9749 H2 Physics Topic 20 — Nuclear Physics

DUNMAN HIGH SCHOOL

Year 6 (2025)

Guiding Questions

- Why are some nuclei radioactive? What happens during radioactive decay? Where does the energy come from?
- What is emitted during radioactive decay, and is it harmful? How do decay products interact with matter?
- What are the uses of radioactivity?
- How are nuclear reactions different from chemical reactions? How are they similar?
- What can conservation laws tell us about nuclear processes?

Content

- The nucleus
- Isotopes
- Mass defect and nuclear binding energy
- Nuclear processes
- Radioactive decay
- Biological effect of radiation

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus

(b) distinguish between nucleon number (mass number) and proton number (atomic number)

(c) show an understanding that an element can exist in various isotopic forms each with a different number of neutrons in the nucleus

(d) use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$

(e) state and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes.

(f) show an understanding of the concept of mass defect

(g) recall and apply the equivalence between energy and mass as represented by $E = mc^2$ to solve problems

(h) show an understanding of the concept of nuclear binding energy and its relation to mass defect

(i) sketch the variation of binding energy per nucleon with nucleon number

(j) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission

(k) show an understanding of the spontaneous and random nature of nuclear decay

(l) infer the random nature of radioactive decay from the fluctuations in count rate

(m) show an understanding of the origin and significance of background radiation

(n) show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required)

(o) show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required)

(p) define the terms activity and decay constant and recall and solve problems using the equation $A = \lambda N$

(q) infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0 \exp(-\lambda t)$ where x could represent activity, number of undecayed particles or received count rate

(r) define and use half-life as the time taken for a quantity x to reduce to half its initial value

(s) solve problems using the relation $\lambda = \dfrac{\ln 2}{t_{\frac{1}{2}}}$

(t) discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the nucleus

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Rutherford's α-particle scattering experiment: Setup

Rutherford's experiment consisted of a beam of alpha particles $\left({}^{4}_{2}\text{He}\right)$, generated by the radioactive decay of radium, directed normally onto a sheet of very thin gold foil (~ 4×10^{-7} m thick) in an evacuated chamber (Fig. 1).

Alpha particles produce a tiny, but visible flash of light when they strike the fluorescent zinc sulphide screen, which is used as a detector, at the focus of a microscope; the screen and microscope could be swivelled around the foil to observe particles deflected at any given angle.

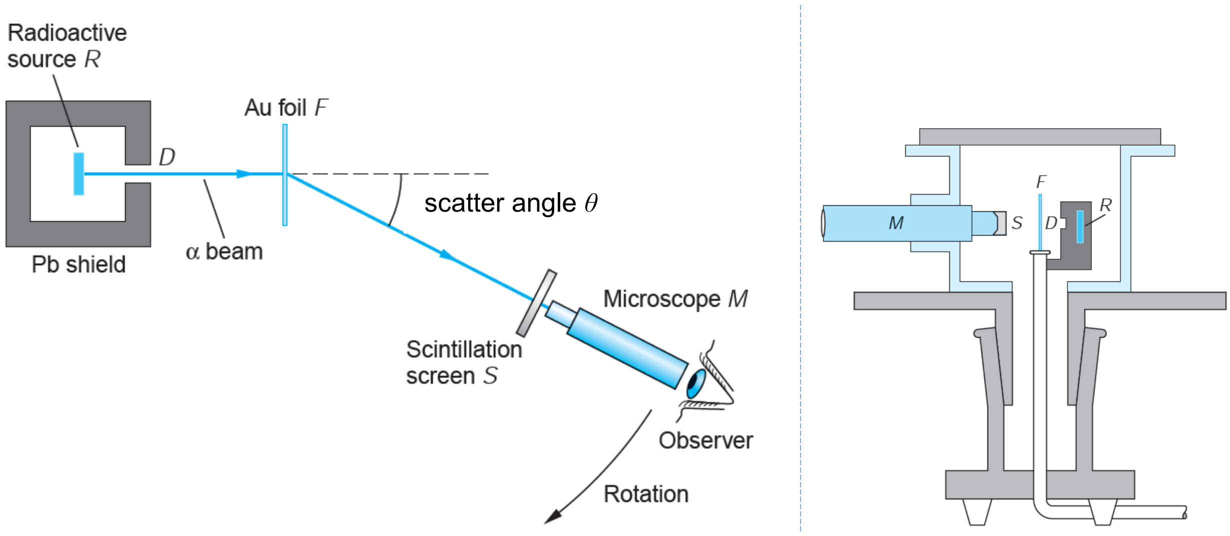


Figure 1: (Left) Schematic diagram of the apparatus used by Geiger and Marsden to test Ernest Rutherford's atomic model. (Right) The actual apparatus.

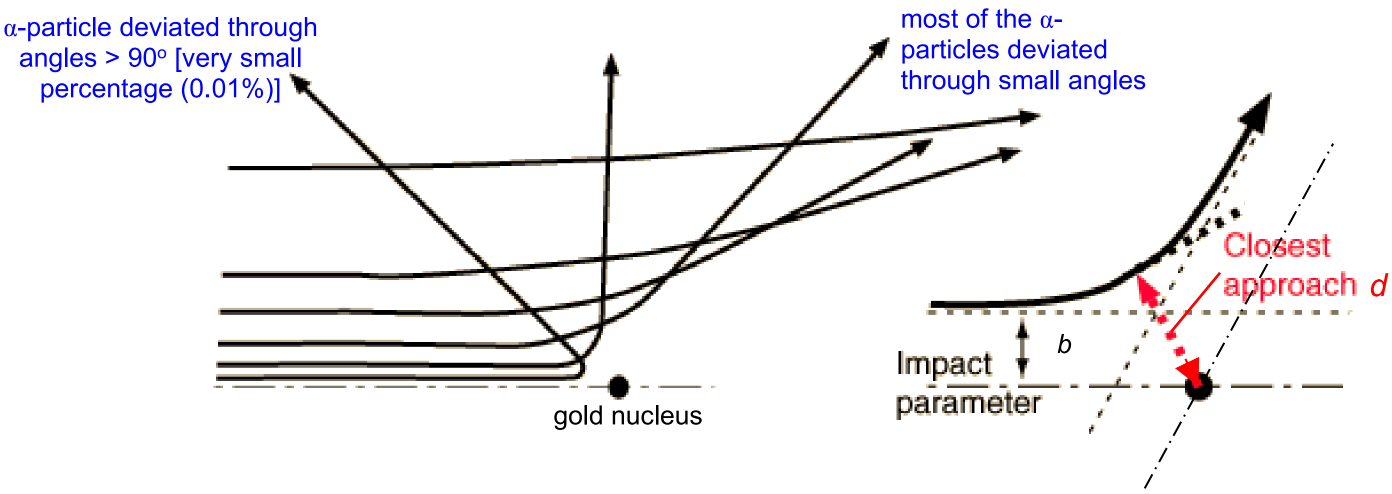


Figure 2: Possible paths of scattered α-particles

Rutherford's α-particle scattering experiment: Observations

1 Most of the α-particles were deviated through small angles.

2 A small but significant percentage of the α-particles were deviated through angles larger than 90°, (Fig. 2).

Rutherford's α-particle scattering experiment: Inferences

Observation	Inference
Most of the α-particles were deviated through small angles.	
A smaller but significant percentage of the α-particles deviated through large angles greater than 90°.	

Models for the atom

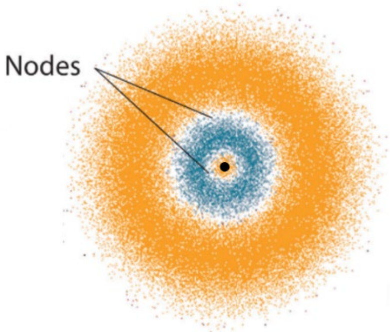


Figure 3a: The quantum mechanical model of an atom, in this case, helium. The quantum mechanical model is based on mathematics. Although it is more difficult to understand than the Bohr model, it can be used to explain observations made on complex atoms.

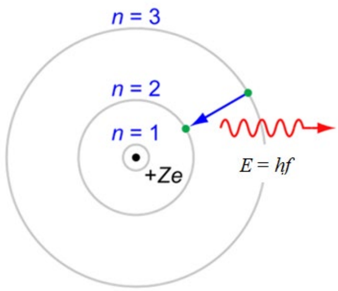


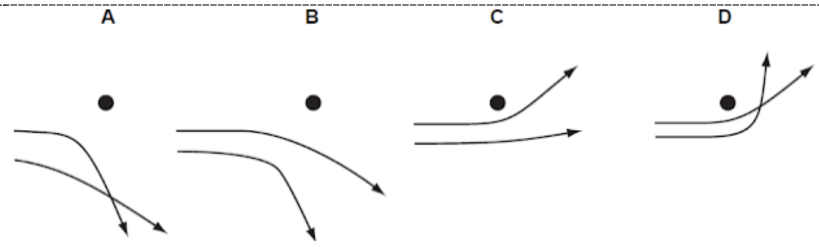
Figure 3b: The Bohr model of an atom. It can make accurate predictions for the behaviour of simple atoms such as hydrogen.

The currently accepted model of an atom (Fig. 3a) consists of a dense central nucleus surrounded by a cloud of negatively charged electrons:

- Over 99.94% of an atom's mass is concentrated in the nucleus with protons and neutrons having roughly equal mass.
- The protons are positively charged while the neutrons have no charge.
- The electrons determine the chemical properties of an element, and strongly influence an atom's magnetic properties.

Example 1 (2013 P1 Q40)

Two α-particles with equal energies are fired towards the nucleus of a gold atom.
Which diagram could represent their paths?

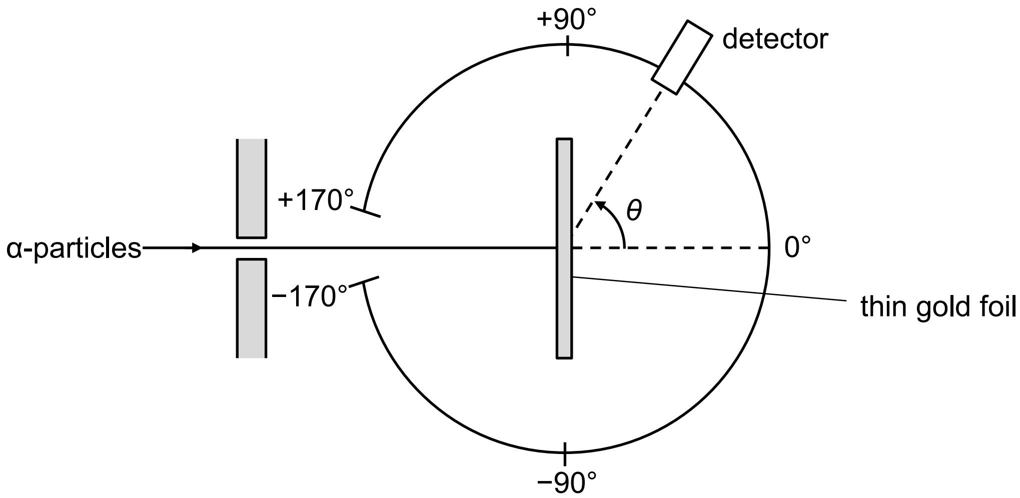


Solution: (Answer: ……….)

