠ on a st. line)
2)(∠ at a point)
3)(∠ sum of Δ)
4)(alt ∠, // sides)
5)(corr. ∠, // sides)
6)(base ∠s of isos Δ)
7)(equilateral Δ)
8)(∠ at centre = twice ∠ at circumference)
9)(∠ in a semicircle = 90°)
10)(∠ in the same segment)
11)(∠ in opp. segment)
12)(tangent ⊥ radius)
13)(tangents from an ext. point)
14)(⊥ from centre bisects chord)
15)(equal chords are equidistant from centre)

Inequalities:

if $-4a \le k$

$a \ge -\frac{k}{4}$

: sign changes

when multiply or divide by a negative number

if $4a \le -k$

$a \le -\frac{k}{4}$

Indices

$a^m \times a^n = a^{m+n}$ $\quad a^0 = 1$

$a^m \div a^n = a^{m-n}$ $\quad a^{-n} = \frac{1}{a^n}$

$\left(a^m\right)^n = a^{mn}$ $\quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

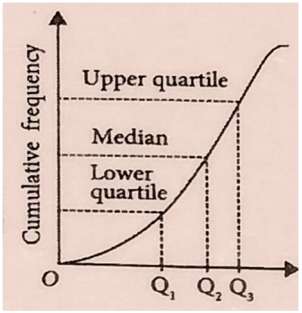
$(ab)^n = a^n b^n$ $\quad$ if $a^m = a^n$, then $m = n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

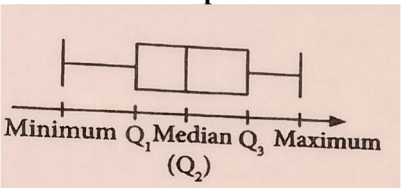
Conversion of units:
For length
1cm = 10mm
1m = 100cm
1km =1000m = 100000 cm
For area
$1m^2 = 10000cm^2$
$1km^2 = 1000\,000m^2$

Cumulative Frequency plots
Q_1: lower quartile (25th percentile)
Q_2: median (50th percentile)
Q_3: upper quartile (75th percentile)

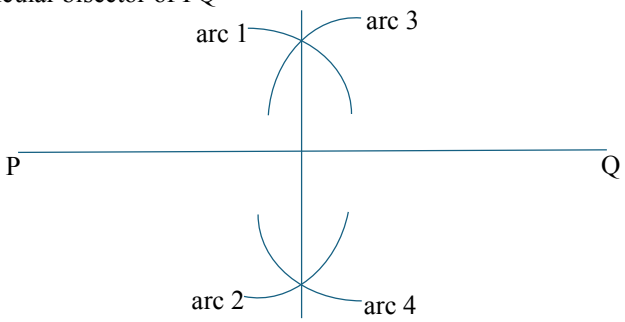
$IQR = Q_3 - Q_1$
Range = Max - Min



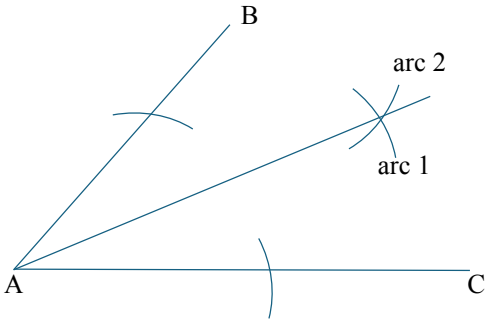
Box-and-whisker plots



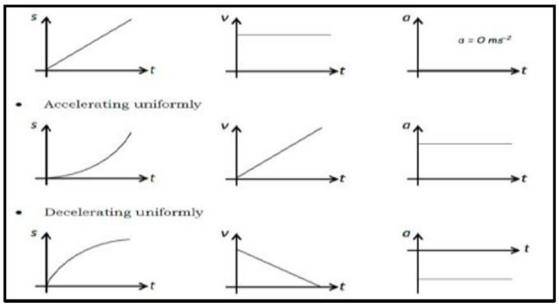
Perpendicular Bisector
To construct the perpendicular bisector of a line segment PQ:
1) Given or draw PQ
2) Using a pair of compasses, choose a radius more than half the length of PQ
3) Using P as the centre, draw arc 1 above PQ and arc 2 below PQ
4) Using Q as the centre, draw arc 3 above PQ and arc 4 below PQ
5) Join the two points on intersection, *X* and *Y*, to get the perpendicular bisector of PQ



