

Inequalities**1. CJC/2013/I/Q4**

(a) Using an algebraic method, solve the inequality $\frac{x+4}{3+2x-x^2} < 1$. [5]

(b) Hence solve the inequality $\frac{x^2-4}{x^4+2x^2-3} < 1$. [2]

2. ACJC/2016/I/1

Without the use of a calculator, solve the inequality $\frac{2w-5}{w^2-3} > 0$. [2]

Hence solve $\frac{(2|y|-5)\sin x}{y^2-3} \leq 0$, given that $\pi < x \leq \frac{3\pi}{2}$. [3]

3. DHS/2016/I/3

(i) Without using a calculator, find the exact solution of the inequality

$$4-x \geq \frac{4}{x+2}. \quad [3]$$

(ii) Hence solve solution of $5-|x| \geq \frac{4}{|x|+1}$. [2]

4. TJC/I/2

Without the use of a calculator, solve the inequality $\frac{3x}{x^2-6x+11} > 2$. [5]

Hence, find the range of values of x which satisfies $\frac{3e^x}{4e^{2x}-12e^x+11} > 1$. [2]

5. ACJC/2010/II/3

Find in terms of a , the range of values of x that satisfy the inequality $\ln\left(2x-\frac{a}{x}\right) \geq 0$, where $a > 1$.

[4]

6. VJC Prelim/2020/01/Q2

Solve the inequality $\frac{x^2+2x+3}{ax^2-(a+1)x+1} < 0$, for $a \in \mathbb{R}$, $a \neq 0$, $a < 1$. You need to distinguish the cases

where a is positive and a is negative. [5]

7. **AJC/2011/1/3**

Solve the inequality $\frac{(x+a)(x-b)}{(x)(x-c)^2} < 0$, where $a, b, c \in \mathbb{R}^+$ and $0 < c < b < a$. Hence, find the range of values of x which satisfy $\frac{(\ln x + a)(\ln x - b)}{(\ln x)(c - \ln x)^2} < 0$. [5]

8. **HCI/2013/Prelim/P1/Q1**

Given that a is a positive real number, solve the inequality

$$\frac{x^2 - x + 2}{x^2 + (1-a)x - a} \leq 0,$$

leaving your answer in terms of a . [3]

9. **DHS/2013/I/Q2**

Without the use of a calculator, solve the inequality $\frac{1}{x} + 1 \leq x(1+x)$. [3]

Hence, solve $1 - e^{2x} + e^{-x} - e^x \leq 0$. [2]

10. **MI/2021/I/6**

(i) Sketch the curve with equation $y = \left| \frac{1}{a-x} \right|$, where a is a positive constant. State, in terms of a , the equations of the asymptotes and coordinates of any intersections with the x -axis and y -axis. On the same diagram, sketch the line with equation $y = b(x-a)$, where b is a positive constant. [4]

(ii) Find, in terms of a and b , the root of the equation $\left| \frac{1}{a-x} \right| = b(x-a)$. [3]

(iii) Hence solve the inequality $\left| \frac{1}{a-x} \right| > b(x-a)$. [2]

11. **MI Prelim/2020/01/Q1**

Sketch the graph of $y = \ln(x^2 - 4)$, stating the equations of any asymptotes. [2]

By adding a suitable curve on your graph above, solve the inequality

$$\ln(x^2 - 4) + x^2 - 8 \leq 0$$

[3]

12. **TJC Prelim/2022/01/Q3**

- (i) On the same axes, sketch the graphs of $y = |x + a|$ and $y = |ax + 1|$ where $0 < a < 1$, and hence solve the inequality $|x + a| < |ax + 1|$. [4]

- (ii) Deduce the solution to the inequality $\left|\frac{1}{x} + a\right| < \left|\frac{a}{x} + 1\right|$. [2]

13. **YIJC Prelim 9758/2023/01/Q3**

- (a) Find the exact roots of the equation $|3x^2 + 8x - 3| = 3 - x$. [3]

- (b) On the same axes, sketch the curves with equations $y = |3x^2 + 8x - 3|$ and $y = 3 - x$.