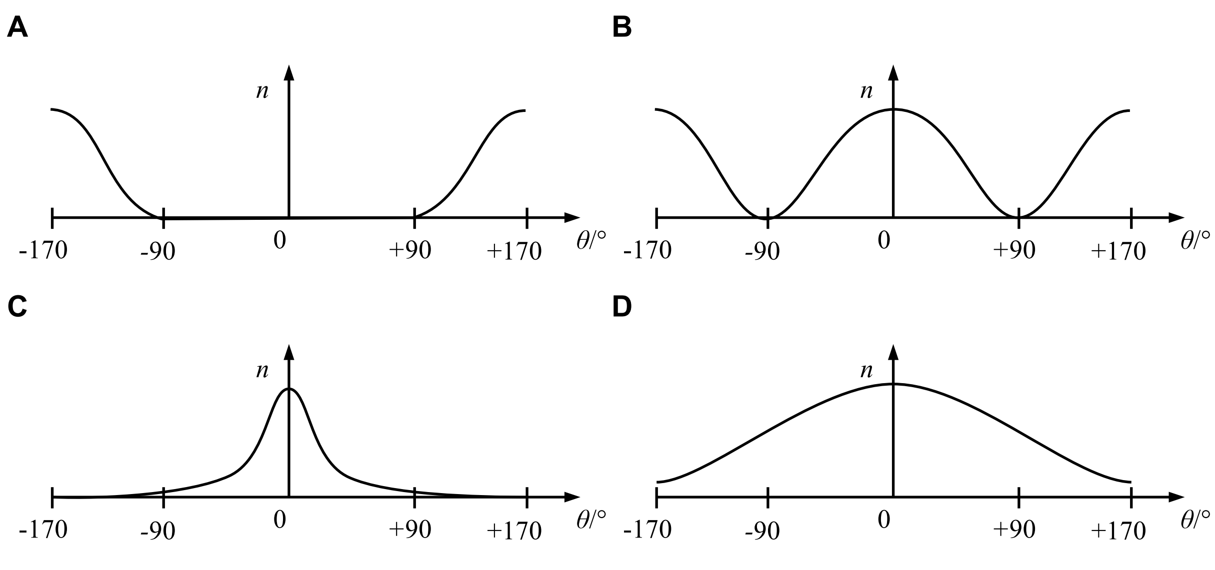
……….. : Both α-particle and the gold nucleus are positively charged and should …………….. each other instead of …………….. each other. The α-particle travelling closer to the gold nucleus should undergo greater deflection.
Thus, the answer is ……... instead of ……...

Example 2 (2002 P1 Q30)

In an α-particle scattering experiment, a student determined the number n of α-particles incident per unit time on a detector held at various angular positions θ.



Which graph best represents the variation of n with θ?



Solution: (Answer: ……….)

Refer to "Rutherford's α-particle scattering experiment: Observations - Point 1".

(b) distinguish between nucleon number (mass number) and proton number (atomic number)

Atomic Structure

Protons and neutrons in a nucleus are collectively known as **nucleons**. The total number of protons and neutrons in a nucleus is equal to its **nucleon number *A*** (which is an integer, without unit). Nucleon number is also known as **mass number**.

Proton number *Z* of a nucleus is equal to the number of protons in it. Each element in the Periodic Table has a unique proton number in the nucleus of its atom, starting with hydrogen having proton number = 1, helium having proton number = 2, etc. It is the proton number in the nucleus of its atom that determines the element. Hence, proton number is also known as **atomic number** of the nucleus of the atom of that element.

All atoms are neutral. In an atom, the proton number of its nucleus and the number of electrons orbiting this nucleus must be the same.

Why is *A* also known as the *mass number*?

	Charge	**Mass**
Proton	$+1.60 \times 10^{-19}$ C or $+e$ (e is elementary charge)	1.6726×10^{-27} kg
Neutron	0	1.6749×10^{-27} kg

As the mass of protons and neutrons is small, it is more convenient to write the mass of a nucleus or atom in *unified atomic mass constant* (symbol u) rather than in kg.

The **unified atomic mass constant**, u, is defined as $\frac{1}{12}$ *of the mass of a carbon-12 atom.*

Thus

$$1\ u = 1.66 \times 10^{-27}\ \text{kg}$$

Nucleons have masses of approximately 1 u (proton mass = 1.007 u, neutron mass = 1.008 u, electron mass = 0.00055 u). Since the mass of protons and neutrons are approximately the same and the electron is much less massive (about 1840 times lighter), the mass of a nucleus (or an atom) is approximately $A\ u$. This is why A is called the *mass number*.

(c) show an understanding that an element can exist in various isotopic forms each with a different number of neutrons in the nucleus

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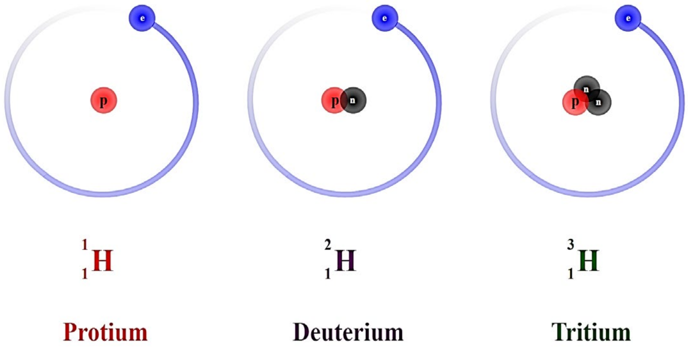
Isotopes

Isotopes are atoms of the same element with the ………………………………………. but ………………………………………. in the nucleus.

Quite a number of elements have isotopes. Hydrogen, for instance, has 3 naturally occurring isotopes. The most common isotope of hydrogen has 1 proton in its nucleus, with no neutron, and is known as protium or hydrogen-1 (i.e. it has 1 nucleon). Another isotope which has 2 nucleons is known as deuterium or hydrogen-2, and yet another which has 3 nucleons is known as tritium or hydrogen-3.



Chemically, these isotopes are identical (since chemical reactions only involve orbital electrons, and all isotopes of the same element have the same number of orbital electrons), but their nuclei have different masses.

Physically, isotopes exhibit different characteristics.

(d) use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$

Symbolic representation of nuclides

Nuclide:

Nuclide is a particular species (type) of nucleus that is specified by its proton number and neutron number.

E.g., Nitrogen-17 $\left(^{17}_{7}N\right)$ and Oxygen-17 $\left(^{17}_{8}O\right)$ are nuclides (but they are not isotopes).

The usual notation used for the representation of a nuclide is as follows:

$$^{A}_{Z}X$$

where: X is the chemical symbol of the element
(e.g. H for hydrogen, He for helium)
A is the nucleon number (mass number); and
Z is the proton number (atomic number).

The number of neutrons in a nuclide of element X can be determined from $^{A}_{Z}X$ by subtracting *Z* from *A* i.e. number of neutrons = $A - Z$

Protium, deuterium and tritium can be represented by notations ^1_1H, ^2_1H and ^3_1H respectively.

A nuclear reaction

In a nuclear reaction, there is a transformation of at least one ……………………….. to another.

A nuclear reaction must be caused by an **external stimulus** such as the collision of two or more nuclides and can be represented by a **nuclear equation** as follows:

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H \quad \text{---- (1)}$$

<u>Conservation laws in nuclear reactions</u>

Nuclear reactions obey 4 important conservation laws:

(i) **Conservation of nucleon number**: *Sum of the mass numbers of the nuclides after reaction is equal to the sum of the mass numbers of nuclides before reaction.*
14 + 4 = 17 + 1 [from (1)]

(ii) **Conservation of proton number**: *Sum of the atomic numbers of the nuclides after reaction is equal to the sum of the atomic numbers of the nuclides before reaction.*
7 + 2 = 8 + 1 [from (1)]

(iii) **Conservation of mass-energy**: *For an isolated system, the total mass-energy of the system remains constant.*

(iv) **Conservation of linear momentum**

The conservation of mass-energy in a nuclear reaction is related to the concept of **mass defect** and **binding energy**.

(f) show an understanding of the concept of mass defect

<u>Mass defect</u>

The mass of a single nucleus is *less* than the total mass of its individual nucleons. E.g., when a single helium nucleus is split up to give 2 protons and 2 neutrons, the mass of the single helium nucleus is *less* than the total mass of its 4 individual nucleons (Fig. 4), so there is a shortage of mass (**mass defect**). The mass defect is 0.03035 u (= 4.03188 u – 4.00153 u).

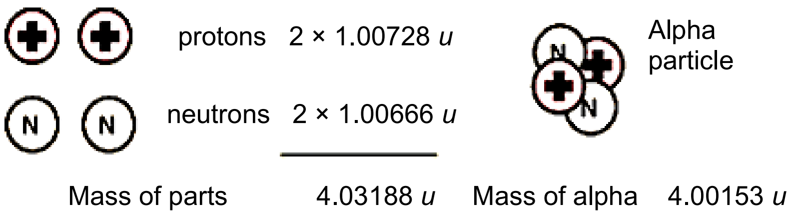


Figure 4: Mass defect of helium nucleus, which is an alpha particle

Since mass is a measure of energy content, any change in mass is accompanied by a corresponding change in energy and vice versa.
A helium nucleus has less mass than its constituent nucleons; in pulling them apart, we do work and so give them energy; hence their mass is greater.
This <u>mass defect</u> of 0.03035 u is a measure of the <u>nuclear binding energy</u>.

Mass defect of a nucleus:
The difference in mass between the mass of a nucleus and the total mass of its constituent nucleons.

mass defect, Δm = (total mass of its constituent nucleons) – (mass of the nucleus)

$$\text{mass defect} = \left(Zm_{\text{p}} + Nm_{\text{n}}\right) - m_{\text{nucleus}}$$

$$\approx \left(Zm_{\text{H-atom}} + Nm_{\text{n}}\right) - m_{\text{atom}} \qquad (*)$$

We use equation (*) to calculate mass defect because it is much more difficult to measure masses of bare nuclei with high precision.

Note:
(a) The mass of a nucleus is always less that the mass of its constituent nucleons.
(b) It is related to the nuclear binding energy according to the equation:

$$\text{nuclear binding energy} = (\text{mass defect})c^2$$

Nuclear binding energy of a nucleus:

……

……

……

or

……

……

……

(g) recall and apply the equivalence relationship between energy and mass as represented by $E = mc^2$ to solve problems.
(h) show an understanding of the concept of nuclear binding energy and its relation to mass defect
(e) state and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes.

Equivalence relationship between energy and mass

After successfully explaining photoelectric effect, Einstein in 1905, through his Special Theory of Relativity, went on to show that mass is a form of energy, via his now-famous equation:

$$E = mc^2$$

where: E represents energy in J;
m represents mass in kg; and
c represents the speed of light in vacuum ($c = 3.00 \times 10^8$ m s^{-1})

Thus, the relationship between the nuclear binding energy E_b and mass defect of a nucleus Δm is given by

$$E_b = (\Delta m)c^2$$

Example 3

A nucleus has a nucleon number A, a proton number Z, and a binding energy B. The masses of the neutron and proton are m_n and m_p, respectively, and c is the speed of light. The mass of the nucleus is given by

A $(A - Z)m_n + Zm_p - \frac{B}{c^2}$

B $(A + Z)m_n + Zm_p + \frac{B}{c^2}$

C $Am_n + Zm_p - \frac{B}{c^2}$

D $Am_n + Zm_p + \frac{B}{c^2}$

Solution: (Answer: ……….)

Putting it altogether

Consider nuclear reaction (1) again as follows:

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H \quad \text{---- (1)}$$

One way of thinking about this nuclear reaction is that it involves

- the breaking up the nitrogen nucleus and helium nucleus into its individual nucleons of 9 neutrons and 9 protons (this process requires absorption of energy) and then
- assembling them back into an oxygen nucleus and a hydrogen nucleus (this process requires a release of energy).

Thus, if the total mass of the reactants is *more* than the total mass of the products, then the nuclear reaction will be *exothermic*; which means there is a net release of energy. So, the total binding energy of the reactants is *less* than the total binding energy of the products.

If the total mass of the reactants is *less* than the total mass of the products, then the nuclear reaction will be *endothermic*; which means it requires a net energy input. So, the total binding energy of the reactants is *more* than the total binding energy of the products.

In a nuclear reaction, **mass-energy is conserved**!

Example 4

When lithium (^{7}Li) is bombarded by a proton, two alpha particles (^{4}He) are produced. Find the net energy released during such a reaction. What is the difference in total binding energies of the reactants and products?

The masses are:

$^{1}_{1}H$: 1.007825*u* $^{4}_{2}He$: 4.002603*u* $^{7}_{3}Li$: 7.016004*u*

Solution

The reaction can be written

$$^{1}_{1}H + ^{7}_{3}Li \rightarrow ^{4}_{2}He + ^{4}_{2}He$$

The initial and final masses:

$^{1}_{1}H$: 1.007825*u*	$^{4}_{2}He$: 4.002603*u*
$^{7}_{3}Li$: 7.016004*u*	$^{4}_{2}He$: 4.002603*u*
8.023829*u*	8.005206*u*

Example 5

By bombarding various gases with alpha particles, in 1919, Rutherford successfully carried out the 1st nuclear reaction:

$$\begin{matrix} {}^{14}_{7}\text{N} & + & {}^{4}_{2}\text{He} & \rightarrow & {}^{1}_{1}\text{H} & + & {}^{17}_{8}\text{O} \\ 14.00307u & & 4.00260u & & 1.00783u & & 16.99913u \end{matrix} \quad \text{--- (1)}$$

How much energy was released or absorbed in this reaction?

