Chapter 1 (quadratic graph)

Sketching graph:

To find if max/min graph

coeff of $x^2 < 0 \to$ max (e.g. $-$)

coeff of $x^2 > 0 \to$ min (e.g. $+$)

To find x-coordinate:

let $y=0$

$0 = 2x^2+4x-6$

$x=-3$ or $x=1$

To find y-coordinate:

let $x=0$

$y=2(0)^2+4(0)-6$

$=-6$

To find turning point /max/min-point

$2x^2+4x-6=0$

$x=-3$ or $x=1$

average $= \frac{1-3}{2}$

$=-1$

sub $x=-1$ into

$2(-1)^2+4(-1)-6=y$

$y=-8$

turning point:

$(-1,-8)$

Completed square:

$-3(x+2)^2-15$

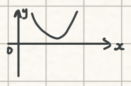
$x+2=0$ → max y-value

$x=-2$

turning point: $(-2,-15)$

Conditions for quadratic curve to lie completely above/below x-axis

Completely above:



1. minimum curve, coeff of $x^2>0$
2. minimum y-value >0

Completely below:

1. maximum curve, coeff of $x^2<0$
2. maximum y-value <0

quadratic equation:

$2x^2+4x-6=0$

$ax^2+bx+c=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Find the range of values of x where $y=2x^2-6x+3$ lies above the line $y=11$.

$2x^2-6x+3>11$

$2x^2-6x-8>0$

$(x+1)(x-4)>0$

$x<-1$ $\quad x>4$

Chapter 2 (Inequalities)

nature of roots	(coeff of $x^2>0$) minimum curve	(coeff of $x^2<0$) maximum curve	how qn would phrase
→ $b^2-4ac>0$ → two distinct real roots → 2 x-intercept			intersects twice, two real, distinct roots
→ $b^2-4ac=0$ → two equal real roots/ one distinct real roots → 1 x-intercept			two real and equal roots, Tangent, touches
→ $b^2-4ac<0$ → no real roots → does not intercept x-axis			no real roots, does not intersect

discriminant ≥ 0 if → has real roots → intersects → two real roots

never negative: coeff of $x^2>0$, discriminant ≤ 0

$b^2-4ac<0$ $\quad b^2-4ac=0$

If qn say find x → inequality

If qn say find ___ → b^2-4ac

Always positive

1. coeff of $x^2>0$ (min graph)
2. $b^2-4ac<0$

Always negative

1. coeff of $x^2<0$ (max graph)
2. $b^2-4ac<0$

number line:

$x<3$ $\quad x>3$

$-2<x<3$

(2.4) Solving quadratic inequalities

1. push all to one side so equation $=0$
2. $b^2-4ac>/=/<0$
3. change to factorised form $(x+1)(x+2)$ / $(x+\sqrt{5})(x-\sqrt{5})$
4. Solve for ___
5. draw graph based on ___-value and coeff of x^2 in b^2-4ac
   - if $b^2-4ac>0$: $x<-2$ or $x>3$
   - if $b^2-4ac=0$: $x=-2$ or $x=3$
   - if $b^2-4ac<0$: $-2<x<3$
6. If qn always positive/negative, rej ___ if not corresponding with coeff of x^2

e.g always positive, $kx^2-5x+4k$

$k<-1.25$ $\quad k>1.25$

(rej as $k>0$)

Solving simultaneous equation (no discriminant)

Qn: Finding coordinate of point of intersection/solve the equation

1. sub line into curve
2. Simplify and push all to one side so equation $=0$
3. use quadratic equation / factorise to find x values
4. Sub x values into equation to find y-values

Points of intersection: (x-value, y-value)

Qn: find the values of ___ for which line $y=$... intersects/intersects once or tangent /does not intersect curve $y=$...

1. sub line into curve
2. Simplify and push all to one side so equation $=0$
3. discriminant >0 / discriminant $=0$ / discriminant <0
4. Solve for ___ / use quadratic equation or factorise

Chapter 3 (surds)

Laws ($a>0, b>0$)

1. $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ $\quad \sqrt{18}=\sqrt{9\times2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$
2. $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ $\quad \frac{\sqrt{216}}{\sqrt{6}}=\sqrt{\frac{216}{6}}=\sqrt{36}=6$
3. $\sqrt{a}\times\sqrt{a}=\sqrt{a^2}=a$ $\quad 5\sqrt{2}\times\sqrt{2}=5(2)=10$

$a+b\sqrt{n}=p+q\sqrt{n}$

$a=p$ $\quad b=q$

$a\sqrt{b}+c\sqrt{b}=a+c\sqrt{b}$

e.g $4\sqrt{3}+5\sqrt{3}=9\sqrt{3}$

Conjugate surds:

$(p+q\sqrt{a})(p-q\sqrt{a})$ (conjugate) $=$ Rational number

Rationalising: multiply surd in denominator by its conjugate surd

$\frac{6}{\sqrt{5}}=\frac{6}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$ $\quad \frac{5}{3+4\sqrt{2}}\times\frac{(3-4\sqrt{2})}{(3-4\sqrt{2})}$

$(p+q\sqrt{3})^2=29-12\sqrt{3}$

$p^2+2pq\sqrt{3}+5q^2=29-12\sqrt{3}$

$p^2+5q^2=29$ —(1) $\quad 2pq=-12$

$pq=-6$

$p=-\frac{6}{q}$ —(2) $\quad$ sub (2) into (1)

e.g. triangle has base of $(2\sqrt{3}+\sqrt{5})$ and area of $(1+\sqrt{15})$ find height in the form of $\frac{a\sqrt{5}-b\sqrt{3}}{c}$

$(1+\sqrt{15})=\frac{1}{2}\times(2\sqrt{3}+\sqrt{5})x$ (height)