Hence solve exactly the inequality $|3x^2 + 8x - 3| \geq 3 - x$. [4]

14. **MI Prelim 9758/2024/02/Q1**

Without the use of a calculator, solve the inequality $\frac{x+5}{(x-3)^2} \geq \frac{x-7}{x(x-3)}$. [4]

Hence solve $\frac{x-5}{(x+3)^2} \leq \frac{x+7}{x(x+3)}$. [2]

15. **EJC Prelim 9758/2024/02/Q1**

It is given that a is a real number such that $a < -1$.

- (a) Factorise $x^3 - ax^2 + a^2x - a^3$. [1]

- (b) Hence solve the inequality $\frac{x^3 - ax^2 + a^2x - a^3}{(x+a)(x+1)} \geq 0$. [4]

- (c) Using your answer to part (b), solve the inequality $\frac{|x|^3 + 2x^2 + 4|x| + 8}{(|x|-2)(|x|+1)} \geq 0$. [3]

16. **NYJC Prelim 9758/2025/P2/Q1**

- (a) Find the set of values of x for which $3|x-3| \leq |1-2x|$. [2]

- (b) Without using a calculator, solve $\frac{x+18}{x^2+5x-14} \geq -1$. [4]

17. **SAJC Prelim 9758/2025/P2/Q1**

- (i) Solve the inequality $3|x^2 - 4| \leq |x + 2|$. [4]

- (ii) Hence, solve $\frac{3}{x^2} |1 - 4x^2| \leq \left| \frac{1+2x}{x} \right|$. [2]

Answers

1. (a) $x < -1$ or $x > 3$; (b) $x < -1$ or $x > 1$
2. $-\sqrt{3} < w < \sqrt{3}$ or $w > \frac{5}{2}$; $-\sqrt{3} < y < \sqrt{3}$ or $y \geq \frac{5}{2}$ or $y \leq -\frac{5}{2}$
3. (i) $x < -2$ or $1 - \sqrt{5} \leq x \leq 1 + \sqrt{5}$ (ii) $-2 - \sqrt{5} \leq x \leq 2 + \sqrt{5}$
4. $2 < x < \frac{11}{2}$; $0 < x < \ln \frac{11}{4}$
5. $\frac{1 - \sqrt{1 + 8a}}{4} \leq x < 0$ or $x \geq \frac{1 + \sqrt{1 + 8a}}{4}$
6. $1 < x < \frac{1}{a}$ $x < \frac{1}{a}$ or $x > 1$.
7. $x < -a$ or $0 < x < b$ and $x \neq c$; $0 < x < e^{-a}$ or $1 < x < e^b$ and $x \neq e^c$
8. $-1 < x < a$
9. $x < 0$ or $x \geq 1$; $x \geq 0$
10. (ii) $x = a + \frac{1}{\sqrt{b}}$, (iii) $x < a$ or $a < x < a + \frac{1}{\sqrt{b}}$
11. Asymptotes: $x = -2, x = 2$ $-2.63 \leq x < -2$ or $2 < x \leq 2.63$
12. (i) $-1 < x < 1$ (ii) $x < -1$ or $x > 1$
13. (a) $\frac{-3 \pm \sqrt{17}}{2}$ or 0 or $-\frac{7}{3}$ (b) $x \leq \frac{-3 - \sqrt{17}}{2}$ or $-\frac{7}{3} \leq x \leq 0$ or $x \geq \frac{-3 + \sqrt{17}}{2}$
14. $x < 0$ or $x \geq \frac{7}{5}, x \neq 3$ Hence: $x > 0$ or $x \leq -\frac{7}{5}, x \neq -3$
15. (a) $(x^2 + a^2)(x - a)$ (b) $a \leq x < -1$ or $x > -a$ (c) $x > 2$ or $x < -2$
16. (a) $\{x \in \mathbb{R} : 2 \leq x \leq 8\}$. (b) $x < -7$ or $-3 - \sqrt{5} \leq x \leq -3 + \sqrt{5}$ or $x > 2$
17. (i) $\frac{5}{3} \leq x \leq \frac{7}{3}$ or $x = -2$ (ii) $\frac{3}{7} \leq x \leq \frac{3}{5}$ or $x = -\frac{1}{2}$