Solution:

Total mass of reactants = 18.00567 *u*
Total mass of products = 18.00696 *u*

Total mass of reactants < Total mass of product,
Thus, this reaction is **………………………………**

Amount of energy **absorbed**

Example 6

A stationary isotope ${}^{226}_{88}\text{Ra}$ decays into ${}^{222}_{86}\text{Rn}$ with the emission of an α-particle and a γ-ray photon of frequency ν.

The principle of conservation of energy is expressed in this decay by the equation

A $[m_{\text{Ra}} - m_{\text{Rn}}]\, c^2 = h\nu$

B $\frac{1}{2}m_{Rn}u_{Rn}^2 = h\nu + \frac{1}{2}m_\alpha u_\alpha^2$

C $[m_{\text{Ra}} - (m_{\text{Rn}} + m_\alpha)]\, c^2 = h\nu + \frac{1}{2}m_\alpha u_\alpha^2 + \frac{1}{2}m_{Rn}u_{Rn}^2$

D $[(m_{\text{Rn}} + m_\alpha) - m_{\text{Ra}}]\, c^2 = h\nu + \frac{1}{2}m_\alpha u_\alpha^2 + \frac{1}{2}m_{Rn}u_{Rn}^2$

where appropriate, *m* represents the mass of a particle, *u* the speed of a particle and *c* the speed of electromagnetic radiation.

Solution: (Answer: ……….)

This process may be represented by

Since ${}^{226}_{88}\text{Ra}$ was originally stationary, it must have higher mass than the products such that $\Delta m = m_{\text{Ra}} - (m_{\text{Rn}} + m_\alpha)$
The energy equivalent of this difference in mass, $E = (\Delta m)\, c^2 = [\, m_{\text{Ra}} - (m_{\text{Rn}} + m_\alpha)\,]\, c^2$
This energy is **released** in the form of the kinetic energies of the product particles and the γ-ray: $h\nu + \frac{1}{2}m_\alpha u_\alpha^2 + \frac{1}{2}m_{Rn}u_{Rn}^2$

By conservation of energy,

(i) sketch the variation of binding energy per nucleon with nucleon number

Binding energy per nucleon of a nucleus: *The binding energy of a nucleus divided by the number of nucleons in the nucleus.*

Thus, it is the average energy per nucleon required to completely separate the nucleons (in the nucleus) to infinity.

Or it is the average energy released per nucleon when nucleons come together (to form nucleus) from infinity.

Note: The higher the binding energy per nucleon, the more stable the nucleus is. The highest binding energy per nucleon (≈ 8.8 MeV) occurs at the region near A ≈ 56. Iron-56 is one of the examples of the most stable nuclei found.

Variation of binding energy per nucleon with nucleon number

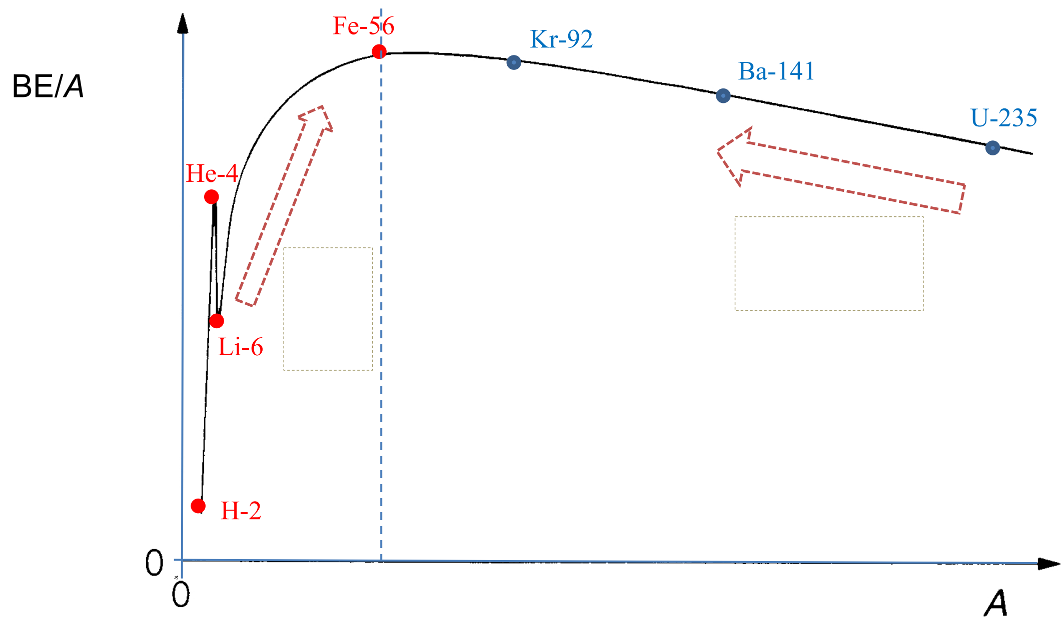


Figure 5: Binding energy per nucleon, BE/*A* (in MeV) vs nucleon number *A* graph

Some important features of the graph (Fig 5):

- The maximum binding energy per nucleon for all nuclei is ~ 8.8 MeV.
- The peak is at $A \approx 56$, so iron-56 is the most tightly bound nucleus. This is the approximate basic reason why iron is a very common metal in planetary cores, since they are produced profusely as end products in the final stages of silicon burning in stars.
- The smallest nucleus with binding energy is deuterium ($^{2}_{1}H$). Its binding energy per nucleon is ~1.1 MeV. The smallest of all nuclei is hydrogen ($^{1}_{1}H$), which has no binding energy (only 1 proton).
- The largest nucleus in this particular graph is uranium-235 ($^{235}_{92}U$), which has a binding energy per nucleon of about ~7.6 MeV.
- The slope of the graph on the left is very steep, and positive; the slope of the graph on the right is gradual and negative.

(j) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission

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Nuclear stability

- *high* binding energy per nucleon → need large amount of energy to break nucleus into individual nucleons → nucleus is *stable*

- *low* binding energy per nucleon → need small amount of energy to break nucleus into individual nucleons → nucleus is *unstable*

Binding energy per nucleon is a **………………………………………………………………………………**

, i.e., how tightly a nucleus is bound (or how strongly bound the nucleons are within the nucleus)

The greater the value of BE/*A*, the greater is the average energy per nucleon required to completely separate the nucleons in a nucleus to infinity, i.e., the more stable is the nucleus.

Nuclear Fission vs Nuclear Fusion

How the graph in Fig. 5 indicates that large amounts of energy are available from both the fission and the fusion processes?

Nuclear reactions occur when the reactants have lower binding energy per nucleon than the products. Fig. 5 shows that this can happen in two ways:

a. two or more light nuclei with very small values of *A* and low BE/*A* join together (fusion) to produce ________________________ that have greater BE/*A* higher up on the graph line (see arrow), or
b. massive nuclei with low BE/*A* can split to form two or more lighter nuclei (fission) each with ___________ that are higher up on the graph line (see arrow)

In both fusion and fission, since the total number of nucleons is conserved, this implies that the **products have greater <u>total binding energy</u> compared to the reactants** and energy is **released** in the form of kinetic energy of the products and radiation, usually gamma rays. The products are **more stable** or **more tightly bound** than the reactants. The products also have _____________ compared to the reactants, and that decrease in mass is equivalent to the energy released, according to the Einstein relationship.

Since

$$\text{binding energy, BE} = \text{binding energy per nucleon} \times A,$$

So

$$\text{energy released} = \text{total BE of products} - \text{total BE of reactants}$$

Nuclear fission:
The splitting of a nucleus of high nucleon number into two smaller nuclei of approximately the same mass with the release of energy and neutrons. The nuclei produced have greater binding energy per nucleon.

Examples of nuclear fission

Nuclear fission (Fig. 6) is usually instigated by a neutron.

E.g., when a large nucleus like uranium-235 ($^{235}_{92}U$) is bombarded by neutrons ($^{1}_{0}n$), one of the many fission reactions that it can undergo is the following:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{148}_{57}La + ^{85}_{35}Br + 3^{1}_{0}n$$

The products are lanthanum ($^{148}_{57}La$) and bromine ($^{85}_{35}Br$), with the emission of 3 neutrons. Another possible nuclear fission with $^{235}_{92}U$, which gives rise to a rhodium nucleus ($^{123}_{45}Rh$), a silver nucleus ($^{111}_{47}Ag$) and 2 neutrons may be represented as follows:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{122}_{45}Rh + ^{112}_{47}Ag + 2^{1}_{0}n$$

These reactions produce additional neutrons, which can then go on and hit other nuclei and cause a chain reaction.

Note:

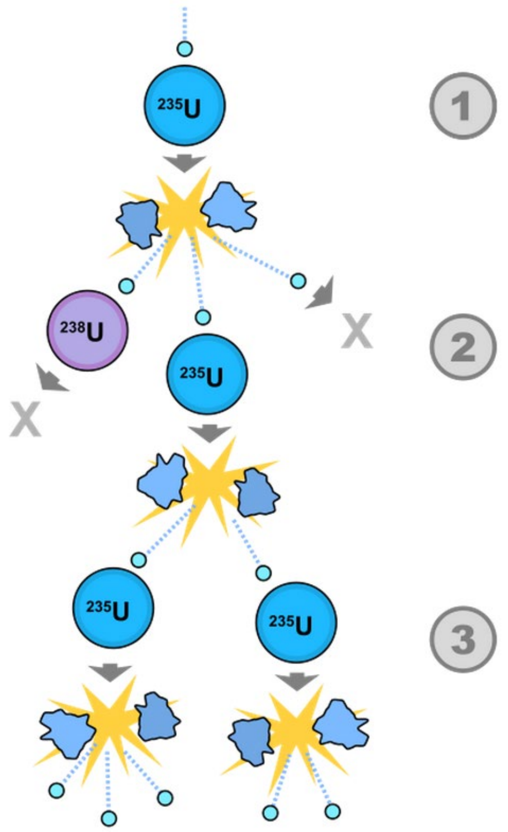
The neutron may be *slow*, with energy in the range of 0.025 eV to 10 eV or *fast*, with energy in the range of 1 MeV to 20 MeV. *Slow* neutrons are used in thermal reactors while *fast* neutrons are used in fast reactors.

Figure 6 (right): A schematic nuclear fission chain reaction.

1. A uranium-235 atom absorbs a neutron and fissions into two new atoms (fission fragments), releasing three new neutrons and some energy.

2. One of those neutrons is absorbed by an atom of uranium-238 and does not continue the reaction. Another neutron is simply lost and does not collide with anything, also not continuing the reaction. However, one neutron does collide with an atom of uranium-235, which then fissions and releases two neutrons and some energy.

3. Both of those neutrons collide with uranium-235 atoms, each of which fissions and releases between one and three neutrons, which can then continue the reaction.



Chain reaction:
A series of similar reactions in which one of the products of a reaction causes further reactions to occur.

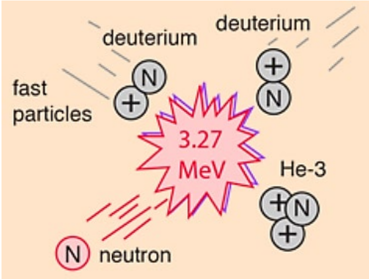
Nuclear fusion:
The combining or fusing of two nuclei of low nucleon number to produce a larger nucleus with the release of energy.

Examples of nuclear fusion

Even though a lot of energy is required to overcome the Coulomb barrier and initiate hydrogen fusion, the energy yields are enough to encourage continued research. Hydrogen fusion on the Earth could make use of the reactions:

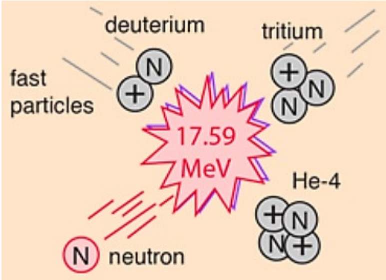
When 2 deuterons (nuclei of deuterium) are fused to form a helium nucleus, a neutron is released in the process, represented by

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n + 3.27\text{ MeV}$$



Another possible nuclear fusion is the formation of a helium nucleus from a deuterium nucleus and a tritium nucleus, represented by

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.59\text{ MeV}$$



There is no chain reaction in nuclear fusion.

Why is nuclear fission 'easier' to initiate than nuclear fusion?

In nuclear fission, the uncharged neutron can be captured easily by the nucleus due to the strong nuclear force. There is ______________________ between the neutron and the positively charged nucleus. In fusion, the charged nuclei (e.g. deuterium) repel each other. Hence, they need to have large KE before they can get close enough ($\approx 10^{-14}$ m) for the strong nuclear force to take effect.