$2(1+\sqrt{15})=(2\sqrt{3}+\sqrt{5})x$

$\frac{2(1+\sqrt{15})}{2\sqrt{3}+\sqrt{5}}=x$

$\frac{2(1+\sqrt{15})(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3}+\sqrt{5})(2\sqrt{3}-\sqrt{5})}=x$

law1. → $\frac{2(2\sqrt{3}-\sqrt{5}+2\sqrt{45}-\sqrt{75})}{12-5}=x$

law 1 → $\frac{2(2\sqrt{3}-\sqrt{5}+2\sqrt{9\times5}-\sqrt{25\times3})}{7}=x$

$\frac{2(2\sqrt{3}-\sqrt{5}+6\sqrt{5}-5\sqrt{3})}{7}=x$

$\frac{2(5\sqrt{5}-3\sqrt{3})}{7}=x$

$\frac{10\sqrt{5}-6\sqrt{3}}{7}=x$

(4.1-4.3) polynomials, cubic equations, (4.4) partial fractions

Division algorithm

$f(x)$ = dividend = divisor × quotient + Remainder (D, Q, R)

$x^3+2x-7=(x-2)(x^2+2x+6)+5$

long division:

$x-2 \overline{) x^3+2x-7}$, quotient x^2+2x+6

$-(x^3-2x^2)$

$2x^2+2x$

$-(2x^2-4x)$

$6x-7$

$-(6x-12)$

5

Remainder Theorem:

let linear divisor $=0$ → degree 1

Remainder $=f(\_)$

let linear divisor $=0$

$x-2=0$

$x=2$

Remainder $=3(2)^3-4(2)^2-5$

$=3$

Not Polynomials

power <0 e.g $3xy^{-2}$ on variables

divide by variable e.g. $\frac{2}{x}$ or $\frac{x}{x}$

$\sqrt{x}$ is not as fraction

factor Theorem, linear divisor is a factor (degree 1)

linear divisor $=0$

$f(\_)=0$

let linear divisor $=0$

$x-3=0$

$x=3$

By factor Theorem, $f(3)=0$

$4(3)^3+6(3)^2-4(3)+2k=0$

$2k=-135$

$k=-67.5$

partial fractions

proper fraction: degree of numerator < degree of denominator

e.g. $\frac{8-x}{x^2-2x+3}, \frac{1}{5-x}$

improper fraction: degree of numerator $\geq$ degree of denominator

e.g. $\frac{x^3+8}{4x^2-x+6}, \frac{x}{x+2}$

1. Proper fractions, distinct linear factors denominator

$\frac{px+q}{(ax+b)(cx+d)}=\frac{A}{ax+b}+\frac{B}{cx+d}$

2. proper fractions, repeated linear factors denominator

$\frac{px+q}{(ax+b)^2}=\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$

3. Proper fraction, quadratic factor x^2+c^2 denominator

$\frac{px+q}{(ax+b)(x^2+c^2)}=\frac{A}{ax+b}+\frac{Bx+C}{x^2+c^2}$

4. Improper fraction

- long division
- use partial fractions formula on remainder
- Final answer: Quotient (from long division) + Partial fractions (from remainder)

factorise cubic expression

$f(x)=2x^3-11x^2-7x+6$

1. First factor (use calcular find 1st factor)

$f(6)=2(6)^3-11(6)^2-7(6)+6$

$=0$

By factor theorem, $x-6$ is a factor

2. Find Remaining factor using this or long division

$f(x)=(x-6)(2x^2+Ax-1)$ → By comparing coeff

$x\times\_=2x^3$

$-6\times\_=6$

By comparing coeff of x

$-6Ax-x=-7x$

$-6Ax=-6x$

$-6A=-6$

$A=1$

3. Factorise (if can)

$f(x)=(x-6)(2x^2+x-1)$

$=(x-6)(2x-1)(x+1)$

Cubic Formula

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Chapter 6 Exponential + logarithm functions

log:

$\lg=\log_{10}$

$\ln=\log_e$

$y=a^x$ $\quad \log_a y=x$

↳ to be defined $a>0, y>0$

$\ln 3x=7$

$e^7=3x$

$x=\frac{e^7}{3}$

e.g. $7^x=5^{x+1}$

↳ log both sides

1. $\log_a 1=0$ / $\lg 1=0$ / $\ln 1=0$

$a^0=1$ $\quad 10^0=1$ $\quad e^0=1$

2. $\log_a a=1$ / $\lg 10=1$ / $\ln e=1$

$a^1=a$ $\quad 10^1=10$ $\quad e^1=e$

3. $\log_a Y=\frac{\log_b Y}{\log_b a}$

4. $\log_a x^r=r\log_a x$ $\quad \boxed{(\lg 2)^3 \neq 3\lg 2}$

$\lg 2^3=\lg(2\times2\times2)$

$=\lg 2+\lg 2+\lg 2$

$=3\lg 2$

5. $\log_a x+\log_a y=\log_a xy$

$\boxed{\log_a(x+y)\neq\log_a x+\log_a y}$

6. $\log_a x-\log_a y=\log_a\frac{x}{y}$

$\boxed{\log_a(x-y)\neq\log_a x-\log_a y \quad \log_a\frac{x}{y}\neq\frac{\log_a x}{\log_a y}}$

7. $\log_a p=\log_a q$

$\therefore p=q$

exponential:

1. $a^m\times a^n=a^{m+n}$
2. $a^m\div a^n=a^{m-n}$
3. $(a^m)^n=a^{mn}$
4. $a^m\times b^m=(ab)^m$
5. $a^m\div b^m=\frac{a^m}{b^m}=(\frac{a}{b})^m$
6. $a^{-n}=\frac{1}{a^n}$ $\quad \frac{a^{-n}}{b^{-m}}=\frac{b^m}{a^n}$
7. $a^0=1$
8. $a^{\frac{1}{n}}=\sqrt[n]{a}$ $\quad a^{\frac{m}{n}}=(\sqrt[n]{a})^m/\sqrt[n]{a^m}$
9. $\sqrt{a}\times\sqrt{a}=a$

$a^m=a^n$

$\therefore m=n$

$a^{\log_a b}=b$

let $x=a^{\log_a b}$

$\log_a x=\log_a a^{\log_a b}$

$\log_a x=(\log_a b)(\log_a a)$

$\log_a x=\log_a b$

$x=b$

e.g. $e^{\ln 5}=5$

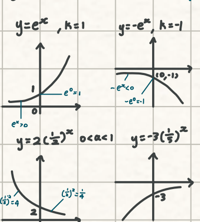
$2^{\log_2 9}=9$

Exponential functions and graphs $k=y$-intercept

$y=ka^x$, $a\neq1$, $a>0$

$y=e^x$, $k=1$ $\quad y=-e^x$, $k=-1$

$y=2(\frac{1}{2})^x$ $0<a<1$ $\quad y=-3(\frac{1}{3})^x$

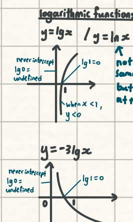


logarithmic functions and graph x-intercept $=1$

$y=\lg x$ / $y=\ln x$

not the same graph but similar at the blue part

$y=-3\lg x$



to draw suitable straight line, make the equation you are trying to solve = graph, draw the other side

e.g. solve equation $\log_4\frac{1}{\sqrt{x-3}}=\frac{1}{2}x$ by drawing a line on graph $y=(\frac{1}{4})^x$

$\log_4\frac{1}{\sqrt{x-3}}=\frac{1}{2}x$ $\quad 4^{\frac{1}{2}x}=\frac{1}{\sqrt{x-3}}$

$x-3=\frac{1}{4^x}$

$x-3=(\frac{1}{4})^x$

$y=x-3$

Binomial (n.o of terms = power + 1)

$(a+b)^4=4C0a^4b^0+4C1a^3b^1+4C2a^2b^2+4C3a^1b^3+4C4a^0b^4$

$C\uparrow b\uparrow$ from 0, $a\downarrow$ from 4

$\binom{4}{1}=4C1$

$T_{(r+1)}={}^nC_r a^{n-r}b^r$ $\quad$ e.g. Term 60 of $(a+b)^{100}=100C59a^{41}b^{59}$

independent of x = no x

$5!=5\times4\times3\times2\times1$

Chapter 7 Coordinate Geometry + Circle

gradient $=\frac{y_2-y_1}{x_2-x_1}$

length $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

midpoint $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

Area of shape $=\frac{1}{2}\begin{vmatrix}x_1 & x_2 & x_3 & x_4 & x_1\\ y_1 & y_2 & y_3 & y_4 & y_1\end{vmatrix}$

$=\frac{1}{2}(\underbrace{\quad}_{\text{sum of downward multiplication}}-\underbrace{\quad}_{\text{sum of upward multiplication}})$

(x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) Anti-clockwise

perpendicular

$m_1m_2=-1$

(m) gradient of AB × gradient of CD $=-1$

e.g m of AB $=3$

$3m=-1$

$m=-\frac{1}{3}$

m of CD $=-\frac{1}{3}$

perpendicular bisector of line = midpoint

parallel

l_1, l_2: m of l_1 = m of l_2

equation (gradient + 1 point)

$y-y_1=m(x-x_1)$

sub y-coordinate, sub x-coordinate

Find the possible equations of the circle which touch both axes and pass through the point A $(-8,1)$

centre of circle: $-x$ coordinate, $+y$ coordinate

$(x+a)^2+(y-a)^2=a^2$

sub $x=-8, y=1$

$(-8+a)^2+(1-a)^2=a^2$

$64-16a+a^2+1-2a+a^2=a^2$

$a^2-18a+65=0$

$(a-13)(a-5)=0$

$a=13$ $\quad a=5$

$(x+13)^2+(y-13)^2=13^2$

$(x+5)^2+(y-5)^2=5^2$

equation of circle

standard form:

$(x-a)^2+(y-b)^2=r^2$ $\quad$ $x^2+y^2=10$, centre: $(0,0)$

centre: (a,b) radius $=r$

complete square

general form:

$x^2+y^2+2gx+2fy+c=0$

centre: $(-g,-f)$ radius: $\sqrt{g^2+f^2-c}$

can apply circle properties from plane theory

tangent at top/bottom is parallel to x-axis

right, left: tangent at left/right is always parallel to y-axis

To show if point inside circle calculate distance from point to centre of circle

(inside) dist < radius

(on) dist = radius

(outside) dist > radius

kite:

rhombus:

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

e.g. $\binom{9}{5}$ without calculator

$\binom{9}{5} = \frac{9!}{5!(9-5)!}$

$= \frac{9\times8\times7\times6\times5\times\cancel{4\times3\times2\times1}}{5!\,\cancel{4\times3\times2\times1}}$

$= \frac{9\times8\times7\times6\times\cancel{5}}{\cancel{5}\times4\times3\times2\times1}$

$= \frac{\cancel{9}^3\times\cancel{8}^2\times7\times\cancel{6}^3}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$

$= 18\times7 = 126$

$(x-1)^{10}$ = descending powers

$(-1+x)^{10}$ = ascending powers

↓ lowest degree

$(3+kx)^{20}$. coeff of $x^4 : x^5 = 3:16$

Find k

$T_r = \binom{20}{r}(3^{20-r})(k^r x^r)$

$\frac{\binom{20}{4}(3^{20-4})(k^4)}{\binom{20}{5}(3^{20-5})(k^5)} = \frac{3}{16}$ → don't calculate separately