Angle Bisector
To construct the angle bisector of angle BAC:
1) Using a pair of compasses, choose a radius less than or equal to the length of AB and AC
2) Using A as the centre, draw an arc to cut AB at X and an arc to cut AC at Y
3) Using X as the centre, draw arc 1
4) Using Y as the centre, draw arc 2, such that arc 1 & 2 meet at R
5) Join A to R and that forms the bisector of angle BAC

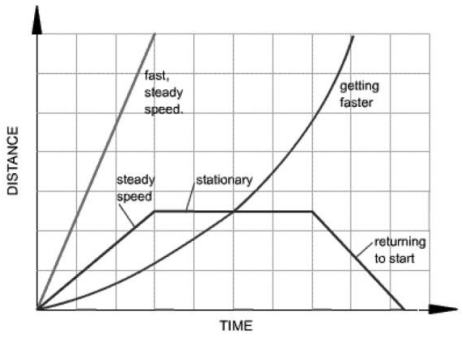


SPEED TIME GRAPH CONVERSION



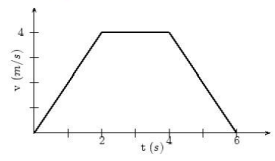
A distance-time graph tells us how far an object has moved with time.

- The steeper the graph, the faster the motion.
- A horizontal line means the object is not changing its position - it is not movi it is at rest.
- A downward sloping line means the object is returning to the start.



Distance-time graph:
Gradient = speed

Speed (Rate of Change of Distance)



Gradient of Speed-Time Graph gives you **ACCELERATION**.
Area under Speed-Time Graph gives you **TOTAL DISTANCE**.

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time}$$

Question: Finding Speed at a Particular Point
Step 1: Find Gradient of the Line
Step 2: Use Gradient = $\frac{y1-y2}{x1-x2}$ to get the Missing Value

Note: DO NOT USE $\frac{Rise}{Run}$ (Prone to Careless)

Acceleration (Rate of Change of Speed)

Deceleration = Retardation. Retardation itself has NEGATIVE connotation. Do not need to add Negative in front of value.

Advance (Must Know for O Levels)

1. If $T_n = 2n^2 + 3n$ $\quad T_{n+1} = 2[n+1]^2 + 3[n+1]$
2. $S_{n+1} - S_n = T_{n+1}$

Number Patterns

2 5 8 11 14

Level 1: General Formula

Common difference (d) = +3 First term (a) – 2

General term, Tn = $dn + (a - d)$ (e.g. $3n + (2-3) = 3n - 1$

Level 2: Typical follow up question

Does 28 lie in the sequence? $3n - 1 = 28$

$n = 9\frac{2}{3}$ since n is not an integer,

28 does not lie in the sequence

Level 3: Level increment *additional info

3 7 13 21

4 6 8

2 2

use $Tn = an^2 + bn + c$

Then create simultaneous equations to find the value of a, b, c

Tests for **CONGRUENT** triangles

1) SSS – 3 pairs of sides equal

2) SAS – 2 sides and 1 included ∠

3) ASA – 2 ∠ s and 1 included side

4) RHS – Right ∠ -Hypothenuse Side

Tests for **SIMILAR** triangles

1) AAA – Show any 2 angles equal

2) SSS – Show 3 pairs of equal ratios (i.e. same scale factor)

3) SAS – Show 2 ratios and 1 angle equal (this angle must be an included angle)