The mean KE of the nuclei is directly proportional to the absolute temperature of the gas. High temperatures ($\approx 10^{8}$ K) are required for the onset of fusion between nuclei, if temperature alone is considered in the process.

Example 7

Data:
mass of ${}^{235}_{92}U$ = 235.04393 u
mass of ${}^{141}_{56}Ba$ = 140.91434 u
mass of ${}^{92}_{36}Kr$ = 91.92625 u
mass of ${}^{1}_{0}n$ = 1.008665 u
1 u = 1.66 × 10^{-27} kg
c = 3.00 × 10^{8} m s^{-1}

The nuclear equation for ${}^{235}_{92}U$ undergoing fission is:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + k\,{}^{1}_{0}n$$

where k is the number of neutrons released in the fission process.
Calculate:

(a) the number k of neutrons released,
(b) the change in mass, Δm, and
(c) the energy released by each ${}^{235}_{92}U$ nucleus after the fission process.

Solution:

(k) show an understanding of the spontaneous and random nature of nuclear decay

Nature of radioactive/nuclear decay

Radioactive decay or **nuclear decay**:
The spontaneous and random disintegration of the **unstable** nucleus. From the nucleus, some or all of the following nuclear radiations may be emitted: alpha particle, beta particle, gamma radiations.

Other particles/types of electromagnetic radiation e.g., neutrino, antineutrino, and positron are emitted too, but these are not in H2 syllabus)

Spontaneous decay:

Random decay:

In a large sample of radioactive nuclide, the probability of decay per unit time is constant and the percentage of nuclei that will decay over time is predictable based on statistical analysis.

(l) infer the random nature of radioactive decay from the fluctuations in count rate

Random nature of radioactive/nuclear decay

Since radiations from radioactive disintegrations cannot be seen with the naked eye, we need detectors. A detector, e.g., a Geiger-Muller tube (G.M. tube), is always connected to a counting device, known as a counter. Whenever an **ionising** radiation is detected, the counter would obtain an **electrical** signal which would be counted.

Of course, not all radiations can be detected and counted. In fact, most of the emitted radiations would be radiated in other directions and would not be counted. The number of counts per unit time recorded by a counter is known as '**count rate**' (Fig. 7). The fact that count rate fluctuates indicates that radioactive decay must be emitting radiations in a fluctuating manner that is unpredictable, i.e., it is random.

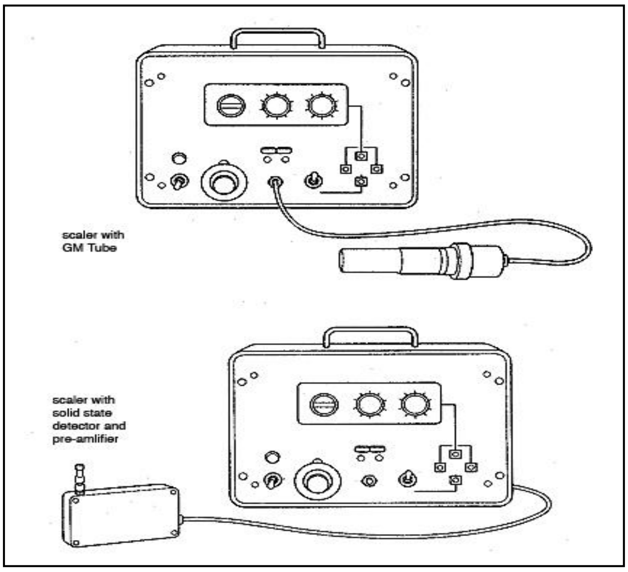


Figure 7: Example of a Geiger-Muller tube, connected to a counting device

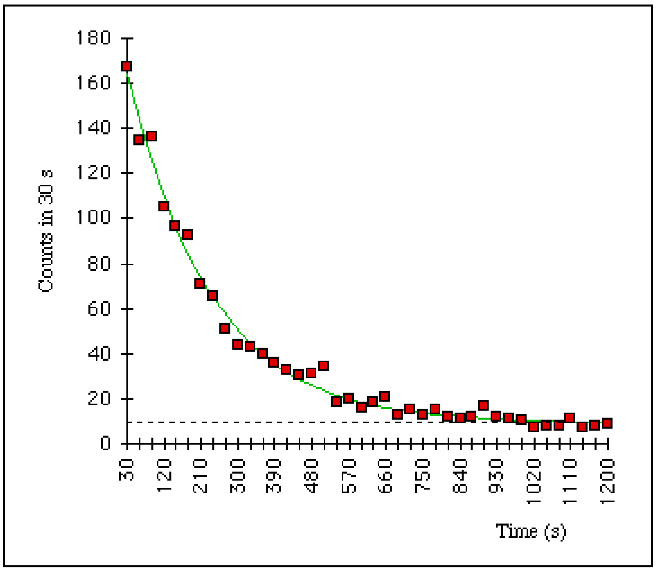


Figure 8: An example of radioactive decay data, counting numbers of decays observed in 30s intervals. The best-fit exponential curve is shown. The points do not fall exactly on the line but shows the fluctuations due to random nature of radioactive decay. For a large sample size, **statistically**, its activity is directly proportional to the number of radioactive nuclei present in the sample.

(m) show an understanding of the origin and significance of background radiation

Background radiation is the radiation detected by a radiation counter when no radioactive source is nearby.

It is an unavoidable radiation arising from natural resources, e.g., cosmic rays, radioactive substances in rocks and the atmosphere.

The background radiation at a place detected by a *Geiger counter* is discrete, erratic/random, and is roughly constant.

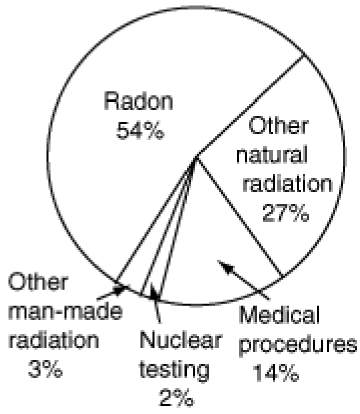
When measuring count-rate from a source, the background count-rate is deducted from the measured count-rate in order to obtain a more reliable measurement of the count-rate from the source.

Count rate: the rate at which emissions from a radioactive source are detected. Its unit: per second (s^{-1})

The count rate is a fraction of the activity *A* (to be discussed) of a radioactive sample.

Radioactive material is found throughout **nature**:

- Detectable amounts (Carbon-14 [1]) occur naturally in soil, rocks, water, air, and vegetation, from which it is inhaled and ingested into the body.



The worldwide average **natural** dose to humans is about four times more than the worldwide average **artificial** radiation exposure.

- Nearly all artificial background radiation comes from medical imaging procedures such as receiving X-rays for X-ray photographs.
- Some artificial background radiation comes from
  - above ground nuclear testing accidents such as the Chernobyl and Fukushima incidents, and
  - cigarettes

Example 8 (2008 P1 Q39)

A detector of ionising radiation gives a background count rate of 24 per minute.
A radioactive source is placed close to the detector and the reading is 532 counts per minute.
What will be the reading of the detector after two half-lifes of the source?

A 127 **B** 133 **C** 151 **D** 157

Solution: (Ans : ……….)

The original activity of the source is

After two half-lives, the activity becomes

The reading of the detector includes the background count rate i.e.

(n) show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required).

There are 3 kinds of radiations emitted from radioactive nuclei: α, β and γ radiations.

α-particles

Alpha particle contains two protons and two neutrons and is the same as a helium nucleus $^{4}_{2}He$. It has a charge of +2*e* and mass 4*u*. It may be emitted from a radioactive nucleus.

Alpha decay:
The spontaneous decay of a nucleus with the emission of an alpha particle.

Note:

(1) Frequently, some of the energy released in the decay is in the form of gamma ray photon.

(2) The parent nucleus is radioactive, but not the alpha particle.

[1] Its presence in organic materials is the basis for radiocarbon dating

In an α-particle decay, a nucleus X, called the parent nucleus, will emit a helium nucleus, and transmute into Y, called the daughter nucleus:

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Since X is unstable compared to Y, the mass of the reactant is more than the mass of the products and the decay is exothermic.

Example 9

A stationary $^{238}_{92}U$ nuclide decays by emitting an α-particle $\left({}^{4}_{2}He\right)$ as follows:

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

Given the atomic mass of

$^{238}_{92}U$ = 238.0507826 *u*, $^{234}_{90}Th$ = 234.0435955 *u* and $^{4}_{2}He$ = 4.0026032 *u*,

calculate the kinetic energy of the α-particle.

Solution

Since $^{238}_{92}U$ was originally stationary, the reactant must have more mass than the products.

Mass difference = Δm =

=

=

Energy released = $\Delta m\ c^2$ =

$m_\alpha v_\alpha = m_{Th} v_{Th}$ ... (1) (from conservation of momentum)

$$\frac{KE_\alpha}{KE_{Th}} = \frac{(1/2)m_\alpha {v_\alpha}^2}{(1/2)m_{Th}{v_{Th}}^2} = \frac{m_\alpha {v_\alpha}^2}{m_{Th}{v_{Th}}^2} = \frac{(m_\alpha v_\alpha)v_\alpha}{(m_{Th}v_{Th})v_{Th}} \quad ... (2)$$

Subs (1) into (2) $\Rightarrow \frac{KE_\alpha}{KE_{Th}} = \frac{v_\alpha}{v_{Th}} = \frac{m_{Th}}{m_\alpha} =$

$$\frac{KE_\alpha}{\Delta E} = \frac{KE_\alpha}{KE_\alpha + KE_{Th}} = \frac{1}{1+(KE_{Th}/KE_\alpha)} =$$

$\Rightarrow KE_\alpha =$

The α – particle carries most of the total kinetic energy.

Observation 1:

Observation 2:

<u>γ-radiation</u>

γ-radiation is short-wavelength electromagnetic radiation.

Gamma decay:

The spontaneous decay of a nucleus with the emission of gamma radiation.

Note:

(1) Frequently, some of the energy released in the alpha and beta decay is in the form of gamma ray photon.

(2) Alpha and beta decay usually leaves the daughter nucleus in an excited state (denoted by *).

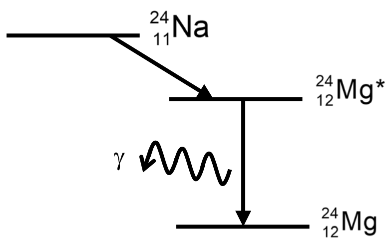
(3) The daughter nucleus emits energy in the form of a gamma ray photon in order to reach its ground state.

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

Example:

$^{24}_{11}Na \rightarrow {}^{24}_{12}Mg^{*} + {}^{0}_{-1}e$

$^{24}_{12}Mg^{*} \rightarrow {}^{24}_{12}Mg + \gamma$



(o) show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required).

<u>β-particles</u>

β-particles are high-speed electrons emitted from within the nucleus of an atom.

Beta decay:

The spontaneous decay of a nucleus with the emission of a beta particle.

Note:

(1) Frequently, some of the energy released in the decay is in the form of gamma ray photon.

(2) The parent nucleus is radioactive, but not the beta particle.

- are electrons emitted in β^{-} decay [2]. This electron does not exist within the parent nucleus and is not any of the orbital electrons in the atom. Instead, the electron originates from the nucleus through a nuclear transmutation in which a neutron changes into a proton and an electron. The new proton is retained in the nucleus (proton number increases from *Z* to *Z*+1 while the nucleon number remained the same) while the electron is ejected as a β-particle.

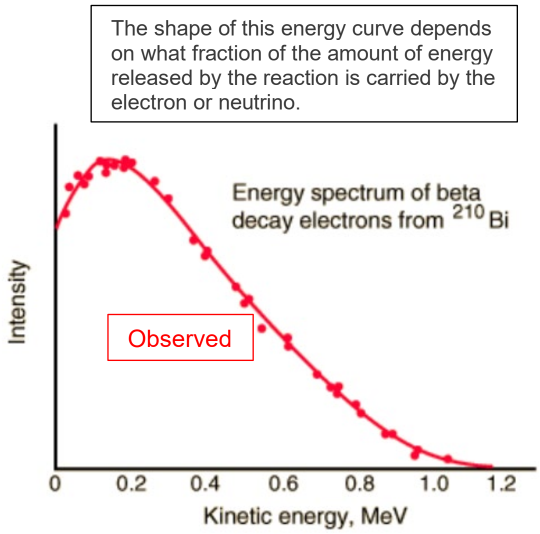
[2] There are two types of beta decay: beta plus and beta minus. The syllabus only covers the latter. No knowledge of antineutrinos is required as well.