$\frac{15k^4}{16k^5} = \frac{3}{16}$ $\quad \frac{15}{16k} = \frac{3}{16}$

Find the general term of $(x^2 - \frac{1}{x})^{12}$

$T_{r+1} = \binom{12}{r}(x^2)^{12-r}(-\frac{1}{x})^r$ factorise out -1

$= \binom{12}{r}(x^{24-2r})(-1)(\frac{1}{x})^r$

$= \binom{12}{r}(-1)(x^{24-2r-r})$

$= \binom{12}{r}(-1)^r(x^{24-3r})$

Find term containing x^9

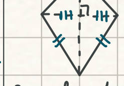
$24-3r = 9$

$-3r = -15$

r must be whole n.o → $r = 5$

$\binom{12}{5}(-1)^5(x^9)$

$= -792x^9$



2 pair of equal sides

diagonals are perpendicular bisector

midpoint of AC = midpoint of BD

opposite ∠s equal

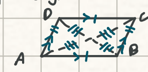
all four sides equal

opposite sides parallel

adjacent ∠s add up to 180° (supplementary)

diagonals are perpendicular bisectors

parallelogram:



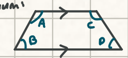
opposite sides equal and parallel

opposite ∠s equal

adjacent ∠s add up to 180°

diagonals bisect each other (not perpendicular)

trapezium:



∠s between parallel sides add up to 180°

A+B , C+D

Chapter 9 Trigo functions and graphs

$\theta =$	$30°/\frac{\pi}{6}$	$45°/\frac{\pi}{4}$	$60°/\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2} = \frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

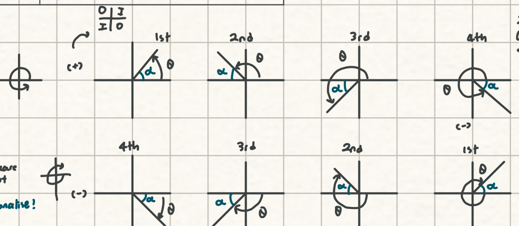
if forget square the value

e.g.

$(\sin 45°)^2 = \frac{1}{2}$) square root

$= \frac{1}{\sqrt{2}}$) rationalise!

$= \frac{\sqrt{2}}{2}$



$\sin\theta = \sin\alpha$

$\cos\theta = \cos\alpha$ (sign is different)

$\tan\theta = \tan\alpha$

e.g $\sin 80° = 0.985$ 1st quadrant sin → (+)

$\sin(180°-80°) =$

$\sin 100° = 0.985$ 2nd quadrant sin (+)

$\sin(180°+80°) =$

$\sin 260° = -0.985$ 3rd quadrant sin (-)

$\sin(360°-80°) =$

$\sin 280° = -0.985$ 4th quadrant sin (-)

θ in 2nd

sin θ (+)
cos θ (-)
tan θ (-)

θ in 3rd

sin θ (-)
cos θ (-)
tan θ (+)

(+) ∠

2nd S	A 1st
3rd T	C 4th

θ in 1st

sin θ (+)
cos θ (+)
tan θ (+)

θ in 4th

sin θ (-)
cos θ (+)
tan θ (-)

obtuse → ∠ more than 90° less than 180°

reflex → ∠ more than 180° less than 360°

without a calculator, find value of the following,

$\sin 210°$

210° is in third quadrant

$210° - 180° = 30°$

$\alpha = 30°$

$\sin 30° = \frac{1}{2}$

$\sin 210° = -\frac{1}{2}$

$\cos\frac{5}{3}\pi$

$\frac{5}{3}\pi$ is in 4th quadrant

$2\pi - \frac{5}{3}\pi = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos\frac{5\pi}{3} = \frac{1}{2}$

$\tan\frac{11\pi}{6}$

$\frac{11\pi}{6}$ is in 4th quadrant

$2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$

$\alpha = \frac{\pi}{6}$

$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\tan\frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

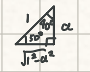
$\sin 50° = a$, express each of the following in terms of a

2nd quadrant $\cos 130°$ → $\alpha = 50°$

$= -\sqrt{1-a^2}$

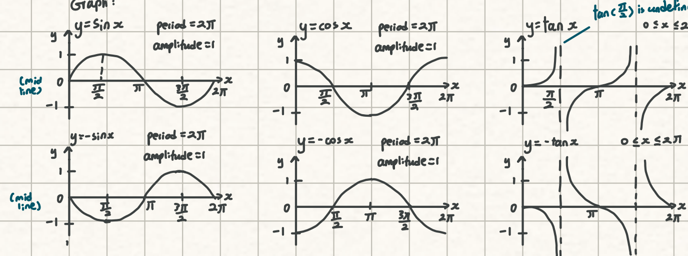
$\tan 40°$ → 1st quad

$= \frac{\sqrt{1-a^2}}{a}$



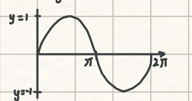
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Graph:



period = π
no amplitude

period = π
no amplitude



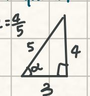
y>1 or y<-1 has no solution

1. $\sin(90°-\theta) = \cos\theta$ 2. $\cos(90°-\theta) = \sin\theta$ 3. $\tan(90°-\theta) = \frac{1}{\tan\theta}$

given that $\sin A = -\frac{4}{5}$ and $90° < A < 270°$. Find without calculator.