Ratios of Areas: $A_1 : A_2 = l_1^2 : l_2^2$

Ratios of Volumes: $V_1 : V_2 = l_1^3 : l_2^3$

Finding the GREATEST ELEVATION

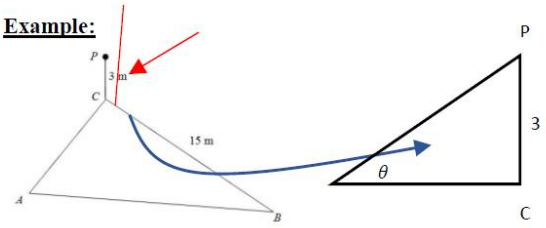
Step 1: Draw your Shortest Perpendicular Line

Step 2: Apply Area Formula to find SHORTEST DISTANCE

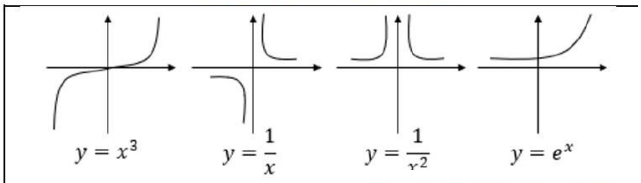
$$\frac{1}{2} \times b \times h = \frac{1}{2} \times a \times b \times \sin C$$

Step 3: Use TOA CAH SOH to find the Angle

Example:

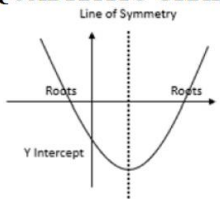


GRAPH FUNCTIONS



<Reflect about $x - a$

QUADRATIC GRAPHS



Graph Sketching Type 2 (Completing the Square)

Step 1. Find your turning point***

$y = (x+4)^2 - 3$ Turning Point: $(-4, -3)$

$y = (2-x)^2 + 3$ Turning Point: $(2, 3)$

$\therefore$ for $y = a(x-h)^2 + k$, the turning point is (h, k)

Step 2. Find Y-Intercept (As Usual)

Sub $x = 0$

Step 3: Find Roots (Square Root Method)

Sub $y = 0$

$(x-4)^2 - 4 = 0$

$(x-4)^2 = 4$ $\quad x - 4 = \pm 2$ $\quad x = 2$ or 6

If x has no real solutions, graph lies entirely above or

Completing the Square:

$$x^2 + bx + c = x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Additional Info about Further Trigonometry

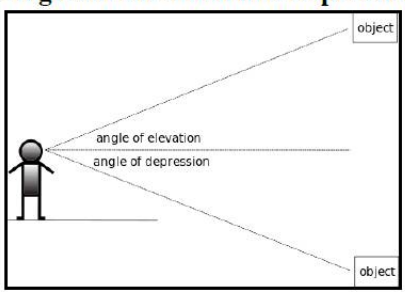
Caution for Obtuse Sine Rule

When you use Sine Rule to Get Your Angle, please double check if the angle should be less or more than 90°.

This is because both $sin30 = 0.5$ *and* $sin150 = 0.5$ and when you use inverse sin (0.5) on your calculator, it will show 30°.

Therefore, look at the diagram and if you know that angle is more than a right angle, take $180° - 30° = 150°$

Angles of Elevation and Depression



SETS

SET/VENN DIAGRAM NOTATION TABLE

SYMBOL	DEFINITION	EXAMPLE	EXPLANATION
∈	**element of**	A ∈ B	A is an element of B
∉	**not an element of**	A ∉ B	A is not an element of B
⊆	**subset of**	A ⊆ B	A is a subset of B
⊄	**not a subset of**	A ⊄ B	A is not a subset of B
∩	**intersection**	A ∩ B	A intersect B (in both sets)
∪	**union**	A ∪ B	A union B (in set A and/or B)
′	**complement**	A′	A complement (not A)

Timeline to O levels

Jun – Complete topical revision (TYS) & notes

Jul – Kickstart Prelim Papers

Aug – Prepare for Prelims

Sep – Complete any outstanding yearly TYS + other schools' Prelim Papers

Oct – Final Revision – aim to ACE O's