- One may expect the β^- decay to proceed in a similar manner as an α-particle decay i.e.:

  INCORRECT β^- decay reaction: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$

- From **Example 9**, it can be concluded that the emitted electron $\left({}^{0}_{-1}e\right)$ is monoenergetic with energy; since the electron is much lighter particle compared to a Helium nucleus, the electron should carry away most of the released energy.

- But in 1911 Lise Meitner and Otto Hahn performed the 1st experiment that showed that the energies of electrons emitted by beta decay had a continuous rather than discrete spectrum.

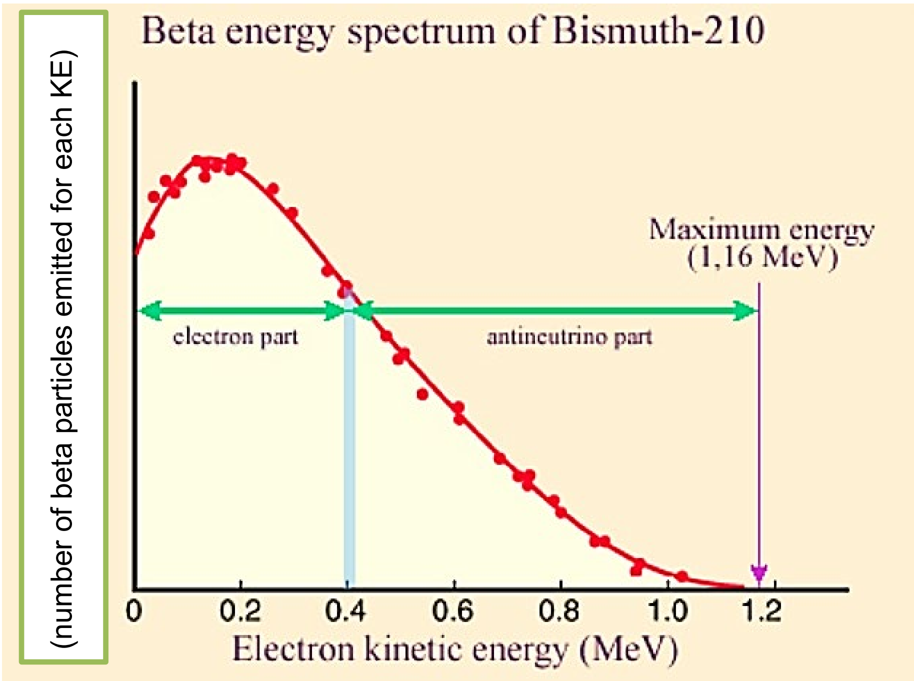


- It was proposed (by Pauli in 1931) that part of the energy released is carried by another particle emitted along with the β-particle. This particle is called the antineutrino (i.e., the little neutral one) $\bar{\nu}$ and its existence was verified in 1956 through experimental detection. The antineutrino has **zero electric charge**, and its **mass is a tiny fraction** [3] of the mass of the electron.

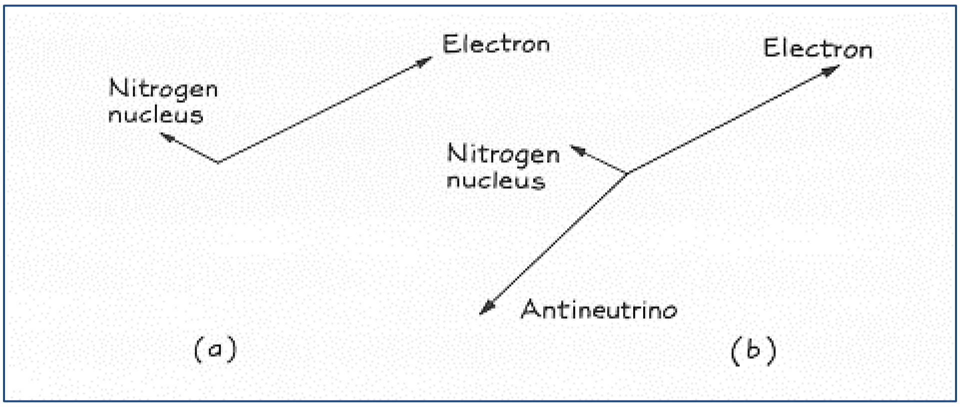
CORRECT β^- decay reaction: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$

<u>Energy and Momentum Spectra for Beta Decay</u>

In the α decay of a particular radionuclide, every emitted α particle has the same sharply defined kinetic energy. However, in the β decay, the total energy Q released (equal to the increase in the total binding energy or decrease in total mass) is shared — in varying proportions — between the emitted electron and neutrino. Sometimes the electron gets nearly all the energy of 1.16 MeV (from the graph) sometimes the neutrino does, with the majority of the electrons being emitted with an energy of about 0.17 MeV. In every case, however, the sum of the electron's energy and the neutrino's energy gives the same value Q.



[3] Here, the sum of the masses of all neutrino flavours put together is considered and it is less than a millionth of the mass of the electron.



Momentum is conserved in beta decay processes.
(a) Measurements showed that the Po-210 nucleus and the electron did not move off in opposite directions.
(b) Momentum would be conserved only if a third particle were released in beta decay.

Another anomaly was that when the Bi-210 nucleus was initially not moving, the Po-210 nucleus and the electron did not move off in exactly opposite directions, as conservation of momentum requires (Fig. (a) above). Neither momentum nor energy appeared to be conserved in β decay.

Rather than abandon the conservation principles, physicists suggested that, yet another particle must be present in the products of beta decay. Conservation laws dictated that this new particle could have no electric charge (electric charge was already conserved), must have a very small mass, and must move off with a speed and a direction that would conserve momentum (Fig. (b) above). Italian physicist Enrico Fermi named the particle the neutrino – little neutral one. For more than 20 years the neutrino hypothesis remained unproved, though generally accepted. Finally, in the mid-1950s physicists detected a particle with properties identical to those predicted for the neutrino.

$$^{210}_{83}\text{Bi} \rightarrow {}^{210}_{84}\text{Po} + {}^{0}_{-1}\text{e} + \overline{\nu_e}$$

The following table summarises the nature of radioactive decays:

	Alpha (α)	Beta (β)	Gamma (γ)
Nature	Helium nucleus $\left({}^{4}_{2}He\right)$ consists of 2 protons and 2 neutrons	Electron ${}^{0}_{-1}e$ High energy electron emitted from nucleus	High energy electromagnetic radiations (photons) of very short wavelength ~ 10^{-12} m
Charge	+2 *e*	– *e*	No charge
Mass	4 *u*	(1/1840) *u*	No mass
Deflection in electric field	Slight (parabolic)	Large (parabolic)	None
Deflection in magnetic field	Slight (Circular)	Large (Circular)	None
Ionising strength	Strong	Weak	Very weak
Number of ion-pairs produced per mm in air (approximate)	10^4	10^2	1
Penetrating ability (Range in air)	Stopped by a piece of paper (……………..)	Stopped by a few mm of aluminium (1 m)	Stopped by about 10 cm of lead; but Bremsstrahlung radiation are emitted from lead in the process if ……………….. ………………………………………… (at least 1 km)
Typical speed	0.05 *c*	0.3 *c* to 0.9 *c*	*c*
Typical energy	characteristic emissions (4 – 9 MeV)	**continuous** spectrum (from 0.01 to 5 MeV)	characteristic emissions (keV – MeV)

Some radioactive nuclei emit one kind of radiation, α, or β, or γ radiation only.
Some nuclei emit two kinds of radiation, either α and β, or α and γ, or β and γ radiations.
Some nuclei emit all 3 kinds.

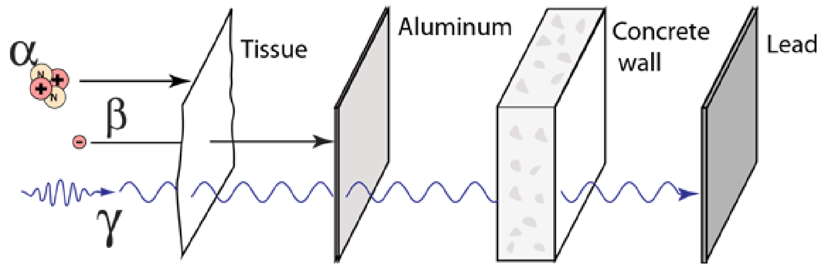


Figure 9 (Right): Illustration of penetrating power of different radiations

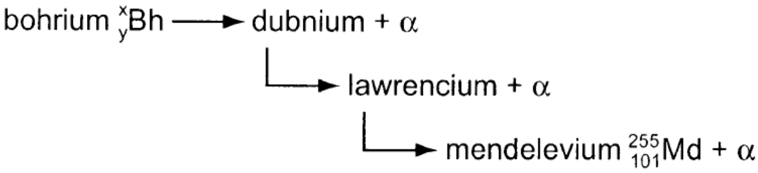
Though the most massive and most energetic of radioactive emissions, the alpha particle is the <u>shortest</u> in range because of its strong interaction with matter.

The electromagnetic gamma ray is extremely penetrating, even penetrating considerable thicknesses of concrete.

The electron of beta radioactivity strongly interacts with matter and has a short range.

Example 10 (2008 P1 Q40)

A nucleus of bohrium ${}^{x}_{y}Bh$ decays to mendelevium ${}^{255}_{101}Md$ by a sequence of three α-particle emissions.



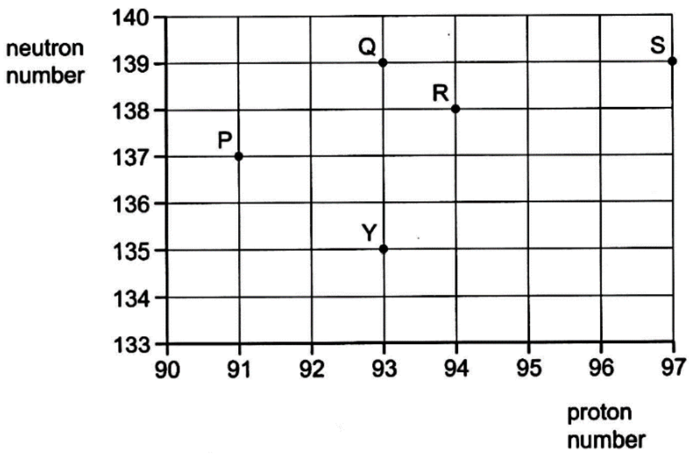
How many neutrons are there in a nucleus of ${}^{x}_{y}Bh$?

A 267 **B** 261 **C** 160 **D** 154

Solution: **(Answer: ……….)**

Example 11 (2012 P1 Q40)

Isotope Y forms after two successive decays. Each decay can be either alpha-emission or beta-emission. Four other isotopes, P, Q, R and S are shown on the diagram.



How many of the isotopes P, Q, R and S could be the initial isotope that, after two successive decays, became Y?

A 1 **B** 2 **C** 3 **D** 4

Solution: **(Answer: ……….)**

Y has proton number 93 and neutron number 135
i.e., nucleon number of 135+93 = 228.

If the two successive decays are both alpha-emissions,
Then the original isotope has proton number of …………………….. and nucleon number of …………………….., which means a neutron number of …………………….. (Isotope **……….**)

If the two successive decays are both beta-emissions,
Then the original isotope has proton number of …………………….. and nucleon number of …………………….., which means a neutron number of …………………….. (Isotope **……….**)

If the two successive decays are one beta-emission and one alpha-emission,
Then the original isotope has proton number of …………………….. and nucleon number of …………………….., which means a neutron number of …………………….. (Isotope **……….**)

(p) define the terms activity and decay constant and recall and solve problems by using $A = \lambda N$

(q) infer and sketch the exponential nature of radioactive decay and solve problems by using the relationship $x = x_0 \exp(-\lambda t)$, where x could represent activity, number of undecayed particles and received count rate

---

Relation between activity, decay constant and received count rate of a radioactive nuclide

Activity:

The activity of the radioactive nuclide is the ………………………………………………………......

………………………………… It is also called the rate of *nuclear disintegration*.

Mathematically, this relationship is expressed as

$$A = -\frac{dN}{dt} \qquad \text{(Unit: Bq}^{4}\text{ equivalent to s}^{-1}\text{)}$$

where the negative sign indicates a decrease in the number of **undecayed particles**, N.

Of course, **activity** is directly proportional to the number of **undecayed particles**.