A in 3rd quadrant

$\sin\alpha = \frac{4}{5}$



$\cos\alpha = \frac{3}{5}$ $\quad \cos A = -\frac{3}{5}$

$\tan\alpha = \frac{4}{3}$ $\quad \tan A = \frac{4}{3}$

(i) $\mathrm{cosec}(90°-A)$

$= \frac{1}{\sin(90°-A)}$

$= \frac{1}{\cos A}$

$= \frac{1}{-\frac{3}{5}}$

$= -\frac{5}{3}$

(ii) $\sec(180°+A)$

$= \frac{1}{\cos(\underbrace{180°+A}_{\theta})}$

$= \frac{1}{\sin[90°-(\underbrace{180°+A}_{\theta})]}$

$= \frac{1}{\sin(-90°-A)}$

$= \frac{1}{\sin[-(90°+A)]}$

$= \frac{1}{-\sin(\underbrace{90°+A}_{\theta})}$

$= \frac{1}{-\cos(90°-(90°+A))}$

$= \frac{1}{-\cos(-A)}$

$= \frac{1}{-\cos A}$

$= \frac{1}{-(-\frac{3}{5})}$

$= \frac{5}{3}$

(iii) $-\cot A$

$= -\frac{1}{\tan A}$

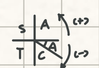
$= -\frac{1}{\frac{4}{3}}$

$= -\frac{3}{4}$

$\sin(-A) = -\sin A$

$\cos(-A) = \cos A$

$\tan(-A) = -\tan A$



$\sin(-\theta) = -\sin\theta$

also determine shape of graph

$y = a\sin bx + c$ / $y = a\cos bx + c$

midline is $y = c$ → average of the two points

amplitude = a (positive)

max min = $c \pm a$

period = $\frac{2\pi}{b}$

$y = a\tan bx + c$

midline is $y = c$

period = $\frac{\pi}{b}$

Refer to class kit + snorkl for examples

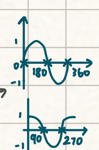
a=1 c=0

a) Explain why $\sin x = 1.1$ has no solution

From graph, maximum value of $\sin x = 1$,

∴ $\sin x = 1.1$ has no solution

b) Explain why the simultaneous equation $\sin x = 0$ and $\cos x = 0$ have no solution.



$\underbrace{\sin x = 0}_{y=0}$, $x = 0°, 180°, 360°$

$\underbrace{\cos x = 0}_{y=0}$, $x = 90°$ or $270°$

∴ simultaneous equation $\sin x = 0$, $\cos x = 0$ have no solution for $0° \le x \le 360°$

10.1 Trigo equations

1. let (___) = x/θ
2. x/θ is in which quadrant(s)
3. range of x/θ
4. find α ∠
5. find x/θ based on quadrants and range
6. find (___)

e.g. $\mathrm{cosec}(x-50°) = 1.35$ $0° < x < 360°$

$\frac{1}{\sin(x-50°)} = 1.35$

$(\sin x - 50°) = 0.74074$

1. let $x - 50° = \theta$
2. θ is in 1st /2nd quadrant — look at range and sign

$0° < x < 360°$

$-50° < x - 50° < 310°$

3. $-50° < \theta < 310°$

$\sin\alpha = 0.74074$

$\alpha = \sin^{-1}(0.74074)$

4. $= 47.794°$

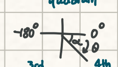
5. $\theta = 47.794°$ or $\theta = 180° - 47.794°$

6. $x - 50° = 47.794°$ $\quad x - 50° = 180° - 47.794°$

$x = 97.794°$ $\quad x = 182.206°$

$= 97.8°$ $\quad = 182.2°$

(-) ∠ is not relevant as θ is in 1st /2nd quadrant



e.g. $2\sin 2x + \sqrt{3} = 0$ $-180° < x < 180°$

$2\sin 2x = -\sqrt{3}$

$\sin 2x = -\frac{\sqrt{3}}{2}$

1. let $2x = \theta$
2. θ is in 3rd or 4th quadrant

$-180° < x < 180°$

$-360° < 2x < 360°$

3. $-360° < \theta < 360°$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{\sqrt{3}}{2}$

$\alpha = \sin^{-1}(\frac{\sqrt{3}}{2})$

4. $\alpha = 60°$

can do negative angles as θ in 3rd/4th quadrant and θ < 180°

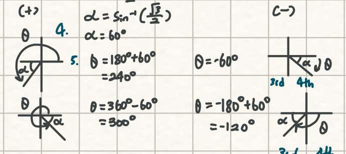
(+) (-)

5. $\theta = 180° + 60°$ $\quad \theta = -60°$

$= 240°$

$\theta = 360° - 60°$ $\quad \theta = -180° + 60°$

$= 300°$ $\quad = -120°$



6. $2x = 240°$ $\quad 2x = -60°$

$x = 120°$ $\quad x = -30°$

$2x = 300°$ $\quad 2x = -120°$

$x = 150°$ $\quad x = -60°$

10.6 R-Formulae

$a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

$a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$

$a, b > 0$, $R = \sqrt{a^2+b^2}$, $\alpha = \tan^{-1}\frac{b}{a}$

max of $\sin x/\cos x = 1$

min of $\sin x/\cos x = -1$

∴ max of $\sin(\theta+\alpha) = 1$

min of $\sin(\theta+\alpha) = -1$

(i) Express $7\cos\theta + 8\sin\theta$ in the form $R\cos(\theta+\alpha)$, $R>0$, $0° < \alpha < 90°$

1. let $\underline{7}\cos\theta + \underline{8}\sin\theta = R\cos(\theta+\alpha)$

$= \underline{R}\cos\theta\,\underline{\cos\alpha} + \underline{R}\sin\theta\,\underline{\sin\alpha}$ addition formulae

$7 = R\cos\alpha$ — (1) $\quad 8 = R\sin\alpha$ — (2)

(2) ÷ (1) $\quad (1)^2 + (2)^2$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{7}$ $\quad \tan\alpha = \frac{8}{7}$ $\quad R^2\cos^2\alpha + R^2\sin^2\alpha = 7^2 + 8^2$

$\alpha = 48.814°$ $\quad R^2(\underbrace{\cos^2\alpha + \sin^2\alpha}_{1}) = 7^2 + 8^2$

∴ $7\cos\theta + 8\sin\theta = 10.6\cos(\theta + 48.8°)$

$R = \sqrt{7^2+8^2}$

$= 10.630$

(ii) Hence find greatest + least values of $7\cos\theta + 8\sin\theta$ and corresponding values of θ. $0° \le \theta \le 360°$

max value of $10.630\underbrace{\cos(\theta+48.8°)}_{\max 1}$ is 10.6

let $\cos(\theta - 48.8°) = 1$

$\theta - 48.814° = 0°$ or $\theta - 48.814° = 360°$

$\theta = 48.8°$ (rej as $\theta \le 360°$)

$\cos 0°/\cos 360° = 1$

min value of $10.630\underbrace{\cos(\theta+48.8°)}$ is -10.6

let $\cos(\theta - 48.814°) = -1$

$\theta - 48.814° = 180°$

$\theta = 228.8°$

$\cos 180° = -1$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$((\cos A - \sin A)(\cos A + \sin A))$

$\cos^2 A = \frac{1}{2}(\cos 2A + 1)$

☆ $\tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

principle values:

$-90° \le \sin^{-1}x \le 90°$

$0° \le \cos^{-1}x \le 180°$

$-90° \le \tan^{-1}x \le 90°$

$\cos^{-1}(\frac{4}{5}) = \theta$

$\cos\theta = \frac{4}{5}$

linear law

$Y = mX + c$

m → gradient

c → constant, y-intercept

X → variable

$y = \frac{px}{x+q}$

$y(x+q) = px$

$x + q = \frac{px}{y}$

$x = \frac{px}{y} - q$

$Y = x$ $\quad X = \frac{x}{y}$

$m = p$ $\quad c = -q$

If x is in index, apply log and ln

$y = x^3 + p^{x-q}$

$y - x^3 = \frac{p^x}{p^q}$

$\ln(y - x^3) = \ln(\frac{p^x}{p^q})$

$\ln(y-x^3) = \ln p^x - \ln p^q$

$\ln(y-x^3) = x\ln p - q\ln p$

$Y = \ln(y-x^3)$ $\quad m = \ln p$

$X = x$ $\quad c = -q\ln p$

practise now 3

$y = \frac{a}{x^2+b}$, graph of $\frac{1}{y}$ (Y) against x^2 (X) drawn

line passes through (1,2) and (4,3)

Find (i) value of a and b (ii) value of y when $2x^2y = 1$

$y = \frac{a}{x^2+b}$

$x^2 + b = \frac{a}{y}$

$x^2 + b = a(\frac{1}{y})$

$\frac{1}{y} = \frac{x^2}{a} + \frac{b}{a}$

$Y = \frac{1}{y}$ $\quad m = \frac{1}{a}$

$X = x^2$ $\quad c = \frac{b}{a}$

gradient $= \frac{3-2}{4-1}$

$= \frac{1}{3}$

$\frac{1}{a} = \frac{1}{3}$

$a = 3$

Sub x=1 and y=2 into $Y = \frac{1}{3}X + c$

$2 = \frac{1}{3}(1) + \frac{b}{3}$

$\frac{5}{3} = \frac{b}{3}$

$b = 5$

$Y = \frac{1}{3}X + 5$

(ii) $2x^2y = 1$

$2x^2 = \frac{1}{y}$

$2X = Y$

$X = \frac{Y}{2}$

Sub $X = \frac{Y}{2}$ into $Y = \frac{1}{3}X + \frac{5}{3}$

$Y = \frac{1}{3}(\frac{Y}{2}) + \frac{5}{3}$

$Y = \frac{Y}{6} + \frac{5}{3}$

$Y = (1 - \frac{1}{6}) = \frac{5}{3}$

$Y = \frac{5}{3} \div (1 - \frac{1}{6})$ $\quad Y = 2$

point of intersection

$\frac{1}{y} = 2$ $\quad y = \frac{1}{2}$

Differentiation

$\frac{d}{dx}(2x^3) = 6x^{3-1}$
$= 6x^2$

$\frac{dy}{dx}$ → differentiate y with respect to x

↑ $\frac{dy}{dx}$ → upward sloping, gradient (+)
↓ $\frac{dy}{dx}$ → downward sloping, gradient (−)

Chain rule:
$y = \frac{6}{(2-5x^4)^3} = 6(2-5x^4)^{-3}$ → differentiate
$\frac{dy}{dx} = -18(2-5x^4)^{-3-1}(-20x^3)$
$= 360x^3(2-5x^4)^{-4}$

product rule:
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule:
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

find range of values of x for $y = \frac{3x}{x^2+1}$ is a decr. function

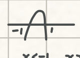
$\frac{dy}{dx} = \frac{(x^2+1)(3) - (3x)(2x)}{(x^2+1)^2}$

$= \frac{-3x^2+3}{(x^2+1)^2}$

for decr. function, $\frac{dy}{dx} < 0$

$\frac{-3x^2+3}{(x^2+1)^2} < 0$, $(x^2+1)^2$ (+) so can multiply with o, sign does not change

$-3x^2+3 < 0$
$-x^2+1 < 0$
$(1+x)(1-x) < 0$



$x < -1$ $x > 1$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \cdot f'(x)$