Mathematically, this relationship is expressed as

where λ is the constant of proportionality, known as the **decay constant**.

The above equations can be rewritten as

$$\lambda N = -\frac{dN}{dt} \Rightarrow \lambda = \frac{\left(-\frac{dN}{N}\right)}{dt}$$

where $\left(-\frac{dN}{N}\right)$ may be interpreted as the fraction of the radioactive nuclei in a sample that has decayed in the small time interval dt.

Decay constant:

The ………………………………………………………………………………………….

$\lambda N = -\frac{dN}{dt}$ is also a first order differential equation with solution:

$$N = N_0 e^{-\lambda t}$$

where N_0 is the original number of undecayed radioactive nuclei in the sample at t = 0 s.

Since $A = \lambda N$, so $A_0 = \lambda N_0$. $N = N_0 e^{-\lambda t}$ can be changed to

$$A = A_0 e^{-\lambda t}$$

where A_0 is the initial activity.

Any detector placed at a certain fixed position to measure the activity of a radioactive sample, will receive a certain fraction of the radiations emitted from that sample, so the received count

---

[4] The symbol 'Bq' is named after Henri Becquerel, who shared the Nobel prize with Marie Curie.

rate, C, must be directly proportional to its activity, i.e., $C \propto A$. (Assume no background count rates.)
Received count rate: the rate at which emissions from a radioactive source are detected.

Hence $A = A_0 e^{-\lambda t}$ can be changed to

$$C = C_0 e^{-\lambda t}$$

where C_o is the initial received count rate (Unit: counts s^{-1}).

In general,

$$x = x_0 e^{-\lambda t}$$

where x can represent the received count rate C, activity A or the number of undecayed particles N; x_0 is the initial value of the corresponding physical quantity.

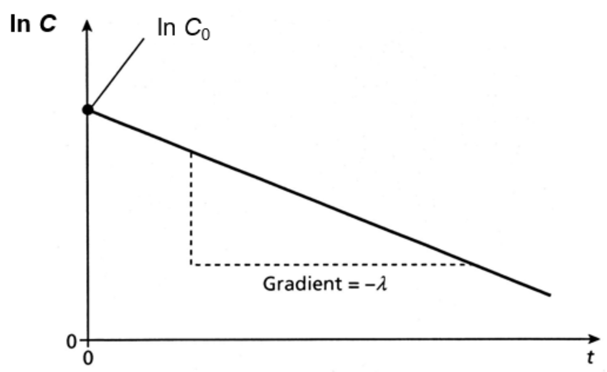
When investigating the decay of a radioactive isotope it is not possible to measure directly the activity of the sample.

The detector detects only a small proportion of the radiation given off by a sample. Radiation is given off in all directions and most of it misses the detector.

Even radiation that enters the detector may pass straight through it without being detected.

What is measured is called the received count rate. The background count rate should be subtracted from the received count rate to give the corrected count rate.

Method for measuring short half-life of a radionuclide



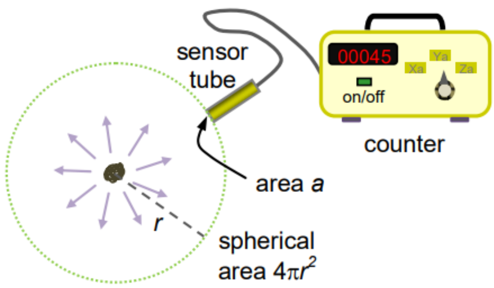
$$C = C_0 e^{-\lambda t}$$

$$\ln C = \ln\left(C_0 e^{-\lambda t}\right)$$

$$\underbrace{\ln C}_{y} = \underbrace{\ln C_0}_{c} \underbrace{- \lambda}_{m} \underbrace{t}_{x}$$

Method for measuring long half-life of a radionuclide

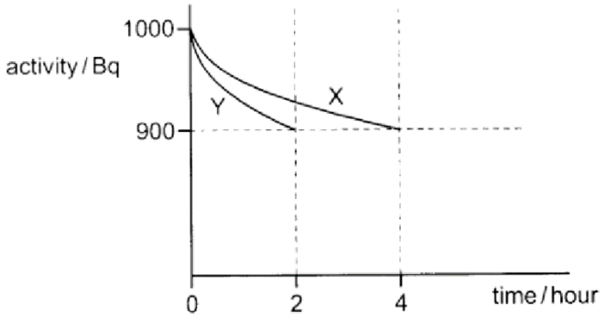
- Determine the number of atoms N in a pure sample from the measured mass of the sample (using weighing machine).
- Use a GM tube, with effective window area a at a distance r from source, connected to a rate meter to measure the count rate C.
- Calculate the activity $A = [(4\pi r^2)/a)]\ C$
- Determine decay constant λ from $A = \lambda N$ and then determine half-life $t_{1/2}$ using equation $t_{1/2} = (\ln 2)/\lambda$



Example 12 (2012 /P1 Q39)

The graph shows how the activities of two isotopes X and Y vary with time.

What will be the ratio $\frac{\text{activity of X}}{\text{activity of Y}}$ after each has decayed for 8 hours?



A $\frac{9}{10}$ **B** $\frac{10}{9}$ **C** $\frac{81}{100}$ **D** $\frac{100}{81}$

Solution: (Answer: ....)

Using $A = A_0 e^{-\lambda t}$,

for X, from time = 0 h to time = 4 h,

from time = 4 h to time = 8 h,

for Y, from time = 0 h to time = 2 h,

from time = 2 h to time = 8 h,

Thus,

Example 13 (2011 P1 Q40)

At time t, a sample of a radioactive substance contains N atoms of a particular nuclide.

At time $t + \Delta t$, where Δt is a short period of time, the number of atoms is $N - \Delta N$. Which expression is equal to the decay constant of the nuclide?

A $\frac{\Delta N}{N}$ **B** $\frac{\Delta N}{\Delta t}$ **C** $\frac{\Delta N}{N \Delta t}$ **D** $\frac{N \Delta N}{\Delta t}$

Solution: (Answer: ....)

Recall:

(r) define half-life

(s) solve problems by using the relation $\lambda = \frac{0.693}{t_{\frac{1}{2}}}$

Half-life $t_{\frac{1}{2}}$:

………

………

There are several ways to define half-life, $t_{1/2}$. It can also be defined as the time taken, *on average*, for

(i) half of the original number of radioactive nuclei in a sample to remain undecayed; or

(ii) the activity of a radioactive sample to decrease to half its original value; or

(iii) the received count rate to reduce to half its original value.
(assume no background count rate)

From $N = N_0 e^{-\lambda t}$, we have $\ln N = \ln(N_0) - \lambda t$

When $t = t_{1/2}$, $N = \frac{1}{2} N_0$, so

$$\ln\left(\frac{1}{2}N_0\right) = \ln(N_0) - \lambda t_{1/2}$$

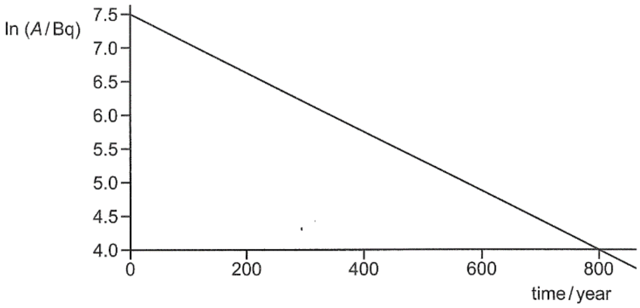
$$\Rightarrow \ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda}$$

$$\Rightarrow \lambda =$$

<u>Example 14 (2007 P1 Q39)</u>

A graph of the natural logarithm of the activity A of a radioactive source plotted against time is given.



What is the half-life of the source in years?

A 0.0044 **B** 160 **C** 400 **D** 860

<u>Solution</u>: (Answer: ……….)

At time = 0 year, $A = e^{7.5}$ Bq

After one half-life, the activity of the source becomes $\frac{e^{7.5}}{2}$ Bq ≈ 904 Bq $\Rightarrow \ln(904) \approx$

From the graph, the answer must be ………………… years.

OR

Gradient of graph =

Quiz: Why nuclide with large decay constant is not suitable to be used to measure activity?

……

……

(t) discuss qualitatively the effects, both direct and indirect, of ionizing radiation on living tissues and cells

Ionizing radiation (α, β, and γ radiation) may interact with orbital electrons of the atoms and molecules of living tissues and cells to cause excitation[5] or ionization[6]. The ionization gives rise to the production of a flux of energetic secondary particles (electrons). Energetic and unbound, they can migrate away from the site of production, interact with other atoms and molecules, and give up or transfer their energy to the surrounding medium. This energy absorption process gives rise to radicals[7] and other reactive molecules that react with nearby atoms and molecules to cause chemical changes, leading to radiation damage of biological systems. The effects of ionizing radiation on living tissues and cells can be both direct and indirect.

Direct Effects:
Direct effects of ionizing radiation occur when the radiation interacts directly with the DNA or other molecules in the cell. The energy from the radiation can cause breaks in the DNA strands, which can lead to mutations or cell death. The severity of these effects depends on the amount and type of radiation exposure.

Indirect Effects:
Indirect effects of ionizing radiation occur when the radiation interacts with water molecules in the cell (involves the removal of an electron from water), which can create highly reactive molecules such as hydroxyl radicals. These reactive molecules can then go on to damage the DNA and other cellular molecules. Indirect effects can occur even at low doses of radiation.

Short-term effects:
Short-term effects of ionizing radiation on living tissues and cells include skin burns, nausea, vomiting, and diarrhea. These effects occur when the radiation exposure is high and can cause immediate damage to the tissues.

Long-term effects:
Long-term effects of ionizing radiation on living tissues and cells include an increased risk of cancer and other diseases. Radiation exposure can cause mutations in the DNA that can lead to the development of cancer. The risk of cancer depends on the amount and type of radiation exposure, as well as other factors such as age and overall health.

Heavy charged particles such as alpha particles have a greater probability of causing direct damage compared to low charged particles such as beta particles which causes most of its damage by indirect effects. However, it's worth noting that the distinction between direct and indirect effects is not always clear-cut, and the precise mechanisms of radiation damage can depend on factors such as the energy of the particles, the type of tissue they encounter, and the exposure conditions.

In summary, ionizing radiation can have direct and indirect effects on living tissues and cells, including DNA damage, mutations, cell death, and an increased risk of cancer and other diseases. The severity of these effects depends on the amount and type of radiation exposure. It is important to minimize exposure to ionizing radiation to reduce the risk of these harmful effects.

---The End---

[5] Raising a bound electron to a higher energy state; the electron does not have sufficient energy to leave the host atom.
[6] The electron receives sufficient energy to be ejected from its orbit and to leave the host atom.
[7] Atoms or molecules with unpaired electrons are referred to as 'radicals'; often term 'free radicals.' The unpaired electrons generally cause radicals to be highly chemically reactive.