$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$

$y = \sin x$, $\frac{dy}{dx} = \cos x$
$y = \cos x$, $\frac{dy}{dx} = -\sin x$
$y = \tan x = \frac{\sin x}{\cos x}$, $\frac{dy}{dx} = \sec^2 x$

$y = \sin^4(x^2+x)$ $\frac{d}{dx}\sin$
$\frac{dy}{dx} = 4\sin^3(x^2+x)\cos(x^2+x)(2x+1)$

$y = (5\tan 4x+2)^2$
$\frac{dy}{dx} = 2(5\tan 4x+2)\underbrace{(5\sec^2 4x + 2 \times 4)}_{\frac{d}{dx}5\tan 4x+2}$ } chain rule

Application

Sub $x =$ __ into $\frac{dy}{dx}$, $\frac{dy}{dx}$ = gradient at $x =$ ___

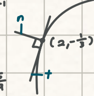
gradient of T × gradient of N = −1 → perpendicular to tangent

Find eqn of tangent and normal to curve, $y = \frac{x-3}{2x-1}$ when $y = -\frac{1}{3}$

Sub $y = -\frac{1}{3}$ into curve
$-\frac{1}{3} = \frac{x-3}{2x-1}$
$-2x+1 = 3x-9$
$x = 2$

$\frac{dy}{dx} = \frac{(2x-1)(1) - (x-3)(2)}{(2x-1)^2}$
$= \frac{5}{(2x-1)^2}$

Sub $x=2$ into $\frac{dy}{dx}$
$\frac{5}{(4-1)^2} = \frac{5}{9}$
at $x=2$, gradient $= \frac{5}{9}$



eqn of tangent
$y + \frac{1}{3} = \frac{5}{9}(x-2)$
$y = \frac{5}{9}x - \frac{13}{9}$

$\frac{5}{9}m = -1$
$m = -\frac{9}{5}$

eqn of normal
$y + \frac{1}{3} = -\frac{9}{5}(x-2)$
$y = -\frac{9}{5}x + \frac{49}{15}$

Rate of change

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

radius of sphere incr. at rate of 0.1 cm/s, find rate of increase of ($\frac{dr}{dt}$)
(i) surface area $\frac{dA}{dt}$
(ii) volume $\frac{dV}{dt}$
when radius is 12cm, ans in π

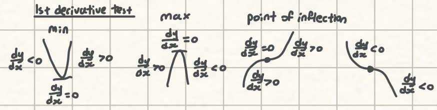
(i) $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
$A = 4\pi r^2$
$\frac{dA}{dr} = 8\pi r$
$\frac{dA}{dt} = 8\pi r \times 0.1$
$= 0.8\pi r$
Sub $r = 12$
$\frac{dA}{dt} = 9.6\pi\ cm^2/s$

(ii) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$
$V = \frac{4}{3}\pi r^3$
$\frac{dV}{dr} = 4\pi r^2$
$\frac{dV}{dt} = 4\pi r^2 \times 0.1$
$= 0.4\pi r^2$
Sub $r = 12$
$\frac{dV}{dt} = 0.4\pi(12)^2$
$= 57.6\pi\ cm^3/s$

Stationary point

$\frac{dy}{dx} = 0$ at $x =$ ___, (___, y) is a stationary point

1st derivative test



$y = x^3 - 3x^2 - 9x + 5$, find stationary point and their nature

$\frac{dy}{dx} = 3x^2 - 6x - 9$
at stationary point, $\frac{dy}{dx} = 0$
$0 = 3x^2 - 6x - 9$
$0 = (3x+3)(x-3)$
$x = -1$ $x = 3$

Sub $x=-1$ and $x=3$ into eqn of curve,
$y = 10$, $y = -22$
$(-1, 10)$, $(3, -22)$

x	-1^-	-1	-1^+
$\frac{dy}{dx}$	>0	0	<0
	/	—	\

x	3^-	3	3^+
$\frac{dy}{dx}$	<0	0	>0
	\	—	/

By 1st derivative test, $(-1, 10)$ is a max point, $(3, -22)$ is a min point

2nd derivative test

$\frac{dy}{dx} = 0$ → $\frac{d^2y}{dx^2} < 0$ max point
→ $\frac{d^2y}{dx^2} > 0$ min point
↓
$\frac{d^2y}{dx^2} = 0$ use 1st derivative test

At max/min gradient → $\frac{d^2y}{dx^2} = 0$

Intergration

$\int x^n = \frac{x^{n+1}}{n+1} + c$

$\int k(ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

e.g. $\int \frac{2}{\sqrt{3x+5}} = 2\int (3x+5)^{-\frac{1}{2}}$
$= 2\left[\frac{(3x+5)^{-\frac{1}{2}+1}}{3(-\frac{1}{2}+1)}\right] + c$
$= 2\left[\frac{(3x+5)^{\frac{1}{2}}}{\frac{3}{2}}\right] + c$
$= 2\left[\frac{2(3x+5)^{\frac{1}{2}}}{3}\right] + c$
$= \frac{4(3x+5)^{\frac{1}{2}}}{3} + c$

Trigo:
$\int \cos(ax+b) = \frac{1}{a}\sin(ax+b) + c$
$\int \sin(ax+b) = -\frac{1}{a}\cos(ax+b) + c$
$\int \sec^2(ax+b) = \frac{1}{a}\tan(ax+b) + c$

only this intergration allowed, if qn say smt else, use conversion

e.g. $\int \tan^2(2x+\frac{\pi}{2}) = \int \sec^2(2x+\frac{\pi}{2}) - 1$ $\sec^2 x = \tan^2 x + 1$
$= \frac{\tan(2x+\frac{\pi}{2})}{2} - \frac{1x^{0+1}}{1} + c$
$= \frac{\tan(2x+\frac{\pi}{2})}{2} - x + c$

Exponential:
$\int e^{ax+b} = \frac{1}{a}e^{ax+b} + c$

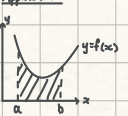
intergration of $\frac{1}{x}$ and $\frac{1}{ax+b}$
$\int \frac{1}{x} = \ln x + c$
$\int \frac{1}{ax+b} = \frac{1}{a}\ln(ax+b) + c$

e.g. $\int \frac{5}{6-9x}$
$= 5\int \frac{1}{6-9x}$
$= -\frac{5}{9}\int \frac{-9}{6-9x}$
$= \frac{-5}{9}\ln(6-9x) + c$
$= \frac{-5\ln(6-9x)}{9} + c$

Examples:
$\frac{dy}{dx}$ of $e^x(\cos x - \sin x) = -2e^x \sin x$
hence find $\int 5e^x \sin$
$\int 5e^x \sin x = 5\int e^x \sin x$
$= -\frac{5}{2}\int -2e^x \sin x$
$= -\frac{5}{2}e^x(\cos x - \sin x)$

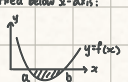
$\frac{7x+3}{(x-1)(2x+3)} = \frac{2}{x-1} + \frac{3}{2x+3}$
hence find $\int \frac{7x+3}{(x-1)(2x+3)}$
$\int \frac{7x+3}{(x-1)(2x+3)} = \int \frac{2}{x-1} + \frac{3}{2x+3}$
$= 2\int \frac{1}{x-1} + \frac{3}{2}\int \frac{2}{2x+3}$
$= 2\ln(x-1) + \frac{3}{2}\ln(2x+3) + c$

Application



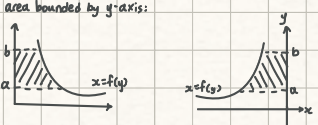
shaded area $= \int_a^b f(x)$

Area below x-axis:



$\int_a^b f(x) \to (-)$
shaded area $= -\int_a^b f(x)$

area bounded by y-axis:



shaded area $= \int_a^b x\,dy$ shaded area $= -\int_a^b x\,dy$

$\underbrace{\int_b^a f(x)}_{(+)} = -\underbrace{\int_a^b f(x)}_{(-)}$

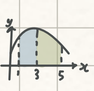
If $a=b$, $\int_a^b f(x) = 0$

$\int_a^b kf(x) = k\int_a^b f(x)$

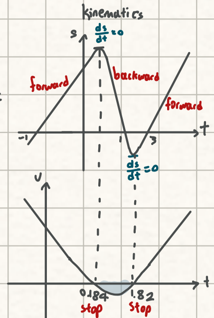
$\int_a^b [f(x) \pm g(x)] = \int_a^b f(x) \pm \int_a^b g(x)$

$\int_1^5 f(x) = 8$
$\int_1^5 \left[\frac{3}{4}f(x)\right] = \frac{3}{4}[f(x)]_1^5$
$= \frac{3}{4} \times 8$
$= 6$

$\int_1^3 f(x) + \int_3^5 [f(x) + 3]$
$= \int_1^5 f(x) + \int_3^5 3$
$= 8 + [3x]_3^5$
$= 8 + [3 \times 5 - 3 \times 3]$
$= 14$



kinematics



$s = t^3 - 3t^2 - t + 3$

displacement → distance travelled from starting point
e.g. $s = 3$, 3m away from start

$\frac{ds}{dt} = v = 3t^2 - 6t - 1$
when $\frac{ds}{dt} = 0$, $v = 0$
area under → displacement

$s < 0$ → ___ forward from start
$s > 0$ → ___ backward from start
$s = 0$ → at start → how far in total distance travelled may be > 0 as can travel backwards

$v < 0$ → obj moving backwards
$v > 0$ → obj moving forwards
$v = 0$ → obj stopped need to stop before change direction

$\frac{dv}{dt} = a = 6t - 6$

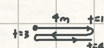
$a = 0$ → constant speed

A particle moves in a straight line and t s after point O, velocity is $v = 3t^2 - 12t + 9$
a) Find value of t when particle at rest
b) Find acceleration of particle when it first comes to rest
c) Find total distance from t=0 to t=4 seconds

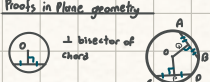
a) $3t^2 - 12t + 9 = 0$
$(t-3)(t-1) = 0$
$t = 3$ $t = 1$

b) $a = 6t - 12$
sub $t = 1$
$a = -6m/s^2$

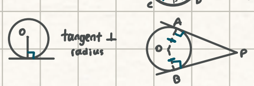
c) $s = \int v = t^3 - 6t^2 + 9t + c$
when $t=0$, $s=0$ ← at point O
∴ $c = 0$
turning pt → $t = 1$, $s = 4m$
← $t = 3$, $s = 0$
$t = 4$, $s = 4m$
distance $= 4 + 4 + 4$
$= 12m$



Proofs in Plane geometry



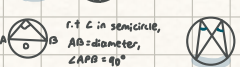
equal chords
AB = CD, OP = OQ
OP = OQ, AB = CD



tangent from ext. pt
AP = BP
OP bisects ∠APB + ∠AOB

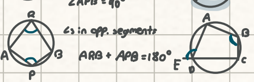
angles

∠ at centre = 2 × ∠ at circumference



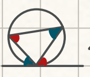
r.t ∠ in semicircle,
AB = diameter,
∠APB = 90°

∠s in same segment



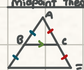
∠s in opp. segments
ARB + APB = 180°

Ext. ∠ of cyclic quadrilateral
∠EDA = ∠ABC



∠s in alt. segments
∠ between chord + tangent = ∠ opp. of same chord

midpoint theorem



if AB = BD,
and AC = CE
then BC // DE