ANNEX

(A) *Methods to determine energy released E_R for a nuclear reaction*

e.g., deuterium-tritium fusion: $^{2}_{1}\text{X} + {}^{3}_{1}\text{Y} \rightarrow {}^{4}_{2}\text{Z} + {}^{1}_{0}\text{n} + \boldsymbol{E}_R$

Method 1: Given the masses

Conservation of Mass-energy:

$$E_\text{X} + E_\text{Y} = E_\text{Z} + E_\text{n} + E_R$$
$$m_\text{X}c^2 + m_\text{Y}c^2 = m_\text{Z}c^2 + m_\text{n}c^2 + E_R$$
$$E_R = \left[\left(m_\text{X} + m_\text{Y}\right) - \left(m_\text{Z} + m_\text{n}\right)\right]c^2$$

Energy released = [Mass of reactants – Mass of products] c^2

Method 2: Given the binding energies

$$E_\text{X} + \text{BE}_\text{X} = E_\text{p} + E_\text{n} \Rightarrow \quad E_\text{X} = E_\text{p} + E_\text{n} - \text{BE}_\text{X}$$
$$E_\text{Y} + \text{BE}_\text{Y} = E_\text{p} + 2\text{E}_\text{n} \Rightarrow \quad E_\text{Y} = E_\text{p} + 2E_\text{n} - \text{BE}_\text{Y}$$
$$E_\text{Z} + \text{BE}_\text{Z} = 2E_\text{p} + 2E_\text{n} \Rightarrow \quad E_\text{Z} = 2E_\text{p} + 2E_\text{n} - \text{BE}_\text{Z}$$

Conservation of Mass-energy:

$$E_\text{X} + E_\text{Y} = E_\text{Z} + E_\text{n} + \boldsymbol{E}_R$$
$$\left(E_\text{p} + E_\text{n} - \text{BE}_\text{X}\right) + \left(E_\text{p} + 2E_\text{n} - \text{BE}_\text{Y}\right) = \left(2E_\text{p} + 2E_\text{n} - \text{BE}_\text{Z}\right) + E_\text{n} + \boldsymbol{E}_R$$
$$\boldsymbol{E}_R = \text{BE}_\text{Z} - \left(\text{BE}_\text{X} + \text{BE}_\text{Y}\right)$$

Energy released = BE of products – BE of reactants

(B) *Radioactive Dating*

This refers to the technique at which the age of ancient materials can be determined. The age of any object made from once-living matter e.g. wood can be determined using the radioactivity of ^{14}C. All living plants absorb carbon dioxide from the air and use it to synthesize organic molecules. The vast majority of these carbon atoms are ^{12}C but a small fraction (about 1.3 × 10^{-12}) is the radioactive isotope ^{14}C. The ratio of ^{14}C to ^{12}C in the atmosphere has remained roughly constant over many thousands of years, in spite of the fact that ^{14}C decays with a half-life of about 5730 years. This is due to the cosmic radiation (consists of highly energetic charged particles and impinges on the Earth from outer space) which strike nuclei of atoms in the atmosphere and releases a shower of secondary particles which includes a large number of neutrons. These neutrons can collide with nitrogen nuclei in the atmosphere to produce the form ^{14}C:

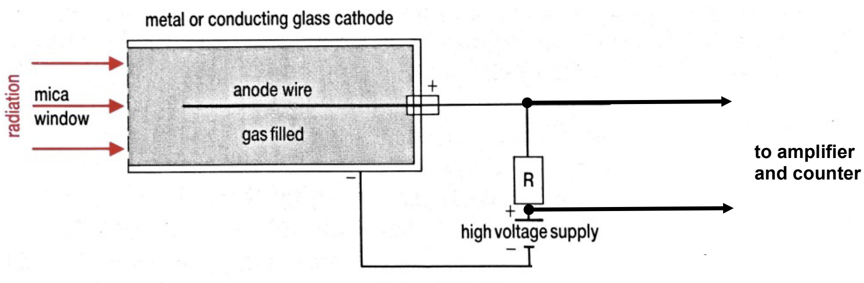
$$^{1}_{0}\text{n} + {}^{14}_{7}\text{N} \rightarrow {}^{14}_{6}\text{C} + {}^{1}_{1}\text{p}$$

This continual production of ^{14}C in the atmosphere roughly balances the loss of ^{14}C by radioactive decay. As long as a plant or tree is alive, it continually uses the carbon from carbon dioxide in the air to build new tissue to replace the old. Animals eat plants, so they too are continually receiving a fresh supply of carbon for their tissues.

When an organism is alive, carbon is exchanged with the environment and since the ratio of ^{14}C to ^{12}C in the atmosphere remains nearly constant, so the organism maintains the same relative abundance of the two isotopes as the environment. When the organism dies however, carbon dioxide is no longer absorbed and carbon exchange with the environment stops. As the ^{14}C present in the organism decays, the ratio of ^{14}C to ^{12}C decreases. The ratio of ^{14}C to ^{12}C in a sample can be measured and used to determine the age of the sample. One way to do this is to measure the ^{14}C activity per gram of carbon.

(C) *Detection of Radiation*

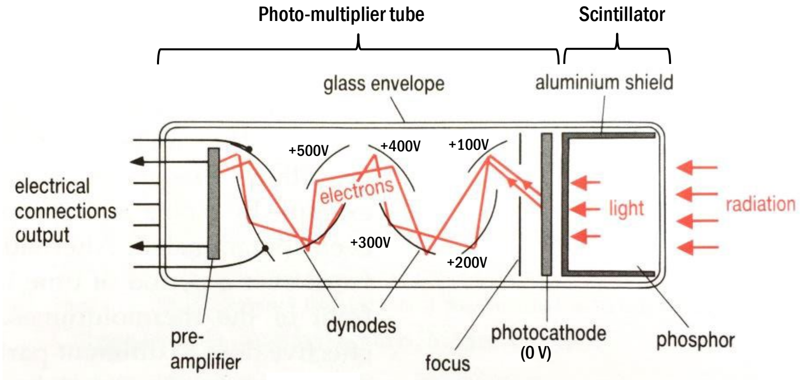
1. Geiger-Muller counter



Geiger–Muller tube.

The Geiger-Muller counter consists of a gas-filled metal. The α-particles, β-particles or γ rays enter the cylinder through a thin window at one end. A wire electrode runs along the centre of the tube and is kept at a high positive voltage (1000 – 3000 V) relative to the outer cylinder. When the decay particles or γ photons enter the cylinder, they collide with and ionise the gas molecules. The electrons produced from the gas molecules accelerate toward the positive wire, ionising other molecules in its path. Additional electrons are formed and an avalanche of electrons rushes towards the wire, leading to a pulse of current through the resistor R. This pulse can be counted or made to produce a "click" in a loudspeaker. The number of counts or clicks is related to the number of disintegrations that produced the particles or γ photons.

2. Scintillation counter

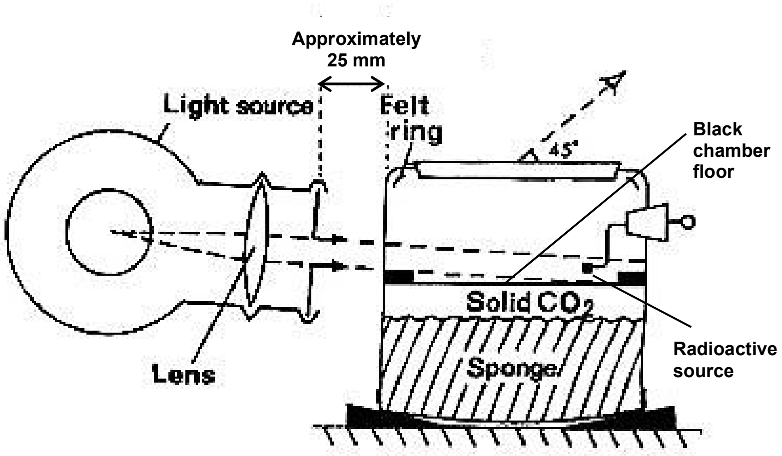


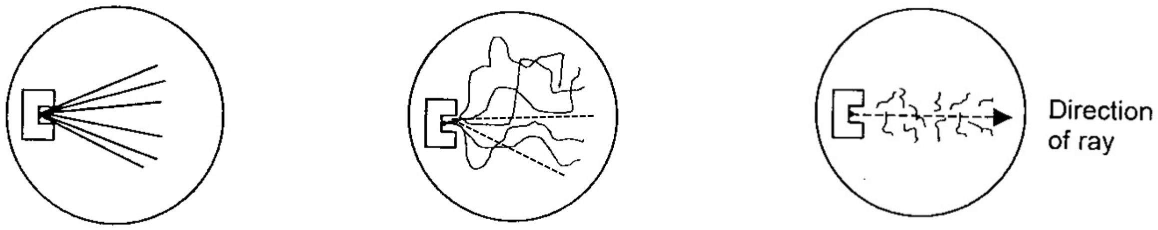
Scintillation counter.

This consists of a scintillator mounted on a photomultiplier tube. Very often, the scintillator is a crystal containing a small amount of impurity. In response to ionising radiation, the scintillator emits a flash of visible light. The photons of the flash then strike the photocathode of the photomultiplier tube. The photocathode tube is made of a material that emits electrons because of photoelectric effect. These photoelectrons are then attracted to a special electrode kept at a voltage of about +100 V relative to the photocathode. The electrode is coated with a substance that emits several additional electrons for every electron striking it. The additional electrons are attracted to a second similar electrode (voltage +200 V) where they generate even more electrons. Commercial photomultiplier tubes contain as many as 15 of these special electrodes, so photoelectrons resulting from the light flash of the scintillator lead to a cascade of electrons and a pulse of current. As in a Geiger-Muller tube, the current pulses can be counted.

3. <u>Wilson cloud chamber</u>

In a cloud chamber, a gas is cooled to a temperature slightly below its usual condensation point (super-cooled) and gas molecules condense on any ionised molecules present. Ions produced when a charged particle passes through serve as centres on which tiny droplets form. Light scatters more from these droplets than from the gas background, so a photo of the cloud chamber at the right moment shows the track of that particle. The paths revealed in a cloud chamber can be photographed to provide a permanent record of the event. α-particles give bright, straight tracks as they cause intense ionisation. Tracks have the same range because α-particles are emitted with characteristic energy. A fast β-particle would show thinner and fainter straight line as it produces fewer ions per mm than an α-particle. The heavier curling and tortuous tracks are due to slow β-particles which are easily deflected when they come close to gas molecules. Diffused ionisation is produced by the interaction of a γ ray photon with gas. The ionisation is produced by electrons which leave the atom after the absorption of high energy from a photon. These tracks start in the path of the beam of γ rays, and are identical with those produced by slow β-particles. So γ rays give short tortuous tracks branching off from its path.





α - particle tracks β - particle tracks γ -ray tracks

4. <u>Solid-state (Semiconductor) detectors</u>

A semiconductor detector consists of a reverse-biased *pn* junction diode. A particle passing through the junction can excite electrons into the conduction band, leaving holes in the valence band. The freed charges produce a short electrical pulse that can be counted just as for Geiger and scintillation counters.

(D) *More information on nuclear fission and fusion*

Since all nuclei are positive, the fusion of 2 nuclei requires some energy to overcome their initial electrostatic repulsion when they are brought together to be fused. This requires the 2 nuclei to have considerable kinetic energy, so they need to be at a considerably high temperature (like 10^8 K), and so it is easier for smaller nuclei to undergo fusion (also known as thermonuclear reaction).

Comparing fission and fusion:

In fission, the binding energy per nucleon increases from about 7.5 MeV to about 8.5 MeV (about 1 MeV) whereas in fusion, the binding energy per nucleon increases from about 1 MeV to about 7 MeV (about 6 MeV). So fusion would result in a release of energy about 6 times the amount released from fission, for the same mass involved in these processes (because the values are all based on 'per nucleon').

At present, atomic bombs and nuclear reactors (which are like atomic bombs, but with the release of energy under control, instead of a blast) involve fission to produce energy. They require nuclear fuels with large nuclei, like uranium and plutonium, which are highly radioactive and limited in supply from ores, mined in places like Russia.

The Sun and all hydrogen bombs use fusion to produce energy (which involves hydrogen nuclei, hence the name ‘hydrogen bomb’ which are reputed to be much more powerful than atomic bombs; so now you should know why. The high temperature required to initiate this thermonuclear bomb may be provided by the explosion of an atomic bomb.) To date, fusion can only be achieved in this uncontrolled way of a bomb. It cannot be done within a nuclear reactor which releases its energy in a controlled manner. If attempts to control the fusion process were to succeed and be used in reactors, the hydrogen fuel required would be virtually inexhaustible, in the form of hydrogen nuclei found in the water of our oceans!

The on-going release of energy from the Sun (and the other stars) due to this fusion process is converted from mass, resulting in the continuing loss of mass from the Sun (many tonnes per second).

The nucleus / Isotope

1 Which of the following can be concluded from the experiment showing the scattering of α-particles by a piece of gold foil?

A Electrons orbit the atomic nucleus in well-defined paths.

B Nuclei of different isotopes contain different numbers of neutrons.

C The atomic nucleus contains protons and neutrons.

D The nucleus is very small compared with the size of the atom.

2 Some properties of four nuclear particles are given in the table.
Which of the particles is a neutron?

particle	affected by electric and magnetic fields	mass in terms of proton mass
A	Yes	$4m_p$
B	No	0
C	Yes	m_p
D	No	m_p

3 Bohrium $^{x}_{y}\text{Bh}$ decays through a series of α-particle emissions to become mendelevium $^{255}_{101}\text{Md}$ as shown. What is the number of nucleons in a nucleus of $^{x}_{y}\text{Bh}$?

$^{x}_{y}\text{Bh} \rightarrow$ dubnium + α ; dubnium $\rightarrow$ lawrencium + α ; lawrencium $\rightarrow$ mendelevium + α

A 267 **B** 261 **C** 160 **D** 154

4 In an α-particle scattering experiment, an α-particle and a gold nucleus (in a piece of gold foil) collide head on and the α-particle rebounds.

(a) Using the data given, write down a numerical expression for the electrostatic force of repulsion, F, acting on the α-particle at the instant of collision when the distance between α-particle and gold nucleus is x. [$3.64\times10^{-26}/x^2$] [2]

{Data: for α-particle, A = 4, Z = 2; for gold nucleus, A = 197, Z = 79.}

(b) The electric potential energy of the α-particle and the gold nucleus at the point of impact is Fx.

(i) If the initial kinetic energy of the α-particle is 1.8 MeV, calculate a value for x, assuming the gold nucleus has no kinetic energy initially. [1.26×10^{-13} m] [2]

(ii) What indication does the value of x give about the size of the two particles involved? [1]

Mass defect and nuclear binding energy/ Nuclear processes

5 The binding energy per nucleon is used as a measure of stability of naturally occurring nuclides. This quantity

A increases uniformly with the atomic mass number.

B is a minimum for nuclides with atomic mass number 56.

C is higher for the daughter nuclides in a nuclear fission reaction.

D is roughly constant for nuclides with atomic mass number less than 16.

6 Nitrogen – 14 can be transformed to oxygen – 17 by high energy α-particles as follows:

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

The nitrogen and helium nuclei have a total rest mass of 18.006 u.
The oxygen and hydrogen nuclei have a total rest mass of 18.007u.
The mass of 0.001u has an energy equivalent of 0.9 MeV.
Which of the following correctly describes the information given?

A Mass of 0.001 u has been converted into 0.9 MeV of energy.

B The kinetic energy of the products exceeds the kinetic energy of the reactants by 0.9 MeV.

C The kinetic energy of the reactants exceeds the kinetic energy of the products by 0.9 MeV.

D The reactants had only 0.9 MeV of kinetic energy and all of this was converted into mass.

7 Radium (Ra) decays to radon (Rn) by the reaction

$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He + \gamma$$

The atomic masses are: Ra: 3.753×10^{-25} kg, Rn: 3.686×10^{-25} kg, He: 0.066×10^{-25} kg.

(a) Estimate the energy released when an atom of ^{226}Ra decays. [9.0×10^{-12} J] [2]

(b) Estimate the wavelength of the γ photon emitted during this decay given that 4% of the energy released turns to γ radiation. [5.53×10^{-13} m] [2]

(c) What happens to the remaining 96% of the energy? [1]

8 The masses of the particles in a nuclear reaction are as follows.

$^{9}_{4}Be$	+	$^{4}_{2}He$	→	$^{1}_{0}n$	+	$^{12}_{6}C$
9.01212 u		4.00260 u		1.00867 u		12.00000 u

(a) Calculate the loss of mass which appears to take place in the reaction. [6.05×10^{-3} u] [2]

(b) Hence find the energy equivalence of this mass. [9.04×10^{-13} J] [2]

9 The first nuclear reaction induced in a laboratory (performed by Rutherford in 1919) can be represented by ${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$, in which a collision takes place between an α-particle travelling at 3.0×10^{7} m s^{-1} and a stationary nitrogen nucleus.

(a) What information can be obtained from the symbol ${}^{17}_{8}O$? [2]

(b) The total rest mass are as follows: ${}^{14}_{7}N + {}^{4}_{2}He$ = 18.00568 *u*,

${}^{17}_{8}O + {}^{1}_{1}H$ = 18.00696 *u*.

Calculate the small change in mass, in kilogram, which takes place in this nuclear reaction, and the minimum kinetic energy needed by the α-particle to cause the nuclear reaction. [2.13×10^{-30} kg, 1.91×10^{-13} J] [4]

(c) The particles move in a straight line. The speed of the proton after the collision is 6.0×10^{7} m s^{-1}. Use the principle of conservation of momentum to calculate the velocity of the oxygen nucleus after the collision (magnitude and direction). [3.53×10^{6} m s^{-1}] [3]

10 The rest mass of the deuteron, ${}^{2}_{1}H$, is equivalent to an energy of 1876 MeV; the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV.

A deuteron may disintegrate to a proton and a neutron if it

A emits a γ-ray photon of energy 2 MeV.

B capture a γ-ray photon of energy 2 MeV.

C emits a γ-ray photon of energy 3 MeV.

D captures a γ-ray photon of energy 3 MeV.

11 The atomic masses, A_r, of ${}^{4}_{2}He$, ${}^{23}_{11}Na$ and ${}^{27}_{13}Al$ are 4.0026 *u*, 22.9898 *u* and 26.9815 *u* respectively. Discuss whether it is possible for ${}^{27}_{13}Al$ spontaneously to emit an alpha particle. [4]

Radioactivity

12 A sample of radioactive material emits α-particles at the rate of 10^{6} per second. The α-particles produce ions in air which are all collected. The ionization current is found to be about 0.01 μA. The charge on an ion may be taken to be about 10^{-19} C.

Which of the following is the best estimate of the average number of ions produced by each α-particle?

A 10^{5} **C** 10^{7}

B 10^{6} **D** 10^{8}

13 An engineer would like to add a radioactive isotope to the water of a pipe that is buried 0.4 m below the surface of a field so that he could trace the pipe line.

What type of isotope should he choose?

	Emitter	***Half-life***
A	β – particle	A few hours
B	β – particle	Several years
C	γ – radiation	A few hours
D	γ – radiation	Several years

14 A certain radioactive isotope has a half-life of 32 hours. After an interval of 16 hours, what is the fraction of a sample remaining?

A 0.25

B 0.29

C 0.50

D 0.71

15 Two samples of radioactive nuclides, **X** and **Y**, both having activity A_0 at time t = 0, are mixed together. The half-lives of **X** and **Y** are 24 years and 16 years respectively.

Which of the following is the total activity of the mixture at t = 48 years?

A $\frac{1}{12}A_0$

B $\frac{3}{16}A_0$

C $\frac{1}{4}A_0$

D $\frac{3}{8}A_0$

16 A sample of nuclide, with a half-life of 2 days, consists of 6.4×10^{11} atoms. A second sample of another nuclide, with a half-life of 3 days, consists of 8×10^{10} atoms. How many days must elapse before the number of active atoms in the two samples becomes the same?

A 6

B 9

C 12

D 18

17 The half-life of an isotope of phosphorus is 14 days. If a sample contains 3.0×10^{16} such nuclei, determine its activity. [1.7×10^{10} Bq] [2]

18 A drug tagged with $^{99}_{43}Tc$ (half-life = 6.05 h) is prepared for a patient. If the original activity of the sample was 1.1 × 10^4 Bq, what is its activity after it has sat on the shelf for 2.0 h?

[8.7 × 10^3 Bq] [2]

19 The half-life of ^{131}I is 8.04 days.

(a) Calculate the decay constant for this isotope. [9.98 × 10^{-7} s^{-1}] [2]

(b) Find the number of ^{131}I nuclei necessary to produce a sample with an activity of 0.50 μCi. (1 Ci is 3.7 × 10^{10} Bq) [1.9 × 10^{10}] [2]

Sample Examination Question(s)

20 2010/ACJC/H2/P3/Q7

(a) Explain what is meant by nuclear decay being *spontaneous* and *random* in nature. [2]

(b) The ^{226}Ra nucleus undergoes alpha decay according to

$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He$$

Particle	Rest mass / *u*
$^{226}_{88}Ra$	226.0254
$^{222}_{86}Rn$	222.0176
$^{4}_{2}He$	4.0026

(i) Show that the energy released in this decay, *Q*, is 4.86 MeV. [2]

(ii) This energy *Q* must be shared by the alpha particle and the daughter nucleus. Use conservation of energy and momentum to show that

$$Q = K_\alpha\left(1+\frac{M_\alpha}{M}\right)$$

where K_α is the kinetic energy of the alpha particle,

M_α is the mass of the alpha particle, and

M is the mass of the daughter nucleus. [3]

(iii) Hence find the kinetic energy of the alpha particle emitted (in MeV) in this decay process. Comment on your answer with reference to **(b)(i)**. [3]

(c) Many radioisotopes have important industrial, medical and research applications. One of these is ^{60}Co, which has a half-life of 5.2 years and each ^{60}Co nucleus decays by emission of a beta particle (energy 0.31 MeV) and two gamma photons (energies 1.17 MeV and 1.33 MeV respectively).

A scientist wishes to prepare a ^{60}Co sealed source that will have an activity of at least 37 × 10^{10} Bq after 30 months of use.

(i) Show that the initial minimum activity of ^{60}Co when the scientist is preparing the radioisotopes is 5.2 × 10^{11} Bq. [2]

(ii) Hence calculate the minimum initial mass in milligrams of ^{60}Co required. [4]

(iii) At what rate will the source emit energy after 30 months if its activity is 37 × 10^{10} Bq after 30 months of use. [3]

(iv) With reference to the type and energies of the radiations emitted, suggest which of the radiations emitted by ^{60}Co decay could be used to detect the uniformity of thickness of metal sheets. [1]

Data Analysis

21 A nuclear reactor throughout its life generates power from fission of uranium-235 by neutrons in a reaction of this type:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{90}_{38}Sr + ^{143}_{54}Xe + 3^{1}_{0}n + 200\ \text{MeV}$$

The fission products, as strontium (Sr) and xenon (Xe) are called, have very small mass and are mostly contained within the fuel cells. The neutrons, on the other hand, have high speed and are uncharged. They are able to escape from fuel cells into the graphite (carbon) moderator where they are slowed down. One of the three neutrons is needed to sustain the chain reaction and the others are absorbed by the moderator, control rods and concrete sides.

The plan in decommissioning nuclear reactors has to take into account the following info:

(i) A reactor has a total mass of 300,000 tonnes and 5% of this may be radioactive. Concrete, graphite and steel take up 8000, 2200 and 3400 tonnes respectively

(ii) The fuel elements can be removed in a normal way. This removes 99.99% of the radioactivity which was on site when the reactor was working.

(iii) The remaining radioactivity is mostly neutron-induced in the steel, concrete and graphite structures.

(iv) Radioactivity is expressed in Becquerels (Bq). 1 Bq is 1 disintegration per second.

(v) British law defines a radioactive substance as one with an activity greater than 400 Bq kg^{-1}.

(vi) There are about 2500 known nuclides. Of these, 79 which have a half-life longer than 1 year may be present in a reactor. Most of these 79 nuclides do not reach the activity of 400 Bq kg^{-1} to necessitate them being called radioactive but some do.

(vii) The table below shows the activities of 13 such nuclides in the 3 main sections, namely concrete, graphite and steel of the reactor 10 years after its shutdown. A dash indicates an insignificant amount of nuclide.

Nuclide	Half-life/ years	Concrete/10^{12} Bq	Graphite/10^{12} Bq	Steel/10^{12} Bq	Total/10^{12} Bq
$^{3}_{1}H$	12	-	110	-	110
$^{14}_{6}C$	5700	-	42	1.7	44
$^{36}_{17}Cl$	310 000	-	1.3	-	1.3
$^{41}_{20}Ca$	130 000	0.10	0.90	-	1.0
$^{56}_{26}Fe$	2.7	0.82	4.0	2400	2400
$^{60}_{27}Co$	5.2	0.28	11	680	690
$^{59}_{28}Ni$	80 000	-	0.096	2.1	2.2
$^{63}_{28}Ni$	92	-	17	230	250
$^{63}_{41}Nb$	20 000	-	-	0.013	0.013
$^{108}_{47}Ag$	130	-	0.0017	0.017	0.019
$^{151}_{62}Sm$	90	0.15	0.051	-	0.020
$^{152}_{63}Eu$	12	0.82	0.11	-	0.93
$^{154}_{63}Eu$	16	0.10	0.61	-	0.71
Total		2.3	190	3300	3500

All figures are given to 2 significant figures. 1 tonne = 1000 kg.

Answer the following questions using the data supplied above.

(a) What is the activity in terms of Bq kg^{-1} of each of the three main sections of the reactor? [3]

[2.88×10^5, 8.64×10^7, 9.71×10^8]

(b) Why is it that over a period of 10 years, the radioactive iron-56 in the reactor is less of a problem than the radioactive nickel? [2]

(c) Why is it likely that the activity of the long half-life nuclides is low? [2]

(d) How long after shutdown will it be before the activity of tritium ($^{3}_{1}H$) in the reactor fall below 0.43 $\times 10^{12}$ Bq? [5]

[106 yrs]

(e) Identify the nuclide(s) that would determine the time required to isolate the reactor before it is deemed safe to demolish. Explain your reasons. [2]

(f) By making appropriate estimations, evaluate numerically the number of years required to isolate the reactor. [4]

[3.3×10^6 years]

~ THE END ~